

DISCRETE OPTIMIZATION OF TRUSSES USING ANT COLONY METAPHOR

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Synopsis: The ant colony optimization [ACO] is a new heuristic, which uses the foraging behavior of ants for finding optimal solutions in the search space. Sizing optimization of trusses using ACO is presented. In structural design, cross sections must be chosen from a limited set of sections commercially available. The objective of the present work is to develop a technique for optimal design of trusses allowing discrete variables alone, making use of the ant colony metaphor. Structural Analysis of trusses is done using FEM. A novel representation scheme for mapping the truss design to the minimum path length formulation is suggested. Stress deflection considerations are handled using constraints in the form of penalty functions. A number of examples are considered whereby the usefulness of the approach is demonstrated. The best design is obtained through a series of iterations. The results obtained through the present study in terms of the optimum volume of material are found to be intuitively optimal suggesting that the proposed method could be used for getting a good truss designs

1.Introduction:

Optimal design of truss-structures has always been an active area of research in the field of search and optimization. Various techniques based on classical optimization methods have been developed to find optimal truss-structures (Deb[1995]). In the sizing optimization of trusses, cross-sectional areas of members are considered as design variables and the coordinates of the nodes and connectivity among various members are considered to be fixed. The resulting optimization problem is a nonlinear programming (NLP) problem. In this work only the sizing optimization of truss structures is considered (Deb[2001]). The use of Genetic algorithms has been reported in literature for optimization of structures(Rajeev et al.[1992],Deb[2001]) using discrete variables. The Ant Colony Optimization [ACO] is a new metaheuristic to solve combinatorial optimization problems by using principles of communicative behaviour found in real ant colonies (for an introduction and overview see Dorigo[1999]).It has been applied successfully to a number of combinatorial optimization problems(Dorigo et al.[1996],Bottee[1998],Stultze[1999]).In this work we study the application of ACO to structural optimization. The paper is organized as follows. In Section 2 the truss sizing optimization problem is formulated. Section 3 provides an introduction to ant colony optimization. Section 4 discusses the methodology for solving the truss optimization problem using ACO. Section 5 reports on experiments and examples. Section 6 gives the scope for further work and conclusions.

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2 Truss optimization problem formulation.

The truss design optimization can be regarded as following problem:

Minimize $f(x)$

Subject to $g_i(x) \geq 0 \quad i=1,2,3,\dots,l$, Where l is the number of constraints.

Since the objective function is defined as selecting the vector of the cross section areas that minimizes the total weight of the structure, the objective function is written as

$$f(x) = \sum_{j=1}^m \rho A_j l_j \dots\dots\dots(1)$$

where A_j is the cross sectional area of the j th member; l_j is the length of the j th member and ρ is the weight density of the material. The constraint functions are expressed in normalised form as follows:

$$\frac{s_a}{s_j} - 1 \geq 0 \quad \text{for } j=1,2,3,\dots,m \dots\dots\dots(2).$$

$$\frac{u_a}{u_k} - 1 \geq 0 \quad \text{for } k=1,2,3,\dots,n \dots\dots\dots(3)$$

where s_j is the stress in the member j and s_a is the allowable stress , u_k is the displacement of joint k and u_a is the allowable displacement. The above mentioned constrained problem is transformed into a unconstrained problem. A penalty based transformation method proposed by Rajiv and Krishnamoorthy[1992] is used in this study. The penalty based modified objective function is written incorporating the constraint violation as

$$P(x) = f(x)[1 + KC] \dots\dots\dots(4)$$

where $f(x)$ is the total weight of the structure , K is the penalty factor(K has to be selected judiciously depending on the required influence of the violated design in the next cycle), and C is the violation coefficient that represents the total violated level of constraints including stress and displacement constraints. The violation coefficient is computed in the following manner.

If $g_i(x) \leq 0$, $c_i = g_i(x)$.
 otherwise $c_i = 0$

and $C = \sum_{i=1}^l c_i \dots\dots\dots(5)$

3.The Ant Colony Optimization metaphor

Standard ACO: ACO is an iterative probabilistic optimization heuristic inspired by the way real ants find short paths between their nest and a food source. The fundamental principle used by ants for communication is stigmergy, i.e. ants use pheromones to mark their trails. Higher pheromone intensity suggests a better path and consequently inclines more ants to take a similar path. Transferring these ideas to the artificial scenario of a TSP with n cities, an ACO approach works as follows: In every iteration, a number of m (artificial) ants construct one solution each through all the given n cities. Starting at a random city, an ant iteratively selects the next city based on heuristic information as well as pheromone information. The heuristic information, denoted by η_{ij} , represents a priori heuristic knowledge w.r.t. how good it is to go from city i to city j . For TSPs, η_{ij}

= $1/d_{ij}$ where d_{ij} is the distance between city i and j . The pheromone values, denoted by τ_{ij} , are dynamically changed by the ACO algorithm and serve as a kind of memory, indicating which choices were good in the past. When having inserted city i in the previous step, the next city j is chosen probabilistically according to the following probabilities:

$$p_{ij}(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l=1}^m [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta} \dots\dots\dots(6)$$

where l, \dots, m is the set of cities that have not been visited yet, and α and β are constants that determine the relative influence of the heuristic and the pheromone values on the ant's decision. After each of the m ants have constructed a solution, the pheromone information is updated. First, some of the old pheromone is evaporated on all edges. Afterwards, a fixed amount $\Delta\tau_{ij}$ of additional pheromone is 'deposited' along all tour edges of the best ant (or all ants) in the iteration. Often, the elitist ant (representing the best solution found so far) is also allowed to deposit pheromone along its path. Each of these positive updates has the form

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + \sum_{k=1}^m \Delta\tau_{ij}^k \text{ where } \Delta\tau_{ij}^k = \frac{Q}{L_k} \dots\dots\dots(7)$$

where cities i and j connected by an edge of the respective tour of ant k with length L_k and Q is a constant. Initially $\tau_{ij} = \tau_0$ for each edge e_{ij} . Here parameter $\rho \in (0, 1)$ specifies the evaporation rate.

4. Ant colony approach to truss optimization

In order to use ACO for truss optimization it is necessary to formulate the truss design problem as a minimum path length problem. Consider a number of ants, ANTS walking along the members of the truss. They have an option to choose any cross section at a member. The number of options is equal to the number of cross sections permitted. However the probability of choosing the j th cross section at member i is calculated according to following equation(8).

$$p_{ij}(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l=1}^m [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta} \dots\dots\dots(8)$$

The number of ants passing through a cross section j at member i in the next state is given by

$$nextstate_{ij}(t) = p_{ij}(t) \cdot ANTS \dots\dots\dots(9)$$

where ANTS is the total number of ants used. However which ant passes through which cross section is decided randomly at each member. This way there is a high probability that the different designs are obtained. After every ant has traversed all the members we get a complete set of cross sectional areas with their lengths fixed a priori. These coded designs are sent to FEM routine for decoding as well as structural analysis. The constraint violation if any is calculated and the modified objective function is evaluated as described in section 2. There after the pheromone trails are updated according to the following rule:.

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + \sum_{k=1}^m \Delta \tau_{ij}^k \dots\dots\dots(10)$$

$$\text{where } \Delta \tau_{ij}^k = \frac{Q}{W_k} \text{ if tour of ant } k \text{ constitutes cross section } j \text{ at member } i.$$

$$= 0 \text{ otherwise}$$

where $W_k = P(x)$ is the objective function for ant k as explained previously.

The modified values of pheromone create bias in the next iteration for the number of ants passing through a particular cross-section at a member. The cross section that corresponded to the best design of previous generation has a greater probability of getting selected. This way after a number of iterations the ants find out good solutions. In fig.1 algorithm is presented in a pseudo code.

```

Procedure_ACO_for_truss_optimization()
Start
Input parameters;
Initialize design variables;
do
    cycle=1;
    initialize trail;
    find number of ants in nextstate(i,j);
    randomly allot cross sections to ants;
    structural analysis of designs();
    compute penalty and evaluate objective function;
    store the best design;
    update trail;
    cycle =cycle +1;
while(termination criteria not satisfied)
print best design;
end

```

Fig.1: The ant colony approach to truss optimization

5 Examples And Discussion Of Results

Example 5.1 41 Bar Truss With 18 Nodes

Fig.2 shows the 41 bar truss and the loading conditions. The assumed data are $E=2100000 \text{ kg/cm}^2$. The objective is to minimize the volume of the structure. The allowable displacement is limited to 8mm and stresses 1250 kg/cm^2 . The member section areas are allowed to take values between 2 and 64 cm^2 in step of 2 cm^2 . The control parameters are $\alpha=1$, $\beta=0.0$, $\rho =0.5$. The number of ants were set as 1000 and the number of cycles were set to 1250. The minimum volume found was 90977.21 cm^3 . Fig.3 summarizes the results for this analysis.

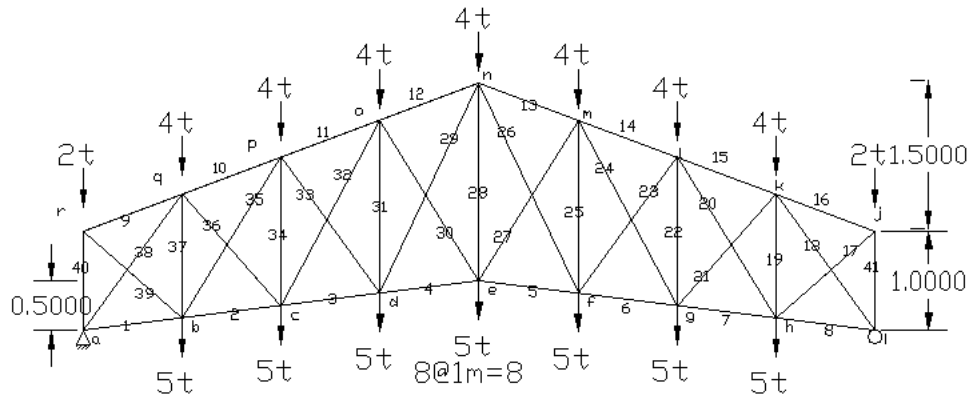


Figure 2. 41 Bar Truss

Fig 3 Stresses and areas for the 41 bar truss

#	A	σ	#	A	σ	#	A	σ
1	28	606.57	15	30	-1187	29	4	1111.04
2	34	818.91	16	6	-956.03	30	2	-647.86
3	44	868.02	17	8	892.10	31	2	102.97
4	60	616.07	18	40	-842.82	32	4	-1008.5
5	36	1024.95	19	4	903.66	33	2	572.05
6	46	829.73	20	6	-844.97	34	2	-1071.1
7	42	668.42	21	12	1021.3	35	4	-1158.2
8	44	454.16	22	2	-950.11	36	14	900.08
9	10	-891.44	23	2	593.4	37	2	147.75
10	32	-1120.4	24	4	-995.2	38	28	-1023.3
11	36	-1087.9	25	2	18.69	39	12	924.2
12	36	-1124.1	26	4	1149.47	40	14	-888.11
13	48	-844.55	27	2	-585.45	41	8	-1089.2
14	38	-1031.4	28	14	1161.04			

Displacements cm

Node	X	Y	Node	X	Y	Node	X	Y
a	00	00	g	0.3516	-0.524	m	0.3341	-0.7550
b	0.0694	-0.3206	h	0.4070	-0.3401	n	0.2287	-0.6368
c	0.1356	-0.5334	i	0.4715	0	o	0.1397	-0.7518
d	0.2063	-0.7636	j	0.2352	-0.0422	p	0.1434	-0.5925
e	0.2341	-0.7474	k	0.2878	-0.3118	q	0.1938	-0.2863
f	0.2829	-0.7533	l	0.3388	-0.6100	r	0.2298	-0.0518

Where, A= area in cm^2 , σ = stress in kg/cm^2 . We can see from the fig.3 for displacements that the displacements for nodes o,d,e,f is very close to the permissible value of 0.8 cm. Hence we can argue that the design obtained is either optimal or near optimal.

6.Conclusions and Scope for Future Work

In this paper we applied the Ant Colony Optimization metaheuristic to the truss sizing optimization problem. As in most real life cases the cross section areas chosen are discrete. A novel

representation scheme is suggested in which a hypothetical ant is imagined to be traveling along the members of the truss. The objective function is defined as the weight of the cross sections which the ant passed at the respective members. Constraints are handled using penalty function approach. The analysis of designs has been done using FEM. The usefulness of the above approach has been proved with two examples in which the ACO finds optimal or near optimal solutions. Future research will try to apply the approach to shape and topology optimization of trusses. Approach suggested by Deb and Gulati[2001] can be considered for shape and topology optimization using ACO. The effect of the evaporation($1-\rho$), α and β values will be studied in greater detail for stagnation and quick convergence.

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