The eigenoperator formalism

Saurav Dwivedi

15 March 2005

Abstract

The operation of classical operators to the well known ψ-wave provides us some similar results as those of eigenvalue formalism. We need to modify this aspect with an interpretation of *eigenoperator formalism*. It describes the classical and quantum mechanical values of operators with an interpretation of *eigenoperator equation* which leads us obtain the Quantum mechanical modification (quantization) of some classical theoretical aspects (equations). The similar study of eigenoperators instead of eigenvalues concerning the eigenvalue uncertainty principle leads us to formulate the eigenoperator uncertainty principle with the similar modification of our results of eigenoperator formalism for higher order eigenoperators.

Keyword(s): Gradient and Laplacian operators; first and second order exact and partial prime operators; d'Alembertian operator; Klein-Gordon and Dirac equation; eigenvalue and eigenoperator uncertainty principle.

When a classical operator (first order) is applied to the well known ψ-wave

$$\psi = \exp\left(\frac{i}{\hbar} S\right) \tag{1}$$

It leaves

$$\mathcal{A}\psi = \frac{i}{\hbar}[\mathcal{A}S]\psi \tag{2}$$

with $\mathcal{A} = (\nabla_\alpha, \partial_t, d_t, ...)$.
and considering the operator analysis, I get the Quantum mechanical $\mathcal{A}$ operator,

$$\hat{\mathcal{A}} \mapsto \frac{i}{\hbar}[\mathcal{A}S] \tag{3}$$

we need a hypothesis (eigenoperator equation) to describe the Quantum mechanical and classical values of $\mathcal{A}$ operator.

Hypothesis: *Let we provide a cap* (^) *to the classical operator $\mathcal{A}$ which may be called the Quantum mechanical $\mathcal{A}$ operator which despite the following eigenoperator equation to describe the phenomena*

$$\hat{\mathcal{A}}\psi = \mathcal{A}\psi \tag{4}$$

where $\mathcal{A}$ *is eigenoperator,* ψ *is eigenfunction and* $\hat{\mathcal{A}}$ *is the Quantum mechanical* $\mathcal{A}$ *operator.*

The gradient and Laplacian eigenoperator equation

Let we obtain the gradient of ψ

$$\nabla_\alpha \psi = \frac{i}{\hbar}[\nabla_\alpha S]\psi \tag{5}$$

with

$$\left.\begin{aligned} \hat{\nabla}_\alpha &= \frac{i}{\hbar}[\nabla_\alpha S] \\ \hat{\nabla}_\alpha \psi &= \nabla_\alpha \psi \end{aligned}\right\} \tag{6}$$

and getting the gradient of eq (5), I get

$$\nabla_\alpha(\nabla_\alpha \psi) = \nabla_\alpha^2 \psi = \frac{i}{\hbar}\left[[\nabla_\alpha^2 S] + \frac{i}{\hbar}[\nabla_\alpha S]^2\right]\psi \tag{7}$$

with

$$\left.\begin{aligned} \hat{\nabla}_\alpha^2 &= \frac{i}{\hbar}\left[[\nabla_\alpha^2 S] + \frac{i}{\hbar}[\nabla_\alpha S]^2\right] \\ \hat{\nabla}_\alpha^2 \psi &= \nabla_\alpha^2 \psi \end{aligned}\right\} \tag{8}$$

and considering the Laplacian eigenoperator equation (8) for the Quantum mechanical Laplacian operator, I get

$$\hbar^2 \nabla_\alpha^2 \psi = \left[i\hbar[\nabla_\alpha^2 S] - [\nabla_\alpha S]^2\right]\psi \tag{9}$$

and considering the momentum eigenvalue equation;

$$\left.\begin{aligned} p_\alpha \psi &= -i\hbar \nabla_\alpha \psi \\ p_\alpha^2 \psi &= -\hbar^2 \nabla_\alpha^2 \psi \end{aligned}\right\} \tag{10}$$

and in the consideration of eq (9) and eq (10), I get

$$[\nabla_\alpha S]^2 - i\hbar[\nabla_\alpha^2 S] - p_\alpha^2 = 0 \tag{11}$$

and as a limiting case $(\hbar \to 0)$ it reduces to classical one.

The first and second order partial prime eigenoperator equation

Let we obtain the partial prime of ψ

$$\partial_t \psi = \frac{i}{\hbar}[\partial_t S]\psi \tag{12}$$

with

$$\left.\begin{aligned} \hat{\partial}_t &= \frac{i}{\hbar}[\partial_t S] \\ \hat{\partial}_t \psi &= \partial_t \psi \end{aligned}\right\} \tag{13}$$

and getting the partial prime of eq (12), I get

$$\partial_t(\partial_t \psi) = \partial_t^2 \psi = \frac{i}{\hbar}\left[[\partial_t^2 S] + \frac{i}{\hbar}[\partial_t S]^2\right]\psi \tag{14}$$

with

$$\left.\begin{aligned} \hat{\partial}_t^2 &= \frac{i}{\hbar}\left[[\partial_t^2 S] + \frac{i}{\hbar}[\partial_t S]^2\right] \\ \hat{\partial}_t^2 \psi &= \partial_t^2 \psi \end{aligned}\right\} \tag{15}$$

and considering the second order partial prime eigenoperator equation (15) for the Quantum mechanical second order partial prime operator, I get

$$\hbar^2 \partial_t^2 \psi = \left[i\hbar[\partial_t^2 S] - [\partial_t S]^2\right]\psi \tag{16}$$

and considering the energy eigenvalue equation;

$$\left.\begin{aligned} H\psi &= i\hbar\partial_t \psi \\ H^2 \psi &= -\hbar^2 \partial_t^2 \psi \end{aligned}\right\} \tag{17}$$

and in the consideration of eq (16) and eq (17), I get

$$[\partial_t S]^2 - i\hbar[\partial_t^2 S] - H^2 = 0 \tag{18}$$

and as a limiting case $(\hbar \to 0)$ it reduces to classical one.

The first and second order exact prime eigenoperator equation

Let we obtain the exact prime of ψ

$$d_t\psi = \frac{i}{\hbar}[d_t S]\psi \tag{19}$$

with

$$\left.\begin{aligned} \hat{d}_t &= \frac{i}{\hbar}[d_t S] \\ \hat{d}_t\psi &= d_t\psi \end{aligned}\right\} \tag{20}$$

and getting the exact prime of eq (19), I get

$$d_t(d_t\psi) = d_t^2\psi = \frac{i}{\hbar}\left[[d_t^2 S] + \frac{i}{\hbar}[d_t S]^2\right]\psi \tag{21}$$

with

$$\left.\begin{aligned} \hat{d}_t^2 &= \frac{i}{\hbar}\left[[d_t^2 S] + \frac{i}{\hbar}[d_t S]^2\right]\psi \\ \hat{d}_t^2\psi &= d_t^2\psi \end{aligned}\right\} \tag{22}$$

and considering the second order exact prime eigenoperator equation (22) for the Quantum mechanical second order exact prime operator, I get

$$\hbar^2 d_t^2\psi = \left[i\hbar[d_t^2 S] - [d_t S]^2\right]\psi \tag{23}$$

and considering the Lagrangian eigenvalue equation;

$$\left.\begin{aligned} L\psi &= -i\hbar d_t\psi \\ L^2\psi &= -\hbar^2 d_t^2\psi \end{aligned}\right\} \tag{24}$$

and in the consideration of eq (23) and eq (24), I get

$$[d_t S]^2 - i\hbar[d_t^2 S] - L^2 = 0 \tag{25}$$

and as a limiting case $(\hbar \to 0)$ it reduces to classical one.

The d'Alembertian eigenoperator equation

Let we consider generalized d'Alembertian operator

$$\Box_{\alpha} = c^{-2}\partial_t^2 - \nabla_{\alpha}^2 \tag{26}$$

and considering the eigenoperator equation (4) and the linear eigenoperator analysis (appendix), I get the Quantum mechanical d'Alembertian operator

$$\hat{\Box}_{\alpha} = c^{-2}\hat{\partial}_t^2 - \hat{\nabla}_{\alpha}^2 \tag{27}$$

and considering the Quantum mechanical Laplacian operator (8) and Quantum mechanical second order partial prime operator (15) for eq (27) in the consideration of linear eigenoperator analysis (appendix), I get

$$\hat{\Box}_{\alpha} = \frac{i}{\hbar}\left[[\Box_{\alpha} S] + \frac{i}{\hbar}\left(c^{-2}[\partial_t S]^2 - [\nabla_{\alpha} S]^2 \right) \right] \tag{28}$$

with the d'Alembertian eigenoperator equation

$$\hat{\Box}_{\alpha}\psi = \Box_{\alpha}\psi \tag{29}$$

and applying the Quantum mechanical d'Alembertian (28) to ψ in the consideration of d'Alembertian eigenoperator equation (29), I get

$$\hbar^2 \Box_{\alpha}\psi = \left[i\hbar[\Box_{\alpha} S] - c^{-2}[\partial_t S]^2 + [\nabla_{\alpha} S]^2 \right]\psi \tag{30}$$

and considering the Klein-Gordon equation (Dirac equation);

$$m^2 c^2 \psi = -\hbar^2 \Box_{\alpha}\psi \tag{31}$$

and in the consideration of eq (30) and eq (31), I get

$$c^{-2}[\partial_t S]^2 - [\nabla_{\alpha} S]^2 - i\hbar[\Box_{\alpha} S] - m^2 c^2 = 0 \tag{32}$$

which is the Quantum mechanical and Relativistic modification (quantization and relativization) of Einstein's mass-energy relation.

The eigenvalue uncertainty principle

The physical quantities commutes while their quantum mechanical values (operator values) do not commute with representation of commutator algebra;

$$[\hat{\mathcal{A}}, \hat{\mathcal{B}}]_{-} \neq 0 \tag{33}$$

with $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$ being operator values of α and β eigenvalues defined with the eigenvalue equation:

$$\left.\begin{aligned}\hat{\mathcal{A}}\psi &= \alpha\psi \\ \hat{\mathcal{B}}\psi &= \beta\psi\end{aligned}\right\} \tag{34}$$

such a transition is referred to the uncertainty principle which creates in its counterpart an essential modification to measure the smallest physical quantities which are immeasurable at an instant due to such an uncertainty modification . The following uncertainty principle presents the modification of classical concepts with an interpretation of quantum concepts of quantities which may be called the *eigenvalue uncertainty principle*

The eigenoperator uncertainty principle

With the similar study of classical operators instead of classical quantities I have obtained the *uncertainty principle of eigenoperators*. Physicists are encouraged to formulate the further consequences of this phenomena. In present study it is obtained that the operation of quantum mechanical operators to ψ-wave with the description of eigenoperator formalism creates an uncertainty in the measurements with higher order eigenoperators, that is, two or more corresponding eigenoperators creates an uncertainty in the measurement with eigenoperator formalism. Such an uncertainty principle may be called the *eigenoperator uncertainty principle* and may be represented as:

$$[\widehat{\mathcal{AB}}] = [\hat{\mathcal{A}}][\hat{\mathcal{B}}] + \text{uncertainty} \tag{35}$$

or

$$[\widehat{\mathcal{AB}}] - [\hat{\mathcal{A}}][\hat{\mathcal{B}}] \neq 0 \qquad ; \qquad [\hat{\mathcal{A}}\hat{\mathcal{B}}] - [\hat{\mathcal{A}}][\hat{\mathcal{B}}] \neq 0 \tag{36}$$

with the classical description (certainty),

$$[\mathcal{AB}] = [\mathcal{A}][\mathcal{B}] \tag{37}$$

with $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$ being the quantum mechanical values of eigenoperators $\mathcal{A}$ and $\mathcal{B}$ defined with eigenoperator equation:

$$\left.\begin{aligned}\hat{\mathcal{A}}\psi &= \mathcal{A}\psi \\ \hat{\mathcal{B}}\psi &= \mathcal{B}\psi\end{aligned}\right\} \tag{38}$$

The eigenoperator uncertainty can be determined with the consideration of eigenoperator formalism and eigenoperator uncertainty principle (35). The eigenoperator uncertainty for the Laplacian may be determined with the comparison of eq (6) and eq (8) in the consideration of eq (35), I get

$$[\hat{\nabla}^2_\alpha]_{\text{uncertainty}} \mapsto \frac{i}{\hbar}[\nabla^2_\alpha S] \tag{39}$$

with the classical certainty

$$[\nabla^2_\alpha] = [\nabla_\alpha][\nabla_\alpha] \tag{40}$$

and considering eq (13) and eq (15) in the consideration of eq (35), I get the partial prime eigenoperator uncertainty:

$$[\hat{\partial}^2_t]_{\text{unceratainty}} \mapsto \frac{i}{\hbar}[\partial^2_t S] \tag{41}$$

with the classical certainty

$$[\partial^2_t] = [\partial_t][\partial_t] \tag{42}$$

and considering eq (20) and eq (22) in the consideration of eq (35), I get the exact prime eigenoperator uncertainty:

$$[\hat{d}^2_t]_{\text{unceratainty}} \mapsto \frac{i}{\hbar}[d^2_t S] \tag{43}$$

with the classical certainty

$$[d^2_t] = [d_t][d_t] \tag{44}$$

and considering eq (26) and eq (28) in the consideration of eq (35), I get the d'Alembertian eigenoperator uncertainty:

$$[\hat{\Box}_\alpha]_{\text{uncertainty}} \mapsto \frac{i}{\hbar}[\Box_\alpha S] \tag{45}$$

with the classical certainty

$$[\Box_\alpha] = [\partial^\mu][\partial_\mu] \tag{46}$$

the eigenoperator uncertainty may be obtained for all second and higher order eigenoperators.

Appendix

The linear eigenoperator analysis

For two corresponding eigenoperators $\mathcal{A}$ and $\mathcal{B}$ with their Quantum mechanical operator values $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$,

$$\hat{\mathcal{C}}\psi = (\hat{\mathcal{A}} + \hat{\mathcal{B}})\psi = (\mathcal{A} + \mathcal{B})\psi = \mathcal{C}\psi \tag{47}$$

with

$$\left.\begin{aligned} \hat{\mathcal{C}} &= \hat{\mathcal{A}} + \hat{\mathcal{B}} \\ \mathcal{C} &= \mathcal{A} + \mathcal{B} \end{aligned}\right\} \tag{48}$$

holds.

And for the two corresponding eigenoperators $\mathcal{A}$ and $\mathcal{B}$ with their Quantum mechanical values $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$, there exists

$$[\hat{\mathcal{A}}\hat{\mathcal{B}}]\psi \neq \hat{\mathcal{A}}[\hat{\mathcal{B}}\psi] \tag{49}$$

with

$$[\mathcal{A}\mathcal{B}]\psi = \mathcal{A}[\mathcal{B}\psi] \tag{50}$$

this holds due to the *Uncertainty principle of eigenoperator* the work which have to be completed.