AJ

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Latest homework

1. Let $a_0, a_1, \dots a_n, x, y$ be strictly positive real numbers such that
$z = [a_0; a_1 \dots a_n, x] = [a_0, \dots a_n, y]$

Prove that $x = y$.

Well, by def $z = \frac{p_{n+1}}{q_{n+1}} = \frac{p_n x + p_{n-1}}{q_n x + q_{n-1}}$

$= \frac{p_n y + p_{n-1}}{q_n y + q_{n-1}}$

So $(p_n x + p_{n-1})(q_n y + q_{n-1}) = (p_n y + p_{n-1})(q_n x + q_{n-1})$

$\hookrightarrow$ $p_n q_n x y + p_{n-1} q_n y + q_{n-1} p_n x + p_{n-1} q_{n-1} =$
$p_n q_n y x + p_{n-1} q_n x + q_{n-1} p_n y + p_{n-1} q_{n-1}$

By cancelling out like terms we get that.

$p_{n-1} q_n y + q_{n-1} p_n x = p_{n-1} q_n x + q_{n-1} p_n y$

Thus, $p_{n-1} q_n (y - x) - q_{n-1} p_n (y - x) = 0$
$= (p_{n-1} q_n - q_{n-1} p_n)(y - x) = 0$
$= (-1)^{n-1} (x - y) = 0$.

So $x = y$. $\square$