3 Find the fundamental solution to $x^2 - 73y^2 = 1$.

$x_0 = \sqrt{73}$ $\qquad a_0 = 8$

$x_1 = \dfrac{1}{\sqrt{73}-8} = \dfrac{\sqrt{73}+8}{9}$ $\qquad a_1 = 1$

$x_2 = \dfrac{1}{\frac{\sqrt{73}+8}{9}-1} = \dfrac{9}{\sqrt{73}-1} = \dfrac{9(\sqrt{73}+1)}{72} = \dfrac{\sqrt{73}+1}{8}$ $\qquad a_2 = 1$

$x_3 = \dfrac{1}{\frac{\sqrt{73}+1}{8}-1} = \dfrac{8}{\sqrt{73}-7} = \dfrac{8(\sqrt{73}+7)}{24} = \dfrac{\sqrt{73}+7}{3}$ $\qquad a_3 = 5$

$x_4 = \dfrac{1}{\frac{\sqrt{73}+7}{3}-5} = \dfrac{3}{\sqrt{73}-8} = \dfrac{3(\sqrt{73}+8)}{9} = \dfrac{\sqrt{73}+8}{3}$ $\qquad a_4 = 5$

$x_5 = \dfrac{1}{\frac{\sqrt{73}+8}{3}-5} = \dfrac{3}{\sqrt{73}-7} = \dfrac{3(\sqrt{73}+7)}{24} = \dfrac{\sqrt{73}+7}{8}$ $\qquad a_5 = 1$

$x_6 = \dfrac{1}{\frac{\sqrt{73}+7}{8}-1} = \dfrac{8}{\sqrt{73}-1} = \dfrac{8(\sqrt{73}+1)}{72} = \dfrac{\sqrt{73}+1}{9}$ $\qquad a_6 = 1$

$x_7 = \dfrac{1}{\frac{\sqrt{73}+1}{9}-1} = \dfrac{9}{\sqrt{73}-8} = \dfrac{9(\sqrt{73}+8)}{9} = \sqrt{73}+8$ $\qquad a_7 = 16$

So $\sqrt{73} = [8; \overline{1, 1, 5, 5, 1, 1, 16}]$

0	1		
1	0		-1
8	1	8	0
9	1	1	1
17	2	1	2
94	11	5	3
~~111~~	~~13~~	~~1~~	~~5~~
~~205~~	~~24~~	~~1~~	~~5~~
~~3397~~	~~397~~	~~16~~	

487	57	5	4
581	68	1	5
1068	125	1	6
17669	2068	16	7
18737	2193	1	8
36406	4261	1	9
200767	23498	5	10
1040241	121751	5	11
1241008	145249	1	12
2281249	267000	1	13
		16	

by Thm 4.2.

a fundamental solution for $x^2 - 73y^2 = 1$ is (p_{13}, q_{13});

where $(p_{13}, q_{13}) = (2281249, 267000)$