Ant Colony Optimization in Earthwork Allocation

Suresh Kataria [1] , Saurabh Arun Samdani [2] and Arvind Kumar Singh [3]

[1,2] Senior Undergraduate,
[3] Asst. Prof.,
Department of Civil Engineering ,
Indian Institute of Technology, Guwahati, India

ABSTRACT

In highway projects, economic considerations are as important as other design controls and elements of design. As a result, the highway designer should consider cut and fill balance along with minimizing earthwork which may significantly decrease construction costs. There is almost an infinite number of feasible grades available to the designer to choose from, all of which satisfy the geometric specification of the road. The roadway grade selection is usually considered as a stage and the earthwork (transportation) is another stage. These two stages have always been treated separately in the literature. The model presented here combines the roadway grade selection stage and the earthwork transportation stage in a single problem. Previous models of earthwork allocation have restricted the excavation cost function to quadratic or stepwise linear function. However, due to varied site conditions, the actual excavation cost function may be nonlinear. To accomodate, such types of cost functions, a robust Ant Colony Optimization(ACO) algorithm has been presented. ACO is based on the indirect communication of a colony of simple agents, called (artificial) ants, mediated by (artificial) pheromone trails. The pheromone trails in ACO serve as a distributed, numerical information which the ants use to probabilistically construct solutions to the problem being solved and which the ants adapt during the algorithm's execution to reflect their search experience.

Keywords: Earthwork allocation; Roadway grades; Ant Colony Optimization

INTRODUCTION

One of the main activities in highway construction is earthwork allocations. The work involved in roadway design is usually handled in two stages. The first stage is related to the selection of roadway grades, while the second stage is that of minimizing the earthwork involved after stage one is completed. These two stages are usually completed separately from each other. A attempt to integrate them into one problem was done by Easa (7). His approach was based on finding the complete enumeration of all technically feasible grades, calculating the cut and fill for each grade and then using linear programming to optimize the earthwork involved. Another work has been done to accomodate nonconstant(three-stepwise) unit cost functions of purchase and excavation for borrow pits using a mixed integer linear programming (MILP) model (6).

Among several methods developed for earthwork allocations, the mass diagram is notable. It is a graphical tool for determining the minimum amount of haul and economic distribution of cut and fill. However, the mass diagram has some shortfalls:

- It is not particularly applicable when hauling cost is not linearly proportional to the hauling distance,
- It does not incorporate all relevant effects of soil characteristics such as the effect of swell (or shrinkage), and
- When additional quantities of materials may be borrowed from borrow pits or wasted at landfills (disposal sites), the capacities and costs associated with the elements cannot be considered simultaneously in the mass diagram.

To overcome the previous limitations of mass diagram, several methods employing linear programming (LP)have been developed by Stark and Nicholas (13), and Mayer and Stark (12). These models generally compute the earthwork allocations that minimize cost and they assume that roadway grades are fixed. However, the Transport and Road Research Laboratory in the United Kingdom established that the cost of earthwork involved in road construction can vary widely based on the roadway grades chosen by the designer. This has started the studies to integrate the preliminary and final stages in order to reach a solution that will find both optimal grades and earthwork allocations.

One of the studies to integrate two stages is by Easa (7) who tried to find the complete enumeration of all technically feasible grades, calculating the cut and fill for each grade and then using linear programming to optimize the earthwork involved. However, his approach cannot be considered an integration of two design stages in a true sense. It is actually a trial and error type of approach.

[1]email:skataria@iitg.ernet.in
[2]email:samdani@iitg.ernet.in
[3]email:arvind@iitg.ernet.in

Because there are too many feasible grades, global optimality is guaranteed only through an exhaustive consideration of all possible grades.

The work by Moreb (11), which is actually based on the model developed by Easa, combines the roadway grade selection stage with earthwork allocation stage in a single linear programming problem, thus guaranteeing global optimality.

Based on a detailed investigation made in (11), it was found that the unit cost of purchase and excavation of borrow pits was the only unit cost that depended on the unknown quantities of earthwork allocation. Therefore, these unit costs required special treatment, while the unit costs of other earthwork activities were used as constants. The unit costs of purchase and excavation typically represents $30 - 40$ % of the total unit cost of earthwork from a borrow pit, and therefore precise representation of the unit cost is warranted. Due to different soil conditions, the cost of excavation varies with depth for different sites. The LP algorithm presented by Easa(8) can handle nonconstant three stepwise unit cost function, however this may not be adequate for real sites. Therefore there is a need to develop an algorithm which can handle nonlinear cost functions. Recently, Ant Colony optimization (3) has been used for optimization in engineering design problems like truss optimization (2), design of water distribution system (10). This algorithm has the advantage of handling any type of nonconstant (quadratic, exponential,. etc) unit cost function. Therefore, Ant Colony Optimization technique can accomodate nonconstant unit cost functions of purchase and excavation. The primary purpose of this paper is to present an extension of the model which can accomodate non-analytic as well as nonlinear cost function. The model and its extension complement the previous models with constant unit costs(12) and step wise unit costs (6).

In the model, the objective is to minimize the total cost of excavation, embankment, and hauling. The excavation costs are defined as between sections and from-borrow-pit costs and the embankment costs as between sections, from-borrow-pit, and disposal-to-landfill. Hauling cost is the cost of moving earth across sections. The folowing section presents the model (8), explores its solution. In section 4, the underlying biological behaviour of ants is elucidated and in section 5, the ACO algorithm is described. A sampled problem is attempted in section 6. Conclusions and references follow these sections.

MODEL

The designer must divide the road into a number of small sections. Following Easa's definition, a section is the portion of the road between two stations (7). Furthermore, the designer should decide on the number of grade lines in the road under study. In Fig. 1, the road under consideration consists of cut sections with volumes $\mathbf{T}_i$; i = 1, 2, ... $\mathbf{N}_c$ and fill sections with volumes $\mathbf{F}_j$:j = 1, 2, ... $\mathbf{N}_f$. Various borrow pits and landfills are available with capacities $\mathbf{B}_p$ and $\mathbf{D}_k$, respectively; p = 1, 2, ... , $\mathbf{N}_d$. Let the (unknown) quantity of material to be moved from cut section i to fill section j and landfill section k denoted by $\mathbf{X}(\mathbf{i},\mathbf{j})$ and $\mathbf{X}_d$(i,k) respectively. Let also the (unknown) quantity of material to be moved from borrow pit p to landfill section j be denoted by $\mathbf{X}_b$(p,j). The unit cost of excavation, haul and embankment (including placing and compaction) for moving one cubic (meter or yard) from cut section i to fill section j is given by

$$\mathbf{C}(\mathbf{i},\mathbf{j}) = \mathbf{u}_e + \mathbf{S}_i \tag{1}$$

in which $\mathbf{S}_i$h = the swell factor in haul for material excavated from cut section i to fill section j; $\mathbf{S}_{ij}$ = distance between midpoints of section i and j; and u_e,u_h, u_c - unit cost of excavation, haul and embankment. The unit cost for landfill,$\mathbf{C}(\mathbf{i},\mathbf{k})$ is defined in a similar manner.

The unit cost of haul and embankment for moving one cubic unit from borrow pit p to fill section j is given by

$$\mathbf{C}(\mathbf{i},\mathbf{j}) = \mathbf{u}_e + \mathbf{S}_i \tag{2}$$

in which the terms in the right-hand side are defined in a similar manner ats those of Eq. 1. After calculating $\mathbf{C}(\mathbf{i},\mathbf{j})$, $\mathbf{C}(\mathbf{i},\mathbf{j})$ and $\mathbf{C}(\mathbf{i},\mathbf{j})$ they are treated as constants in the model. The unit cost of purchase and excavation for a borrow pit, however, requires a special formulation because it depends on the quantity of material moved from the borrow pit.

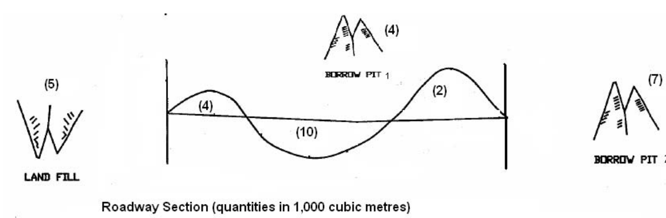

FIG. 1. Typical Roadway Segment

Model Formulation

The problem reduces to fitting each grade line (straight line) to some scattered points in the x-y plane using linear programming. This is very similar to using linear programming as a line fitting technique as done by (9). The components of such linear program is as follows: The objective function that minimizes the total cost of embankment, excavation and haul is: Minimize

$$Z = \sum_i \sum_j \mathbf{C}(\mathbf{i},\mathbf{j})\mathbf{X}(\mathbf{i},\mathbf{j}) + \sum_i \sum_k \mathbf{C}_\mathbf{d}(\mathbf{i},\mathbf{k})\mathbf{X}_\mathbf{d}(\mathbf{i},\mathbf{k})$$

$$+ \sum_p \sum_j \mathbf{C}_\mathbf{b}(\mathbf{p},\mathbf{j})\mathbf{X}_\mathbf{b}(\mathbf{p},\mathbf{j}) + \sum_p \mathfrak{c}_p(\mathbf{x}_p)\mathbf{x}_p \tag{3}$$

The first and second terms are the total earthwork costs of moving materials from the cut sections to fill section and landfills, respectively. The third term is the total earthwork cost of haul and compaction of materials from borrow pits to fill sections. The last term is the total cost of purchase and excavation of materials from borrow pits. $\mathfrak{c}_p(\mathbf{x}_p)$ represents the cost of purchasing material from borrow pits, which is a function of the amount material purchased from the borrow pit. The decision variables are $\mathbf{X(i,j)}, \mathbf{X}_d(i,k), \mathbf{X}_b(p,j)$ and x_p, which are limited by the following five constraints:

1. The total quantities moved from cut sections i to fill sections and landfills should be equal to the quantity available at cut section i, T_i

$$\sum_j \mathbf{X(i,j)} + \sum_k \mathbf{X}_d(i,j) = \mathbf{T}_i; i = 1,2,\ldots,\mathbf{N}_c \quad (4)$$

2. The total quantities moved from fill sections j from all cut sections and borrowpits should be equal to the quantity available at fill section j, F_j

$$\sum_j \mathbf{S}^f_{ij}\mathbf{X(i,j)} + \sum_j \mathbf{S}^f_{pj}\mathbf{X}_b(p,j) = \mathbf{F}_j; i = 1,2,\ldots,\mathbf{N}_f \quad (5)$$

3. The total quantities moved to landfill k from all cut sections should be equal to or less than the landfill capacity D_k

$$\sum_j \mathbf{S}^f_{ik}\mathbf{X}_d(i,k) \le \mathbf{D}_k; k = 1,2,\ldots,\mathbf{N}_D \quad (6)$$

4. The total quantities moved from borrow pit p to all fill sections should be equal to or less than the borrow pit capacity, $\mathbf{B}_p$

$$\sum_j \mathbf{X}_b(i,j) \le \mathbf{B}_p; p = 1,2,\ldots,\mathbf{N}_b \quad (7)$$

5. The quantity used from borrow pit p, $\mathbf{x}_p$, should be equal to the sum of the quantities moved from this pit to all fill sections

$$\mathbf{x}_p = \sum_j \mathbf{X}_b(p,j) \quad (8)$$

The following nonnegativity restrictions complete the formulation

$$\mathbf{X(i,j)}, \mathbf{X}_d(i,k), \mathbf{X}_b(p,j), and x_p >= 0 \quad (9)$$

The formulation of Eqs.3-9 represents a quadratic programming (QP) model and a summary of the complete model is given below Minimize

$$Z = \sum_i \sum_j \mathbf{C(i,j)X(i,j)} + \sum_i \sum_k \mathbf{C_d(i,k)X_d(i,k)} +$$

$$\sum_p \sum_j \mathbf{C_b(p,j)X_b(p,j)} + \sum_p (\alpha_p + \beta_p \mathbf{x}_p)\mathbf{x}_p \quad (10)$$

Subject to

$$\sum_j \mathbf{X(i,j)} + \sum_k \mathbf{X}_d(i,j) = \mathbf{T}_i; i = 1,2,\ldots,\mathbf{N}_c \quad (11)$$

$$\sum_j \mathbf{S}^f_{ij}\mathbf{X(i,j)} + \sum_j \mathbf{S}^f_{pj}\mathbf{X}_b(p,j) = \mathbf{F}_j; i = 1,2,\ldots,\mathbf{N}_f \quad (12)$$

$$\sum_j \mathbf{S}^f_{ik}\mathbf{X}_d(i,k) \le \mathbf{D}_k; k = 1,2,\ldots,\mathbf{N}_D \quad (13)$$

$$\sum_j \mathbf{X}_b(i,j) \le \mathbf{B}_p; p = 1,2,\ldots,\mathbf{N}_b \quad (14)$$

$$\mathbf{x}_p = \sum_j \mathbf{X}_b(p,j) \quad (15)$$

$$\mathbf{X(i,j)}, \mathbf{X}_d(i,k), \mathbf{X}_b(p,j), and x_p >= 0 \quad (16)$$

The foregoing model has $(\mathbf{N}_c\mathbf{N}_f + \mathbf{N}_c\mathbf{N}_d + \mathbf{N}_b\mathbf{N}_f + \mathbf{N}_b)$ decision variables and $(\mathbf{N}_c + \mathbf{N}_f + \mathbf{N}_d + 2\mathbf{N}_b)$ constraints.

REAL ANTS BEHAVIOUR

Ants are social insects, that is, insects that live in colonies and whose behavior is directed more to the survival of the colony as a whole than to that of a single individual component of the colony. Social insects have captured the attention of many scientists because of the high structuration level their colonies can achieve, especially when compared to the relative simplicity of the colony's individuals. An important and interesting behavior of ant colonies is their foraging behavior, and, in particular, how ants can find the shortest paths between food sources and their nest (4).

Although individual ants move in a quasi-random fashion, performing relatively simple tasks, the entire colony of ants can collectively accomplish sophisticated movement patterns and can find the shortest route between their nest and a food source. Ants accomplish this by depositing a substance called *pheromone* as they move. This chemical trail can be detected by other ants, which are probabilistically more likely to follow a path rich in pheromone. This chemical trail can be detected by other ants, which are probabilistically more likely to follow a path rich in pheromone.To show how trail information can be utilized to adapt to sudden unexpected changes in the terrain, a brief example is given next (Fig.2). In Fig.2a a colony of ants is traveling in both directions between point A and point B. Each ant knows which direction to take because of the pheromone trail that is present from point A to point E (Fig.2a). Fig.2b shows what happens when an object is placed in the middle of the path. Since the object is not placed symmetrically on the trail, the path B-C-D is shorter than the path B-F-G-H-D. The ants moving from point B to point D and vice versa, will have to make a decision whether to turn left or right. Since there is no pheromone in either direction, the ant has an equal probability of turning right or left. Initially the first ants turn left or right equally, which means that the equal number of ants are taking path B-C-D and path B-F-G-H-D. The ant traveling along any path will leave pheromone along it. The ants traveling the B-C-D path will arrive at point D earlier than the ones traveling along path B-F-G-H-D. An ant traveling in the opposite direction and is at point D will detect more pheromone along the D-C path, since not only are ants going from D to C, but ants also have started to arrive from C to D. On the other hand the path B-F-G-H-D being longer, the ants have not yet arriving on path D-H. The exact same thing is occurring at point B. Therefore due to more pheromone deposition, probabilistically more ants will begin taking path D-C-B. Eventually the pheromone level on the D-C-B path will become so dominant that all of the ants will choose this path as in Fig.2c. This will also hold true for the ants traveling from point B to D. Hence the ants, using their highly effective pheromone based communication method, are able to find the shortest path between point B and D.

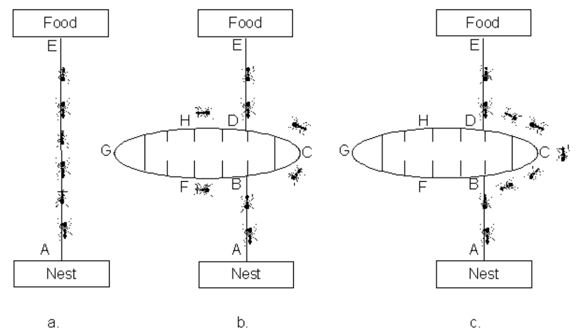

FIG. 2. Behaviour of biological ants.

This particular behaviour of ant colonies has inspired the Ant Colony Optimization (ACO) meta-heuristic algorithm, in which a set of artificial ants co-operate to find solutions to a given optimization problem by depositing pheromone trails throughout the search space.

ANT COLONY OPTIMIZATION

In the early 90's, ant colony optimization (ACO) (5; 3) emerged as a novel nature-inspired metaheuristic method for the solution of combinatorial optimization problems. The inspiring source of ACO is the foraging behavior of real ants. When searching for food, ants initially explore the area surrounding their nest in a random manner. As soon as an ant finds a food source, it evaluates quantity and quality of the food and carries some of the food found to the nest. During the return trip, the ant deposits a chemical pheromone trail on the ground. The quantity of pheromone deposited, which may depend on the quantity and quality of the food, will guide other ants to the food source. The indirect communication between the ants via the pheromone trails allows them to find shortest paths between their nest and food sources. This behaviour of real ant colonies is exploited in artificial ant colonies in order to solve discrete optimization problems.

ACO algorithms are metaheuristic methods for tackling combinatorial optimization problems (4). The central component of an ACO algorithm is the pheromone model, which is used to probabilistically sample the search space. As outlined in (1), the pheromone model can be derived from a model of the CO problem under consideration. A model of a CO problem can be stated as follows.

Definition 5.1 *A model $P = (S, \Omega, f)$ of a CO problem consists of:*

- *a **search (or solution) space** $\mathcal{S}$ defined over a finite set of discrete decision variables and a set Ω **constraints** among the variables;*
- *an **objective function** $f : \mathcal{S} \rightarrow \mathcal{R}^+$ to be minimized.*

*The search space $\mathcal{S}$ is defined as follows: Given is a set of n **discrete variables** $\mathcal{X}_i$ with domains $D_i = v_i^1, \cdots, v_i^{|Di|}, i = 1, \cdots, n$. A variable instantiation, that is, the assignment of a value v_i^j to a variable $\mathcal{X}_i$, is denoted by $\mathcal{X}_i = v_i^j$. A feasible solution $s \in \mathcal{S}$ is a complete assignment (i.e., an assignment in which each decision variable has a domain value assigned) that satisfies the constraints. If the set of constraints Ω is empty, then each decision variable can take any value from its domain independently of the values of the other decision variables. In this case we call $\mathcal{P}$ an **unconstrained** problem model, otherwise a **constrained** problem model. A feasible solution $s^* \in \mathcal{S}$ is called a **globally optimal solution**, if $f(s^*) \leq f(s) \quad \forall s \in \mathcal{S}$ The set of globally optimal solutions is denoted by $\mathcal{S}^* \subseteq \mathcal{S}$ To solve a CO problem one has to find a solution $s \in \mathcal{S}^*$*

A model of the CO problem under consideration implies the finite set of solution components and the pheromone model as follows (1). First, we call the combination of a decision variable $\mathcal{X}_i$ and one of its domain values v_i^j a *solution component* denoted by $\mathfrak{c}_i^j$. Then, the pheromone model consists of a *pheromone* trail parameter $\mathcal{T}_i^j$ for every solution component $\mathfrak{c}_i^j$. The value of a *pheromone* trail parameter $\mathcal{T}_i^j$ called pheromone value – is denoted by τ_i^j. The set of all pheromone trail parameters is denoted by $\mathcal{T}$. As a CO problem can be modelled in different ways, different models of the CO problem can be used to define different pheromone models. Alg. 1 captures the framework of a basic ACO algorithm , as outlined in 1). It works as follows. At each iteration, n_a ants probabilistically construct solutions to the combinatorial optimization problem under consideration, exploiting a given pheromone model. Then, optionally, a local search procedure is applied to the constructed solutions. Finally, before the next iteration starts, some of the solutions are used for performing a pheromone update. The details of this framework (1) are explained in more detail in the following.

InitializePheromoneValues($\mathcal{T}$). At the start of the algorithm the pheromone values are all initialized to a constant value $c > 0$.

ConstructSolution($\mathcal{T}$). The basic ingredient of any ACO algorithm is a constructive heuristic for probabilistically constructing solutions. A constructive heuristic assembles solutions as sequences of elements from the finite set of solution components $\mathfrak{C}$. A solution construction starts with an empty partial solution $\mathfrak{s}_p = \langle\rangle$. Then, at each construction step the current partial solution $\mathfrak{s}_p$. is extended by adding a feasible solution component from the set $\mathfrak{N}(s_p) \subseteq \quad \mathfrak{C} \ \setminus \ \mathfrak{s}_p$, which is defined by the solution construction mechanism. The process of constructing solutions can be regarded as a walk (or a path) on the so-called construction graph $\mathcal{G}_C = (\mathfrak{C}, \mathfrak{L})$, which is a fully connected graph whose vertices are the solution components $\mathfrak{C}$ and the set $\mathfrak{L}$ are the connections. The allowed walks on $\mathcal{G}_C$ are implicitly defined by the solution construction mechanism that defines the set $\mathfrak{N}(s_p)$ with respect to a partial solution $\mathfrak{s}_p$. We denote the set of all solution that may be constructed in this way by $\mathfrak{S}$. The

Algorithm 1: Framework of a basic Ant Colony Optimization Algorithm

Algorithm:ACO procedure

Input: An instance of the problem $\mathcal{P}$ of a CO problem model $\mathcal{P} = (\mathcal{S}, f, \Omega)$

```
InitializePheromoneValues(T) ;
s_bs ← NULL ;
while Termination criteria not true do
    S_iter ← φ ;
    for each ant k do
        s ← ConstructSolution(T) ;
        s ← LocalSearch(s) ;
        if (f(s) < f(s_bs)) or (s_bs == NULL ) then
            s_bs = s_bs
        end
        S_iter ← S_iter ∪ {s};
    end
    ApplyPheromoneUpdate(T, S_iter, s_bs) ;
end
```

Output: The best– so– far solution $\mathfrak{s}_{bs}$

choice of a solution component from $\mathfrak{N}(s_p)$ is, at each construction step, done probabilistically. In most ACO algorithms the probabilities for choosing the next solution component – also called the *transition probabilities* – are defined as follows (1):

$$p(\mathfrak{c}_i^j|\mathfrak{s}_p) = \frac{{\tau_i^j}^{\alpha} \eta(\mathfrak{c}_i^j)^{\beta}}{\sum_{\mathfrak{c}_k^l \in \mathfrak{N}(s_p)} {\tau_i^l}^{\alpha} \eta(\mathfrak{c}_i^l)^{\beta}} \quad \forall \mathfrak{c}_k^l \in \mathfrak{N}(s_p) \tag{17}$$

where η weighting function that assigns, at each construction step, a heuristic value $\eta(\mathfrak{c}_i^j)$ to each feasible solution component $(\mathfrak{c}_i^j)$. The values that are given by the weighting function are commonly called the *heuristic information*. α and β are positive parameters whose values determine the relative importance of pheromone and heuristic information.

ApplyPheromoneUpdate$(\mathcal{T}, \mathfrak{S}_{iter}, \mathfrak{s}_{bs})$ Most ACO algorithms use the following pheromone value update rule:

$$\tau_i^j \leftarrow (1-\rho)\tau_i^j + \rho \sum_{\{\mathfrak{s} \in \mathfrak{S}_{upd} | \mathfrak{c}_i^j \in \mathfrak{s}\}} F(\mathfrak{s}), \tag{18}$$

for $i = 1, \cdots, n$, and $j \in \{1, \cdots, |\mathcal{D}_i|\}$. $\rho \in (0, 1]$ is a parameter called evaporation rate. $F : \mathfrak{S} \mapsto R^+$ is a function such that $f(\mathfrak{s}) < f(\mathfrak{s}\prime) \Rightarrow F(\mathfrak{s}) \geq F(\mathfrak{s}\prime), \quad \forall \mathfrak{s} \neq \mathfrak{s}\prime \in \mathfrak{S}$. $F(\mathfrak{s})$ is commonly called the quality function. Instantiations of this update rule are obtained by different specifications of $\mathfrak{S}_{upd}$, which is a subset of $\mathfrak{S}_{iter} \bigcup \{\mathfrak{s}_{bs}\}$, where $\mathfrak{S}_{iter}$ is the set of solutions that were constructed in the current iteration, and $\{\mathfrak{s}_{bs}\}$ is the best-so-far solution. A well-known example of update rule Eq.18 is the AS-update rule (3) (i.e., the update rule of Ant System (AS)) which is obtained from Eq.18 by setting

$\mathfrak{S}_{upd} \leftarrow \mathfrak{S}_{iter}$ The goal of the pheromone value update rule is to increase the pheromone values on solution components that have been found in high quality solutions. In AS the quality function $F(\mathfrak{s})$ is defined as follows,

$$F(\mathfrak{s}) = \frac{Q}{f(\mathfrak{s})}, \quad \forall \mathfrak{s} \in \mathfrak{S}_{iter} \tag{19}$$

where Q is a constant. In (3) elitist strategy for trail update was also suggested as per Eq.20.

$$F(\mathfrak{s}_{bs}) = n_e . \frac{Q}{f(\mathfrak{s}_{bs})} \tag{20}$$

where $\mathfrak{s}_{bs}$ is the the best–so–far solution and n_e is the number of elitist ants, a parameter of the algorithm.

APPLICATION

Consider the simple roadway segment shown in Fig.1, which consists of two cut and one fill section (the estimated quantities are shown in parentheses). The shrinkage factor in embankment for all cut sections and borrow pits is 0.8. There are two borrow pits and one landfill with the capacities shown in parentheses. The estimated unit costs of excavation, haul, and embankment($ per 1,000 m^3) for earthwork from cut section 1 to fill section and landfill are 7,400 and 3,700, respectively and from cut section 2 are 5,000 and 3,000, respectively. The unit cost of haul and embankment ($ pre 1,000 m^3) for earthwork from borrow pits 1 and 2 to the fill section are 2,500 and 3,000, respectively. The intercept and slope of the linear unit costs are 1,200, 0.08 and 800, 0.04 for borrow pits 1 and 2 respectively. The respective setup costs are $4,000 and $6,000 (the landfill does not have a setup cost).

Solution with linear excavation cost function.

Substituting the previous data into the model of Eqs. 10 - 16, we have (the decision variables are in 1,000 m^3): Minimize

$$Z = 7,400X(1,1) + 5,000X(2,1) + 3,700X_d(1,1)$$
$$+3,000X_d(2,1) + 2,500X_b(1,1) + 3,000X_b(2,1)$$
$$+ (1,200 + 80x_1)x_1 + (800 + 40x_2)x_2 \tag{21}$$

Subject to

$$X(1,1) + X_d(1,1) = 4 \tag{22}$$

$$X(2,1) + X_d(2,1) = 2 \tag{23}$$

$$0.8X(1,1) + 0.8X(2,1) + 0.8X_b(1,1) + 0.8X_b(2,1) = 10 \tag{24}$$

$$0.8X_d(1,1) + 0.8X_d(2,1) \leq 5 \tag{25}$$

$$X_b(1,1) \leq 4 \tag{26}$$

$$X_b(2,1) \leq 7 \tag{27}$$

$$x_1 = X_b(1,1) \tag{28}$$

$$x_2 = X_b(2,1) \tag{29}$$

$$X(i,j), \mathbf{X}_d(i,k), \mathbf{X}_b(p,j), x_p \geq 0 \tag{30}$$

$$i = 1, 2, j = 1, k = 1, p = 1, 2 \quad (31)$$

The ACO algorithm was implemented in C programming language on Linux based platform. Parameters of the algorithm were as follows: Number of ants : 50, Maximum number of iterations: 50, evaporation factor $\rho = 0.2$. The best solution of the above model obtained in simulation (in 1,000 m^3) was X(1,1) = 4; X(2,1) = 2; X_d(1,1) = 0; X_d(2,1) = 0; X_b(1,1) = 2.58; X_b(2,1) = 3.92; x_1 = 2.58, and x_2 = 3.92. This solution was obtained at the 8 th. The total minimum cost is $65,189. This solution is depicted graphically in Fig. 3. It is noted that the landfill is not used.

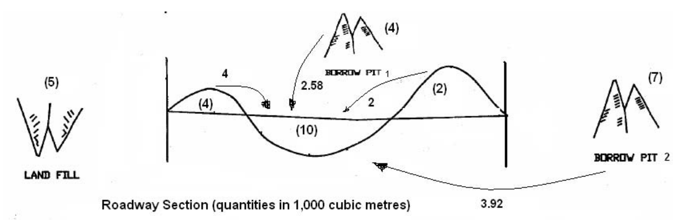

FIG. 3. Optimized Earthwork allocations.

Solution with nonlinear excavation cost function.

The unit cost, $\mathfrak{c}_p$ was assumed to be a linear function of the quantity used from borrow pit p , x_p in the previous case. The ACO algorithm presented here can also accomodate, nonlinear unit cost $\mathfrak{c}_p(x_p)$. $\mathfrak{c}_{p1}(x_p)$, the unit cost of purchasing from borrow pit 1 is

$$\mathfrak{c}_{p1}(x_p) = (1200 + 80x_1 + 15x_1^2)x_1 \quad (32)$$

$\mathfrak{c}_{p2}(x_p)$, the unit cost of purchasing from borrow pit 2 is

$$\mathfrak{c}_{p2}(x_p) = (800 + 40x_2 + 9x_2^2)x_2 \quad (33)$$

So the cost function to be minimized becomes

$$Z = 7,400X(1,1) + 5,000X(2,1) + 3,700X_d(1,1)$$
$$+3,000X_d(2,1) + 2,500X_b(1,1) + 3,000X_b(2,1)$$
$$+ (1200 + 80x_1 + 15x_1^2)x_1 + (800 + 40x_2 + 9x_2^2)x_2 \quad (34)$$

Using the modified objective function, the ACO algorithm was again implemented. Parameters of the algorithm were as in previous section. The best solution of the above model obtained in simulation (in 1,000 m^3) was X(1,1) = 4; X(2,1) = 2; X_d(1,1) = 0; X_d(2,1) = 0; X_b(1,1) = 2.74; X_b(2,1) = 3.76; x_1 = 2.74, and x_2 = 3.76. This solution was obtained at the 5 th iteration. The total minimum cost is $65,979. Compared to the previous case, it can be noted that lesser material is now taken from borrow pit 2 and more material is taken from borrow pit 1.

CONCLUSIONS

A method for optimizing earthwork allocation using Ant Colony Optimization was presented. Roadway grade selection and earthwork allocation (transportation) has been treated in a integrated way. These two stages have always been treated separately in the literature. Four types of costs in earthwork have been considered as cost of:

- cut sections to fill sections.
- cut sections to landfills.
- haul and compaction from borrow pits to fill sections.
- purchase and excavation from borrow pits.

Hauling cost is the cost of moving earth across sections. Cost of purchase and excavation is the only cost that depends on amount of earthwork, since it depends on the soil stratum. Previous models of earthwork allocation can only accomodate linear or constant unit cost functions of purchase and excavation for borrow pits. To overcome the inadequacy of previous models, a robust Ant Colony Optimization(ACO) algorithm has been presented. ACO technique does not assume anything about the continuity as well as linearity of objective function. Since the nature of function is not required to be known beforehand, proposed method can be used for all types of borrow pits. The efficacy of the method was demonstrated with a linear purchase and excavation cost function as well as a quadratic (nonlinear) one, although the applicability of the model is not restricted by the type of function.

REFERENCES

[1]C. Blum and M. Dorigo. The hyper-cube framework for ant colony optimization. *IEEE Transactions on Systems, Man, and Cybernetics – Part B*, 34(2):1161–1172, 2004.

[2]Charles V. Camp and Barron J. Bichon. Design of space trusses using ant colony optimization. *Journal of Structural Engineering*, 130(5):741–751, 2004.

[3]M. Dorigo, V. Maniezzo, and A. Colorni. The ant system: Optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics*, 26:29–41, 1996.

[4]M. Dorigo and Thomas Stützle. *Ant Colony Optimization.* MIT Press, Cambridge , MA, 2004.

[5]Marco Dorigo. Optimization, learning and natural algorithms. Technical Report MI-92-03, PhD thesis, Dip. Elettronica e Informazione, Politecnico di Milano, Italy, 1992.

[6]Said M. Easa. Earthwork allocations with nonconstant unit costs. *Journal of Construction Engineering and Management*, 113-1:34–50, 1987.

[7]Said M. Easa. Earthwork allocation with linear unit costs. *Journal of Construction Engineering and Management*, 114-2:641–655, 1988.

[8]Said M. Easa. Selection of roadway grades that minimize earthwork cost using linear programming. *Transportation Research Journal*, 22A-2:121–136, 1988.

[9]J. P. Ignizio. *Linear programming in single and multiple objective systems.* Prentice-Hall,Englewood Cliffs, NJ, 1982.

[10]Holger R. Maier, Angus R. Simpson, Aaron C. Zecchin, Wai Kuan Foong, Kuang Yeow Phang, Hsin Yeow Seah, and Chan Lim Tan. Ant colony optimization for design of water distribution systems. *Journal of Water Resources Planning and Management*, 129(3):200–209, 2003.

[11]Ahmad A. Moreb. Linear programming model for finding optimal roadway grades that minimize earth-

work cost. *European Journal of Operation Research*, 93:148–154, 1996.

[12] R. Stark and R. Mayer. *Quantitative construction management:Uses of linear optimization*. John Wiley and Sons, New York, N. Y., 1983.

[13] R. Stark and R. Nichollas. *Mathematical foundations for design: Civil engineering systems*. McGraw-Hill Publishing Co., Inc., New York N.Y., 1972.