

## Boundary Value Problem

- A thin rod made of uniform material
- Surrounded by a blanket of insulation so that temperature changes along the length of the rod are a result of
  - Heat transfer at the ends of the rod and
  - Heat conduction along the length of the rod
- The ends of the rod are in contact with ice water (0 degrees C).
- The initial temperature at distance  $x$  from the end of the rod is  $y(100\sin(\pi x))$
- Over time the rod cools

- A partial difference equation (PDE) can be used to model the temperature at any point on the rod as a function of time.
  - We use finite difference equations to solve a partial difference equation on a computer.
  - Gives an approximate solution to the PDEs

## Partitioning

- There is one data item per grid point
- So, one primitive task with each grid point.
- 2D domain decomposition.

## Communication

- Communication pattern between the tasks
- If task A needs a value from task B, draw a channel from B to A.
- $u(i,j+1)$  requires values from  $u(i-1,j)$ ,  $u(i,j)$  and  $u(i+1,j)$ 
  - For position  $i$  and time  $j$ , requires values from  $i-1,i$  and  $i+1$ .

## Agglomeration

- Even if enough processors were available it is impossible to compute each task concurrently because
- The tasks computing temperatures later in time depend on the results produced by tasks computing rod temperatures earlier in time.
  - Look at the vertical lines from bottom task to top task.
  - Sequential computation
- So, agglomerate the tasks in the same column.

## Agglomeration



## Mapping

- We will still have more tasks than we need.
- Group the tasks and map to a processor.
  - Balance workload
  - Minimize communication
- A task is now responsible for computing, over all time steps, the temperatures for a contiguous group of rod locations.