

Answer to the comments of the author to my report on the paper

GENERAL GEOMETRY AND GEOMETRY OF ELECTROMAGNETISM,
by Shervgi S. Shahverdiyev.

In what follows, I will transcript some of the author's comments, in italic, and immediately after each one of these comments, I will give my own opinion.

The author claims that he has found a way of unifying the derivation of such equations like the one governing the motion of a relativistic particle subject to gravity, and that of a charged particle moving in an electromagnetic field.

In the papaer there is no such a claim.

Every main result is stated in the abstract in separate sentences and there is nothing relevant to any kind of unification.

My own opinion 1: *It might be true that "there is no such a claim", as the author says, made in a very explicit way. But "such a claim" is implicit everywhere in the paper. There are sentences and formulas in the paper where the motion of charged particles is described as being a geodesic in "General Geometry". For instance, in the introduction, it says "...we show that equations for geodesics in geometry underlying electromagnetism coincides with the equation of motion for a charged particle...". This is done in Section 4, where the "most simplest case of General Geometry", is developed. On the other hand, the author describes in Section 2 that a free particle in relativity, is given essentially by a geodesic. This is also stated at the beginning of the Introduction. For all this, I thought that part of the justification for introducing a "new geometry" called "General Geometry", would be to unify the motion of particles under different types of forces. For instance, at the end of Section 5, for a combined electromagnetic and gravitational force, a unified treatment for the action—the action of a free particle, I suppose—in terms of "General Geometry", is proposed, although not explained in detail.*

About the notion of "unification", mentioned at the end of the author's previous comment, I think that it appears all over the paper, I suppose, in order to justify the introduction of a "new geometry", called "General Geometry". For instance, it is said in Section 5, that "In this way we will be able to find electroweak model and unify all interactions". Here, the author does not specifies what does it means "all interactions". It is not explained in the paper what is the precise meaning of phrases, also related obviously to the notion of unification, like, "It is worth expecting that we can construct geometries of weak and strong interactions using correspondence between Physics and Mathematics", in Section 5, which sounds very important, provided that it is really developed. Kaluza-Klein theory is also mentioned at the end of section 5, but its exact connection with "General Geometry", and specially with "Geometry of Electromagnettism", is not explained.

I must say that I do not have any objection, in principle, to the introduction of such entities like $g_{\mu\nu}(x, x_u)$ or $\Gamma_{\lambda}^{\sigma}(x, x_u)$, and then try to generalize Riemannian Geometry by writing equations of geodesics like equation (7) or equation (10). But, I must say that, as far as I can see, the author does not explicitly says what is the advantage of, for instance, interpreting the

well known equation for the motion of a charged particle considered in Section 4, as part of his formalism of "General Geometry". I would say something similar about the equation of a geodesic in Riemannian Geometry. How important general equations like (7) and (10) could be, is not proven in the paper, in my opinion.

This unification, seems to consists in the assumption that one has a connection-like object $\Gamma_{\sigma}^{\nu}(x, x^{\mu})$. Then the curvature of this connection-like object gives a force. An appropriate choice of $\Gamma_{\mu}^{\nu}(x, x^{\mu})$, namely, the usual connection associated to the standard pseudometric $g_{\mu, \nu}$ in relativity, reproduces the gravitational field.

Actually, it is demonstrated that a new geometry called "General Geometry" includes Riemannian geometry as a special case in sec 3. There is no discussion of gravitational field, and force in Sec 3 and in the rest of the paper.

My own opinion 2: *Gravitational field is mentioned immediately before equation (1) and electromagnetic field is mentioned immediately before equation (4).*

The electromagnetic field is obtained by choosing, somehow, the connection-like structure using the electromagnetic tensor F .

In Sec. 4, Whole electromagnetism is obtained by choosing $\Gamma(x, x_u)$ as being $F(x)$. Functions $\Gamma(x, x_u)$ are general functions of coordinates. Therefore it is possible to choose them as $F(x)$.

My own opinion 3: *No comment.*

I must say that, in my opinion, the interpretation of curvatures as being fields seems to be well known.

Interpretation of equations of motion for gravitational field in terms of curvature of Riemannian geometry is well known for gravitation only. We show that equations of motion and Lagrangian for electromagnetic field can be derived from curvature characteristic of a new geometry called Geometry of Electromagnetism, which is completely different from Riemannian geometry and is the most simplest special case of General geometry.

My own opinion 4: *The idea, essentially Kaluza-Klein theory, that electromagnetic forces are curvatures of connections on principal bundles with group S^1 , is well known. The tensor F there is the curvature of A . The author uses this tensor to write the equation of motion of a charged particle, which is also well known, in Section 4. I do not see anything new added to this well known results in the paper. Of course, reinterpretations, even trivial ones, could be very important. But, at the moment, I do not see any real advantage in reinterpreting some well known equations of motion as being part of "Geometry of Electromagnetism".*

Now, about the Lagrangian for electromagnetism in terms of curvatures in the sense of "Geometry of Electromagnetism". First of all—although this might be only a technical point—it is not clearly explained in Section 3 what is the meaning of such symbols like $\Gamma_{\lambda}^{\sigma}(x, x_u, x_v)$, and how the curvature R_{λ}^{σ} is derived. This makes it difficult to check the correctness of the formulas in the paper. Assuming that all the formulas are correct, then the formula $R_{\mu\lambda}^{\sigma} = \partial_{\mu}F_{\lambda}^{\sigma}$ in Section 4 would be correct, in the context of "Geometry of Electromagnetism". Then, Maxwell

equations, it is said in the paper, are obtained as $R_\lambda = 0$. I could believe that, this way, at least part of Maxwell equations would be obtained, and may be this is interesting. However, regarding the Lagrangian for electromagnetism, which is given by R , and which differs from the standard Lagrangian by a total derivative, according to the author, the meaning of it is not clearly stated. This R does not look like a curvature, on the contrary, it is the contraction of R_λ with A^λ . Then one gets the impression that it has been constructed with the elements at hand provided by the formalism of "Geometry of Electromagnetism", but not by a kind of universal procedure, valid in a diversity of cases, like the ones mentioned in Section 5: "It is worth expecting that we can construct geometries of weak and strong interactions using correspondence between Physics and Mathematics".

And what is done in the paper is essentially well known too.

To the best of my knowledge,...,etc.

My own opinion 5: See **My own opinion 4**.

The author does not describe any example, other than gravity and electromagnetism, in which his structure could be useful.

Description of such structures are under investigation.

The main result of the paper is to show that electromagnetism has its own underlying geometry and it turns out that this geometry is different from Riemannian geometry. And is a special case of General Geometry.

My own opinion 6: Here, it is clearly stated what is the main result of the paper. Then it is not "General Geometry". I must confess that it was difficult for me to realize that.

In any case, the "General Geometry", that is, the "geometry" associated to that structure, does not look, at first sight, really "general", it seems to leave out too many branches of geometry. The author does not explain the motivation for this name.

There are almost no references in the paper.

In conclusion, unfortunately, I cannot recommend the paper for publication in JMP.

Again, I am sorry, but I do not see any reason to change my previous conclusion. Let me add my personal opinion, that introducing a "new geometry", called "General Geometry", as it is proposed in the Introduction, requires more foundation.