Numerical Analysis of Type I Settling

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Abstract: Sedimentation and clarification are used interchangeably for potable water; both refer to the separating of solid material from water. Various types of sedimentation exist, based on characteristics of particles. In type 1 settling, particles whose size, shape, and specific gravity do not change over time are suspended in dilute solution In dilute solutions number of particles is insufficient to cause displacement of water (most potable water sources). Terminal settling velocity can be found out theoretically from the equilibrium of a particle settling through a column and used to calculate the removal efficiency. However many parameters are not easy to measure and therefore empirical analysis of type I sedimentation is carried out to determine the efficiency of removal for a particular loading rate. A mathematical model is formulated and a numerical solution technique have been used for calculating removal efficiency from the time and concentration data of settling analysis.

Keywords: Numerical analysis, Sedimentation, Type I Settling

INTRODUCTION

Sedimentation and clarification are used interchangeably for potable water; and both refer to the separating of solid material from water (Peavy et al. 1985). Since most solids have a specific gravity greater than 1, gravity settling is used remove suspended particles. When specific gravity is less than 1, floatation is normally used. Various types of sedimentation exist, based on characteristics of particles

Discrete or type 1 settling particles are those particles whose size, shape, and specific gravity do not change over time.

Flocculating particles or type 2 settling particles are those particles that change size, shape and perhaps specific gravity over time.

Above types have both dilute and concentrated suspensions

Dilute: In dilute suspension, number of particles is insufficient to cause displacement of water (most potable water sources).

Concentrated: In concentrated suspension, number of particles is such that water is displaced (most waste waters)

Sedimentation has many applications in preparation of potable water as it can remove suspended solids and dissolved solids that are precipitated. Examples:

- Plain settling of surface water prior to treatment by rapid sand filtration (type 1)
- Settling of coagulated and flocculated waters (type 2)
- Settling of coagulated and flocculated waters in lime-soda softening (type 2)
- Settling of waters treated for iron and manganese content (type 1)

PARTICLE SETTLING

Particle settling, or sedimentation, may be described for a singular particle by the Newton's equation for terminal settling velocity of a spherical particle. A knowledge of this velocity is basic in the design and performance of a sedimentation basin. The rate at which discrete particles will settle in a fluid of constant temperature is given by the Stokes equation for laminar flow:

$$v_t = \frac{g(\rho_p - \rho_w)d^2}{18\mu} \quad (1)$$

where

g = accelaration due to gravity
ρ_p = density of the particle
ρ_w = density of the water
d = diameter of particle
μ = coefficient of viscosity

However direct application of above equation is not seldom possible in water treatment because the size of particles d mus t be known and the correction factor ϕ to account for departure from sphercity has to be determined. An indirect method of measuring velocities of discrete particles in dilute susupensions and of determining setling characteristics of a suspension, was devised by T.R.Camp 1943(cited

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from (Peavy et al. 1985)). Settling column is constructed as shown in Fig.1 Suspension is placed in column and completely mixed and then allowed to settle quiescently.

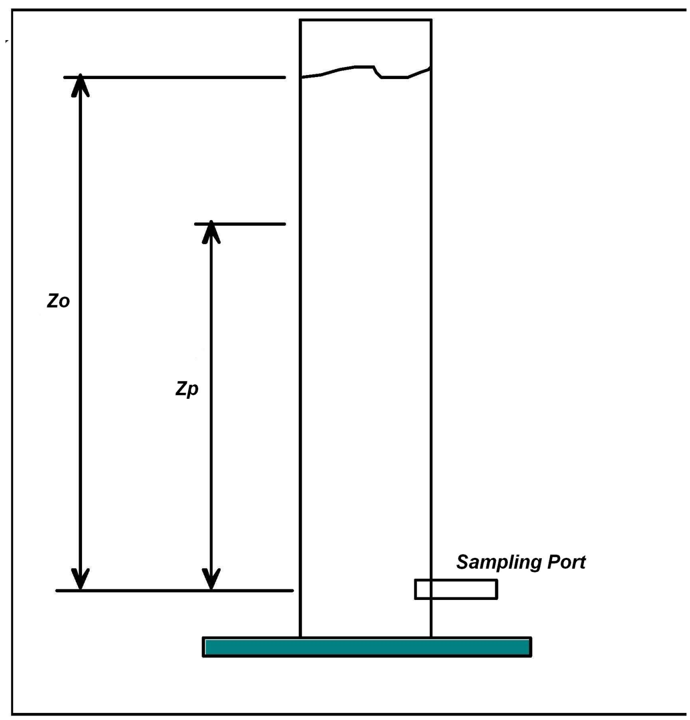

FIG. 1. Settling column for analyzing type 1 suspension

Particle placed at the surface has an average settling velocity of:

$$v_0 = \frac{\text{distance travelled}}{\text{time of travel}} = \frac{Z_0}{t_0} \tag{2}$$

Another particle placed at distance Z_p, terminal velocity less than the surface particle, but arrives at the same time, has a settling velocity of:

$$v_p = \frac{Z_p}{t_0} \tag{3}$$

which is less than v_0; but arrives at sampling port a same time. Thus, the travel time for both particles is equal, where $t_0 = Z_0/v_0 = Z_p/v_p$ and $v_p/v_0 = Z_p/Z_0 = h/H$ Thus some generalized statements can be made concerning the above relationships.

1. All particles having a diameter equal to or greater than d_0, i.e. have settling velocity greater than v_0, will arrive at or pass the sampling port in time t_0.
2. article with diameter $d_p < d_0$ will have a settling $v_p < v_0$, will arrive at or pass the sampling port in time t_0, provided its position is below Z_p.
3. If the suspension is uniformly mixed, i.e. particles are randomly distributed,. then the fraction of particles with size d_p having settling velocity v_p which will arrive at or pass the sampling port in time t_0 will be $Z_p/Z_0 = v_p/v_0$. Thus the removal efficiency of any size particle from suspension is the ratio of the settling velocity of that particle to the settling velocity v_0 defined as Z_0/t_0. These principles can be used to determine the settleability of any given suspension, using shown apparatus. Theoretically depth is not a factor but for practical reasons, 2 m is usually chosen.

PROCEDURE

1. Determine C_0 of completely mixed suspension at time zero.
2. Measure C_1 at time t_1. All particles comprising C_1have a settling velocity less than Z_0/t_1, where $v_1 = Z_o/t_1$. Thus, the mass fraction of particles removed with $v_1 < Z_o/t_1$ is given by $x_1 = C_1/C_0$.
3. Repeat process with several times t_i, with the mass fraction of particles being $v_i < Z_0/t_i$
4. Values are then plotted on a graph to obtain Figure 2, where the fraction of particles remaining for any settling velocity can be determined
5. For any detention time t_0, an overall percent removal (X) can be obtained. That is, all particles having a settling velocity greater that $v_0 = Z_0/t_0$, will be removed 100% (un-hatched area in Figure 2). Thus $1 - x_0$ particles are completely removed in time t_0 The remaining particles have a $v_i < v_0$ (hatched area in Figure 2), and will be removed according to ratio v_i/v_0.
6. If the equation relating v and x are known, then the area can be found through integration using Eq. 4

$$X = 1 - x_0 + \int_0^{x_0} \frac{v_i}{v_0} dx \tag{4}$$

where X is the total mass fration removed by sedimentation.

NUMERICAL SOLUTION

Eq. 4 is usually solved graphically by plotting x_i vs v_i and then finding the area under the curve till limits of integration. The method is tedious and prone to errors. The area under the curve can be found out numerically. However since the point v_0 upto which area is to be determined is generally not an element of the set of available (calculated) points, the curve has to be interpolated smoothly upto v_0 to find the value of x_0. Interpolation formulas (Scarborough 1966) are available for

- Start of the interval – Newtons's forward difference interpolation formula.
- End of the interval – Newtons's backward difference interpolation formula.
- Middle of the interval – Gauss's central difference interpolation formula, Stirling's formula, Bessel's formula.

However above formulas are valid only for equal intervals. In the problem at hand the intervals of x_i and v_i are not uniform, so none of the above formulas could be used. Among the interpolation formulas for unequal intervals, Lagrange's and Newtons' general interpolation formulas are different forms of the same equation with lagrange's formula best suited for computer implementation.

In Lagrange's formula, values of $f(x)$ at $x_0, x_1, x_2, \ldots, x_n$ points are known and denoted as $y_0, y_1, y_2, \ldots, y_n$. The aim is to find a value of y for a particular value of x. Lagrange's formula for y is given by

$$y = x_0 \frac{(y-y_1)(y-y_2)(y-y_3)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)\ldots(y_0-y_n)} \tag{5}$$

$$+x_1 \frac{(y-y_0)(y-y_2)(y-y_3)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\ldots(y_1-y_n)}$$

$$+x_2 \frac{(y-y_0)(y-y_1)(y-y_3)\ldots(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\ldots(y_2-y_n)}$$

$$+\ldots$$

$$+x_n \frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_2)\ldots(y_n-y_{n-1})}$$

An implementation of the lagrange's formula was tried but the interpolated points are undulating as shown in Fig.3. Lagrange's formula works well for the upper portion where the original points and the interpolated points overlap, but in the lower portion, the graph is an undulating one with some of the values even negative which is not physically possible.

In the numerical interpolation of this point (x_0, v_0), there are many hurdles and many standard methods cannot be used because:

1. Intervals are unequal for v_i or x_i so solution cannot be obtained by usual integration techniques like simpson's rule, weddle's rule (Scarborough 1966).
2. Lagrange interpolation is undulating.
3. To use splines, first and second derivatives at the start and end of the interval is required. Hence in this case splines also could not be used.

The only option available was to use linear interpolation between two data points. Correspondingly integration is performed using Trapezoidal rule(Scarborough 1966) which is applicable to any number of subintervals, whether even or odd and for all interval sizes. Using the trapezoidal rule

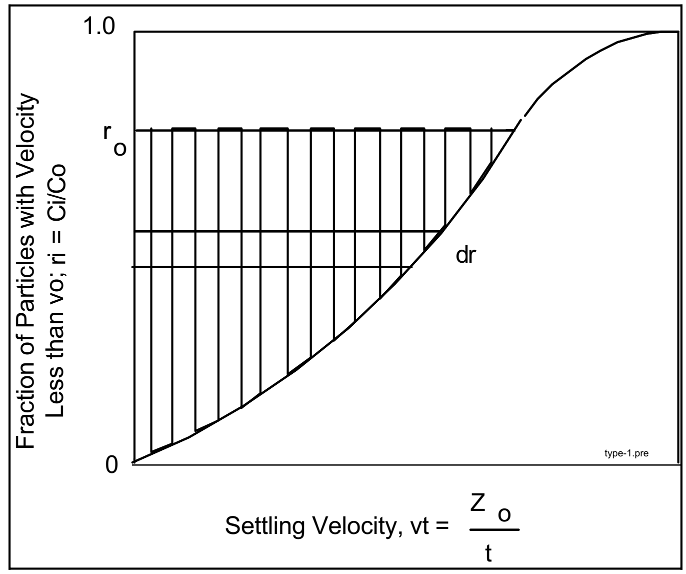

FIG. 2. Removal efficiency as a function of settling time

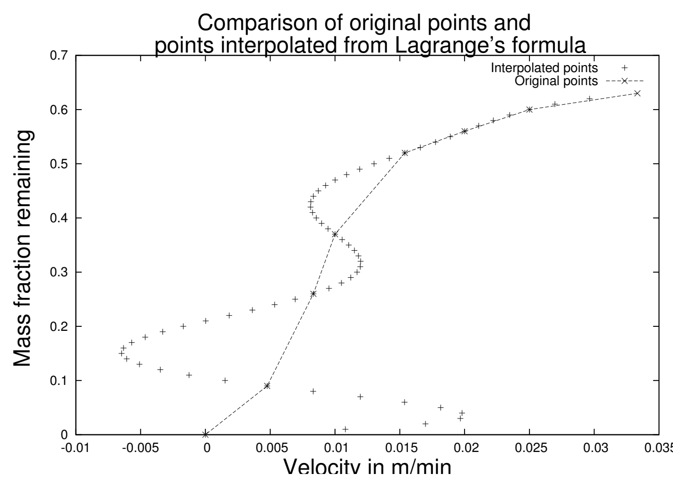

FIG. 3. Interpolated points from Lagrange's formula

removal efficiency X is calculated for all the available values of v_i X_i denotes the removal efficiency for $v_0 = v_i$.

$$X = 1 - x_0 + \frac{\sum_{x_i=0}^{x_0} v_i \cdot \Delta x}{v_0} \tag{6}$$

Further calculations are done on the basis of the problem type.

Type a: v_0 is given

In this case value of X is required to be found out for given v_0. From the stored values X_i, value of X is interpolated as

$$X = X_i + (v_0 - v_i)\frac{X_{i+1} - X_i}{v_{i+1} - v_i} \tag{7}$$

such that

$$v_{i+1} \geq v_0 \geq v_i \tag{8}$$

Type b: X is given

In this case value of v_0 is required to be found out for given X. From the stored values v_i, value of v_0 is interpolated as

$$v_0 = v_i + (X - X_i)\frac{v_{i+1} - v_i}{X_{i+1} - X_i} \tag{9}$$

such that

$$X_i \geq X \geq X_{i+1} \tag{10}$$

ALGORITHM AND FLOWCHART

The flowchart for the algorithm is shown in Fig.7 Program recieves the input in form of a input file from which all the required information is extracted. The next step involve the calculation of velocities and the mass fraction. Once the velocities and mass fractions at various instants are known removal efficiencies at all these instants are calculated using equation 4. After deciding the problem type, case specific interpolation is done to get the desired result in any of the two cases.

IMPLEMENTATION

The algorithm was coded in the C programming language on a Linux based platform. For graph plotting Gnuplot software has been used and the output of the program is a pdf file processed by LaTeX. To run the program, an input file is required. In settling analysis two kinds of problems can be defined

1. When the loading rate (v_0) is known and removal efficiency is required to be found out. In this case option `X` is used in the input file.
2. When the removal efficiency (X) is known and corresponding loading rate (v_0)is required to be found out. In this case option `v` is used in the input file.

A sample input file to be used with the program is shown. All numerical entries are typed after the colon. Input Units:
Loading rate v_0 in m / min
Removal Efficiency X in decimals (not %)
Concentration, C_i in mg/ L
Time, t_i in min
Height of column,H in m

```
Type your entries after the colon(ie :) :
If you want to calculate   X
with v given take option X :
If you want to calculate   v0
 with X given take option v:
Your option  Problem type ( X or v): X
Value of v if X  or X if v:0.0174
Height of the column: 2
No of data points:8
Time concentration:
0  300
60  189
80  180
100  168
130  156
200  111
240  78
420  27
```

TABLE 1. Calculations for `ex1.inp`.

Time (min)	C_i (mg/L)	x_i	Velocity v_i
0	650	1.00	inf
58	560	0.86	0.034483
77	415	0.64	0.025974
91	325	0.50	0.021978
114	215	0.33	0.017544
154	130	0.20	0.012987
250	52	0.08	0.008000

For analysing the data from this input file, following command is used at the prompt ($)
`$ settlingtypeI exmpl`
In this case exmpl is the name of the input file. The output of the program is found in the file `exmpl.inp.report.pdf`.

CASE STUDIES

Example 1

A Settling column analysis is run on a Type-1 suspension. The settling column is 2 m tall. The initial concentration of well mixed sample is 650 mg/l. Result of the analysis are shown below

Time,	min	58	77	91	114	154
Conc.,	mg/L	560	415	325	215	130

What is the theoretical efficiency in a settling basin with a loading rate of 2.4 x $10^{-2} m^3/m^2 d (m/d)$? The input file for this problem is ex1.inp

```
Problem type ( X or v):X
Value of X if v  or v0 if X:0.024
Height of the column:2
No of data points:7
0 650
58 560
77 415
91 325
114 215
154 130
250 52
```

After analysing this data with `settlinganalysisI` program, the efficiency X is found to be 78.49 % for given loading rate $v_0 = 0.024000$ m/ min

Calculations are shown in Table 1 and the graph of v_i vs. x_i is shown in Fig. 4. In Fig. 4, it can be seen that the vertical line of v_0 cuts the curve at an intermediate point. The area of the curve is found out only upto this point. Please note that in the table the entries are given in increasing order of time, while calculations are done in increasing order of v_i, with the assumption that at $t = \infty$, $v_i = 0$ and $x_i = 0$.

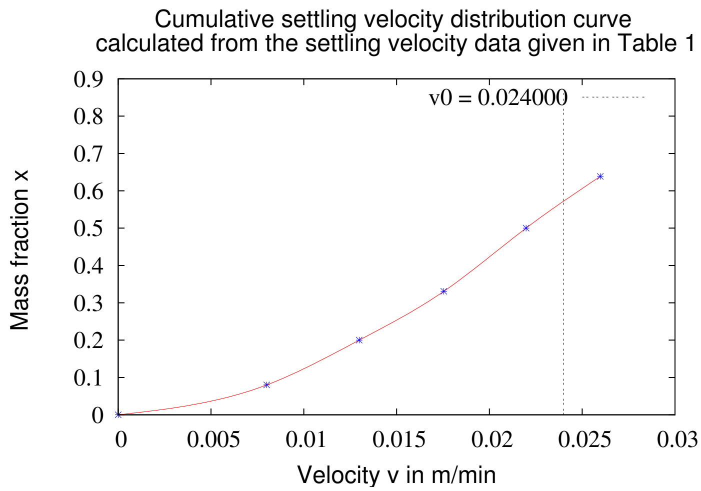

FIG. 4. Graph for `ex1.inp`

Another possibility is with a given value of X the loading rate is required to be determined. For the same data if one wishes to find the value of loading rate for efficiency 80 %, the input file will be

```
Problem type ( X or v):v
Value of X if v  or v0 if X:0.80
Height of the column:2
No of data points:7
0 650
58 560
77 415
91 325
114 215
154 130
250 52
```

After analysing this data, for the given efficiency $X = 80(\%)$ the loading rate v_0 is found to be 0.022982 m/ min. The plot for this input file is shown in Fig.5. Note that the value of v_0 is different than in the previous case.

Example 2

Settling analysis is run on a Type-1 suspension in a typical 2 m column. The initial concentration is 300 mg/l. Data as follows.

Time, min	60	80	100	130	200	240	420
Conc, mg/L	189	180	168	156	111	78	27

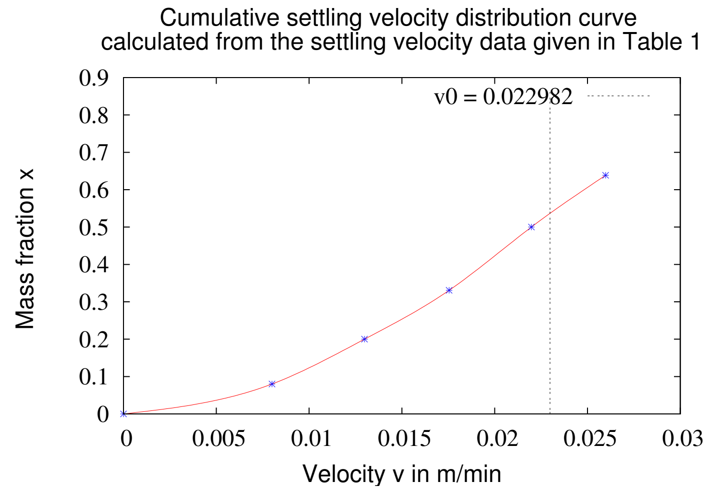

FIG. 5. Graph for `inv.inp`

What is the removal efficiency in a settling basin with a loading rate of 25 m3/m2*d (m/d)? The input file for this problem is ex2.inp

```
Problem type ( X or v):x
Value of X if X  or v0 if v:0.0174
Height of the column: 2
No of data points:8
Time concentration:
0   300
60   189
80   180
100   168
130   156
200   111
240   78
420   27
```

An analysis was run using this input file and the the efficiency X is 72.58 % for velocity $v_0 = 0.017400$ m/ min The In the inverse problem X is assumed to be 65 % and the input file is prepared as

```
Problem type ( X or v ):v
Value of X if X  or v0 if v:0.65
Height of the column: 2
No of data points:8
Time concentration:
0   300
60   189
80   180
100   168
130   156
200   111
240   78
420   27
```

TABLE 2. Calculations for `ex2.inp`.

Time (min)	C_i (mg/L)	x_i	Velocity v_i
0	300	1.00	inf
60	189	0.63	0.033333
80	180	0.60	0.025000
100	168	0.56	0.020000
130	156	0.52	0.015385
200	111	0.37	0.010000
240	78	0.26	0.008333
420	27	0.09	0.004762

Again, analysis was run using this input file and the velocity v_0 is obtained as 0.023494 m/ min for given efficiency X of 65.00 (%).

Calculations are shown in Table 2 and the graph of v_i vs. x_i is shown in Fig. 6.

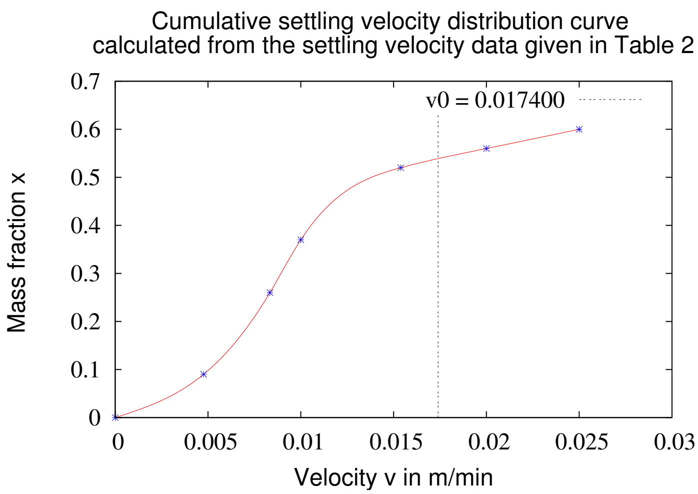

FIG. 6. Graph for `ex2.inp`

CONCLUSIONS

Removal efficiency is a critical parameter in the design of sedimentation tank. However calculating removal efficieny manually is a tedious job that is best left to the computer. The data obtained from settling analysis experiment is the concentration of solids in various samples taken at different times. Even though the samples may be taken at equal intervals, the main variables in finding the removal efficiency are velocity and mass fraction left. Since the interval in velocity and mass fraction are unequal, many popular numerical interpolation and integration methods are not applicable and the simple method of trapezoidal rule has to be used. The algorithm is implemented on a Linux based machine and the output file contains the plot of mass fraction vs. velocity. This software can be used study the variation of loading rate on the efficiency and the designing of a sedimentation tank for a particular removal efficiency.

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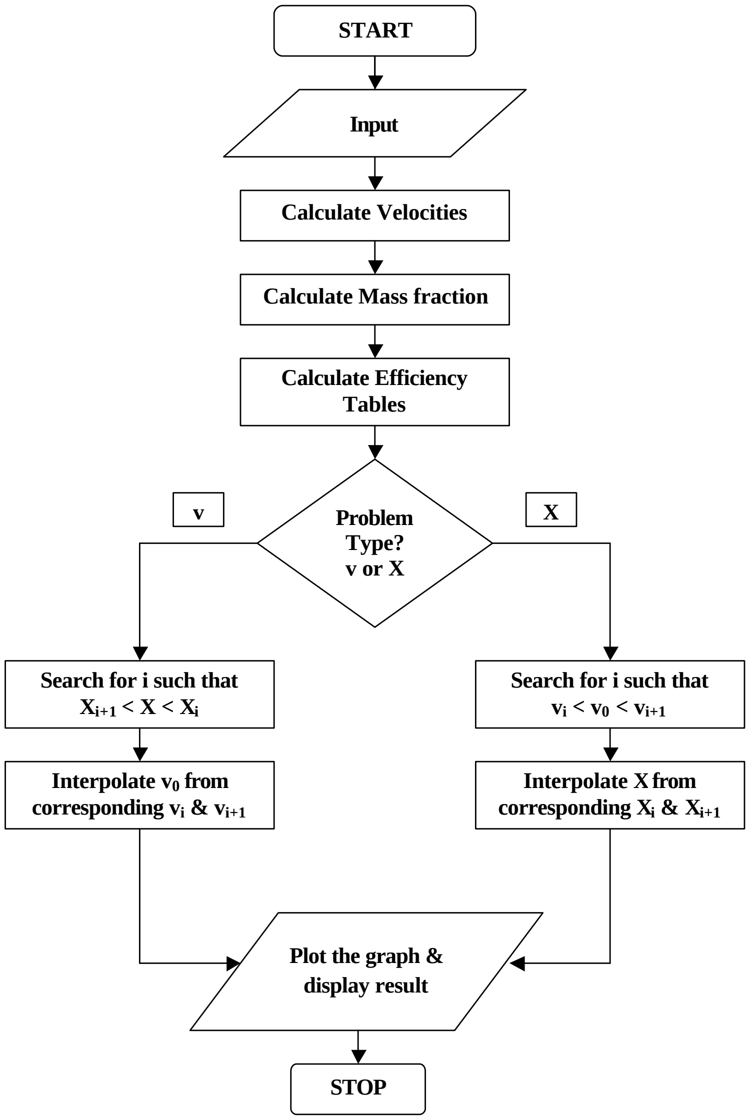

FIG. 7. Flowchart for Type 1 settling analysis

Output file of Settling analsis of type–1 sedimentation for input file : `inv.inp`

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The Efficiency X is 80.00 (%) for velocity $v_0 = 0.022982$ m/ min

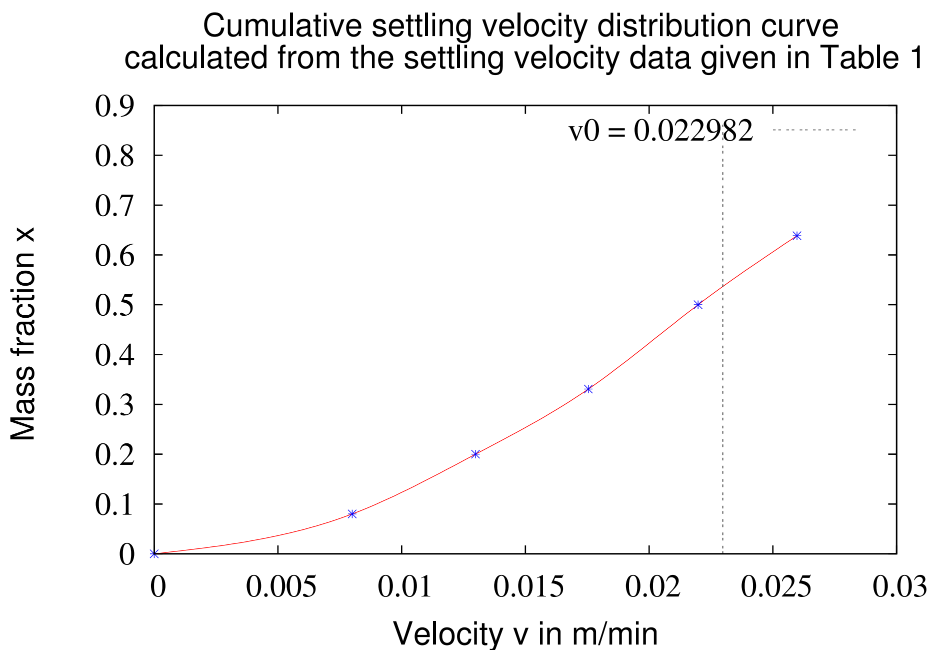

Figure 1: Cumulative settling velocity distribution curve calculated from the settling velocity data given in Table 1

Table 1: Data from a Type I sedimentation analysis as per `inv.inp`.

Time (min)	C_i (mg/L)	x_i	Velocity v_i
0	650	1.00	inf
58	560	0.86	0.034483
77	415	0.64	0.025974
91	325	0.50	0.021978
114	215	0.33	0.017544
154	130	0.20	0.012987
250	52	0.08	0.008000