The author's comments to the referee's answer. My comments are given in windows in appropriate places of the referee's opinions.

Answer to the comments of the author to my report on the paper

GENERAL GEOMETRY AND GEOMETRY OF ELECTROMAGNETISM, by Shervgi S. Shahverdiyev.

In what follows, I will transcript some of the author s comments, in italic, and immediately after each one of these comments, I will give my own opinion.

The author claims that he has found a way of unifying the derivation of such equations like the one governing the motion of a relativistic particle subject to gravity, and that of a charged particle moving in an electromagnetic field.

In the paper there is no such a claim. Every main result is stated in the abstract in separate sentences and there is nothing relevant to any kind of unification.

My own opinion 1*: It might be true that there is no such a claim, as the author says, made in a very explicit way. But such a claim is implicit everywhere in the paper.*

There are sentences and formulas in the paper where the motion of charged particles is described as being a geodesic in General Geometry. For instance, in the introduction, it says ...we show that equations for geodesics in geometry underlying electromagnetism coincides with the equation of motion for a charged particle.... This is done in Section 4, where the most simplest case of General Geometry, is developed.

No comments.

On the other hand, the author describes in Section 2 that a free particle in relativity, is given essentially by a geodesic. This is also stated at the beginning of the Introduction.

This is done for stressing that the same thing holds to be true for a particle interacting with electromagnetic field provided that we consider it in the framework of a new geometry different from Riemannian geometry. I do not consider this as a unification.

For all this, I thought that part of the justification for introducing a new geometry called General Geometry, would be to unify the motion of particles under different types of forces.

Unfortunately, this is not the case. "General Geometry is introduced first of all as a new mathematical object(sec.3), second as a source for geometrization of known physical theories (sec.4). In the paper this is realized for electromagnetism only.

For instance, at the end of Section 5, for a combined electromagnetic and gravitational force, a unified treatment for the action of a free particle, I suppose in terms of General Geometry, is proposed, although not explained in detail.

Yes, it is not explained. It is done with the aim to inform a reader that a unified model of electromagnetism and gravitation is being developed by using "general geometry" in a different paper.

About the notion of unification, mentioned at the end of the author s previous comment, I think that it appears all over the paper, I suppose, in order to justify the *introduction of a new geometry, called General Geometry.*

In my opinion there must not be any attempt to justify introduction of a new geometry. We may introduce them because they are new geometries. In our case it is also proven that this new geometry has its physical application as including geometries underlying physical theories (in our case electromagnetism and gravitation). As far as I know most mathematical objects have been introduced in the literature, although they have not found any physical applications yet.

For instance, it is said in Section 5, that In this way we will be able to find electroweak model and unify all interactions. Here, the author does not specifies what does it means all interactions.

Yes it is true. I thought it might be obvious from the context. I will change "all interactions" to "all known interactions"

It is not explained in the paper what is the precise meaning of phrases, also related obviously to the notion of unification, like, It is worth expecting that we can construct geometries of weak and strong interactions using correspondence between Physics and Mathematics, in Section 5, which sounds very important, provided that it is really developed.

The meaning of that sentence is the following. As it is demonstrated in sec. 2 and sec. 5. electromagnetism can not be geometrized in the framework of Riemannian geometry, because of the properties of electromagnetic interaction mentioned in sec. 5. Realization of this, led us to look for a different type of geometries with appropriate properties for geometrization of electromagnetism. Because of this, we can expect that we may be able to construct geometries of weak and strong interactions using correspondence between physical properties of these interactions and mathematical properties of some unknown at this moment geometries in the future along the same lines as it is done for electromagnetism.
I will add this to the paper.

Kaluza-Klein theory is also mentioned at the end of section 5, but its exact connection with General Geometry, and specially with Geometry of Electromagnetism, is not explained.

It is well known that Kaluza-Klein theory is based on 5-dimensional Riemannian geometry. In the paper it is shown that Riemannian geometry is a special case of General Geometry. It is also shown that Riemannian geometry is completely different from Geometry of Electromagnetism. Therefore, I thought that connection with the Kaluza-Klein theory was obvious.

I must say that I do not have any objection, in principle, to the introduction of such entities like $g_{\mu\nu}(x, x_u)$ or $\Gamma^{\sigma}_{\lambda}(x, x_u)$, and then try to generalize Riemannian Geometry by writing equations of geodesics like equation (7) or equation (10).

No comments.

But, I must say that, as far as I can see, the author does not explicitly says what is the advantage of, for instance, interpreting the well known equation for the motion of a charged particle considered in Section 4, as part of his formalism of General Geometry. I would say something similar about the equation of a geodesic in Riemannian

Geometry.

From the point of view of obtaining some new predictions there is no advantage. But, from the point of view, that behind of all known physical interactions there might be their own geometries like gravitation, it is the advantage that the well known equation of motion for a particle interacting with electromagnetic field coincides exactly with the equation for geodesics of Geometry of Electromagnetism. This is one of the requirements for geometrization. As it is also well known that Kaluza-Klein theory failed to satisfy this because of its charge/mass problem. In our theory there is no such a problem. In my opinion this must be considered as one of the advantages of our theory. I will add this to the paper.

How important general equations like (7)

The important role of (7) follows from (8), where it is seen that although ${}^{A}g_{\mu\nu}$ is different from $\eta_{\mu\nu}$ the role of a tensor for lowering indices plays $\eta_{\mu\nu}$.

and (10) could be, is not proven in the paper, in my opinion.

The important roles of (10) are

1. It allows us to formulate more general geometry than Riemannian geometry,

2.By representing $\Gamma^{\sigma}_{\lambda}(x, x_u)$ as

$$\Gamma^{\sigma}_{\lambda}(x, x_u) = F^{\sigma}_{\lambda}(x) + \Gamma^{\sigma}_{\lambda\nu}(x)x^{\nu}_{u} + \Gamma^{\sigma}_{\lambda\nu\mu}(x)x^{\nu}_{u}x^{\mu}_{u} + ...$$

and considering each order in x_u or their combinations separately we realize that there are many more geometries different from Riemannian geometry, which can be underlying geometries for physical theories.

This unification, seems to consists in the assumption that one has a connection-like object $\Gamma^{\sigma}_{\lambda}(x, x_u)$. Then the curvature of this connection-like object gives a force. An appropriate choice of $\Gamma^{\sigma}_{\lambda}(x, x_u)$, namely, the usual connection associated to the standard pseudometric $g_{\mu\nu}$,. in relativity, reproduces the gravitational field.

Actually, it is demonstrated that a new geometry called General Geometry includes Riemannian geometry as a special case in sec 3. There is no discussion of gravitational field, and force in Sec 3 and in the rest of the paper.

My own opinion 2*: Gravitational field is mentioned immediately before equation (1) and electromagnetic field is mentioned immediately before equation (4).*

No comments.

The electromagnetic field is obtained by choosing, somehow, the connection-like structure using the electromagnetic tensor F.

In Sec. 4, Whole electromagnetism is obtained by choosing $\Gamma(x, x_u)$ as being $F(x)$. Functions $\Gamma(x, x_u)$ are general functions of coordinates.Therefore it is possible to choose them as $F(x)$.

My own opinion 3*: No comment.*

I must say that, in my opinion, the interpretation of curvatures as being fields seems to be well known.

Interpretation of equations of motion for gravitational field in terms of curvature of Riemannian geometry is well known for gravitation only. We show that equations of motion and Lagrangian for electromagnetic field can be derived from curvature *characteristic of a new geometry called Geometry of Electromagnetism, which is completely different from Riemannian geometry and is the most simplest special case of General geometry.*

My own opinion 4: *The idea, essentially Kaluza-Klein theory, that electromagnetic forces are curvatures of connections on principal bundles with group S1, is well known.*

> In the paper we do not consider any kind of bundles.

The tensor F there is the curvature of A.

> We do not use this kind of notion for curvatures.

The author uses this tensor to write the equation of motion of a charged particle, which is also well known, in Section 4.

> Of course, equation of motion for a charged particle, is well known, and we do not claim that this a new result.

I do not see anything new added to this well known results in the paper. Of course, reinterpretations, even trivial ones, could be very important. But, at the moment, I do not see any real advantage in reinterpreting some well known equations of motion as being part of Geometry of Electromagnetism.

> From the point of view, that behind of all known physical interactions there might be their own geometries like gravitation, it is the advantage that the well known equation of motion for a particle interacting with electromagnetic field coincides exactly with the equation for geodesics of Geometry of Electromagnetism. In my opinion this must be considered one of the advantages of this reinterpretation. Therefore at this point one of the requirements of geometrization (see sec. 1) of a theory is satisfied. As it is fairly noted by Kaluza in his original paper that his theory fails to satisfy this requirement even for electrons because of charge/mass problem. Therefore, at this point of our paper one of the main problems of Kaluza-Klein theory is cured in our theory, although we do not use the same ideas as Kalusa-Klein.

Now, about the Lagrangian for electromagnetism in terms of curvatures in the sense of Geometry of Electromagnetism. First of all, although this might be only a technical point, it is not clearly explained in Section 3 what is the meaning of such symbols like $\Gamma^{\sigma}_{\lambda}(x, x_u)$

> The meaning of the $\Gamma^{\sigma}_{\lambda}(x, x_u)$ is the same as the meaning $\Gamma'^{\sigma}_{\lambda\nu}(x)x^{\nu}_{u}$ in Riemannian geometry (see(9)). More precisely, it shows how coordinates of a vector changes alog a curve.
> It is up to you to require to add this to the paper. As far as I know its meaning is well known.

and how the curvature R^{σ}_{λ} *is derived.*

> I decided not to give simply, but long calculations of R^{σ}_{λ} in the paper. It is derived by using definition of derivative for a function of two variables. All the long calculations are available and can be added to the paper or sent you upon request.

This makes it difficult to check the correctness of the formulas in the paper.

> No comments.

Assuming that all the formulas are correct, then the formula $R^{\sigma}_{\mu\lambda} = \partial_{\mu} F^{\sigma}_{\lambda}$ *in Section*

4 would be correct, in the context of Geometry of Electromagnetism. Then, Maxwell equations, it is said in the paper, are obtained as $R_\lambda = 0$. I could believe that, this way, at least part of Maxwell equations would be obtained, and may be this is interesting.

No comments.

However, regarding the Lagrangian for electromagnetism, which is given by R, and which differs from the standard Lagrangian by a total derivative, according to the author, the meaning of it is not clearly stated.

Its meaning is the following: As we can construct Lagrangian of gravitational field as scalar curvature in Riemannian geometry by contracting Ricci tensor with $g_{\mu\nu}$ (which originates from the length of a curve (metric)), we can construct Lagrangian of electromagnetic field as scalar curvature of Geometry of Electromagnetism by contracting curvature vector R_λ (analog of Ricci tensor) and vector A_λ, (analog of $g_{\mu\nu}$, because it also originates from the length of a curve(metric) as $g_{\mu\nu}$ in Riemannian geometry).
I will add this information to sec 4.

This R does not looks like a curvature, on the contrary, it is the contraction of R_λ with A_λ.

R is an analog of curvature of Riemannian geometry, because it is obtained by contracting R_λ and vector A_λ. (see also previous comment)

Then one gets the impression that it has been constructed with the elements at hand provided by the formalism of Geometry of Electromagnetism, but not by a kind of universal procedure, valid in a diversity of cases, like the ones mentioned in Section 5: It is worth expecting that we can construct geometries of weak and strong interactions using correspondence between Physics and Mathematics.

In addition to the last two comments, I would like to add that because A_λ appears in our theory as $g_{\mu\nu}$ appears in Riemannian geometry (both comes from the lengh of a curve(metric)), all the constructions of our theory are done along the same lines as in Riemannian geometry. Accordinly, it is a universal procedure.

And what is done in the paper is essentially well known too.

To the best of my knowledge,...,etc.

My own opinion 5*: See My own opinion 4.*

The author does not describe any example, other than gravity and electromagnetism, in which his structure could be usefull.

Description of such structures are under investigation. The main result of the paper is to show that electromagnetism has its own underlying geometry and it turns out that this geometry is different from Riemannian geometry. And is a special case of General Geometry.

My own opinion 6*: Here, it is clearly stated what is the main result of the paper. Then it is not General Geometry. I must confess that it was difficult for me to realize that.*

As a physicist, I consider geometrization of electromagnetism as the most important main result of the paper. Of course, General geometry is also one of the main results. A mathematicisian, who is not interested in physics can consider General Geometry as the most important main result of the paper. In my opinion it is up to a reader to choose which main result of the paper is more important.

Again, I am sorry, but I do not see any reason to change my previous conclusion. Let me add my personal opinion, that introducing a new geometry, called General Geometry, as it is proposed in the Introduction, requires more foundation.

I do not know what do you mean by "more foundation"?
If you mean that introduction of a new geometry, requires many reasons, then I can say that General Geometry is introduced for informing readers that
1) there exist geometries including Riemannian geometry as a special case and many more geometries different from Riemannian geometry.
2) the most simplest case of General Geometry is geometry underlying Electromagnetism
In my opinion this two properties of General Geometry are more than enough for bringing it to the attention of readers.

If you mean more mathematical foundation then, I can say that General Geometry is introduced along the same lines as Riemannian geometry (see for example W. Pauli, Theory of Relativity, Pergamon press 1958.), except that we do not require existence of a parallel transport or equivalently we do not restrict ourselves to the function in the right hand side of (9) for the right hand side of (10). This allows us to discover many interesting geometries as it is demonstrated in the paper.

Resuming our correspondence, I conclude that only derivation of R^{σ}_{λ} remains not proven by me. I can add to the paper its derivation or send you all detailed calculation of R^{σ}_{λ} upon request. All other points of your answer are overcomed by adding more information to the paper.
A revised vesion of the paper is sent to Professor Roger. G. Newton.