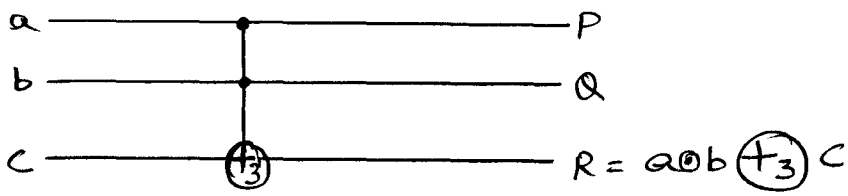
ADVANCED LOGIC SYNTHESIS

HOMEWORK 3

1. Prove that the ternary toffoli gate is reversible.

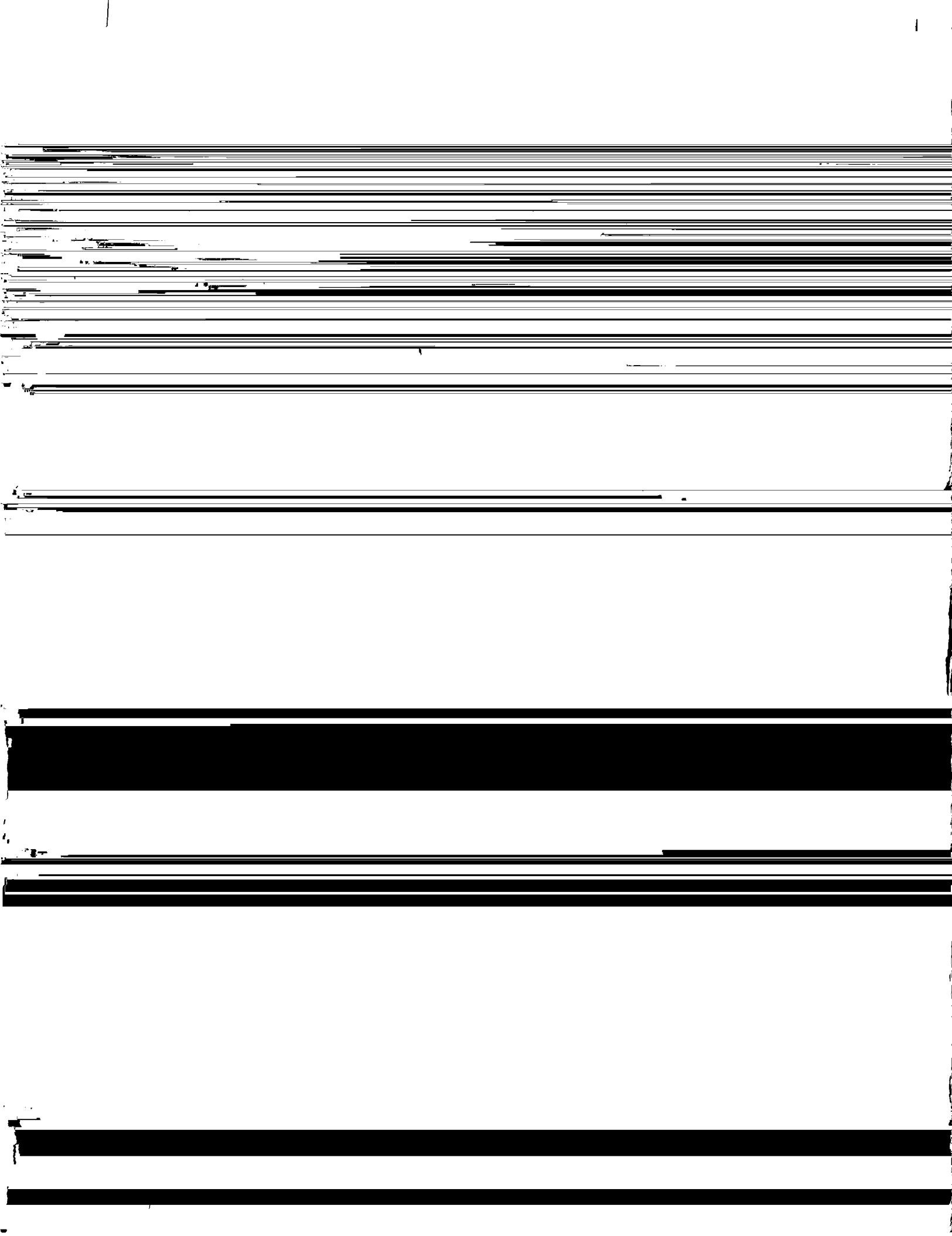
Sol: Reversible gate: A gate is said to be reversible if the no. of input bits is equal to the no. of output bits. i,e the output bits are unique.

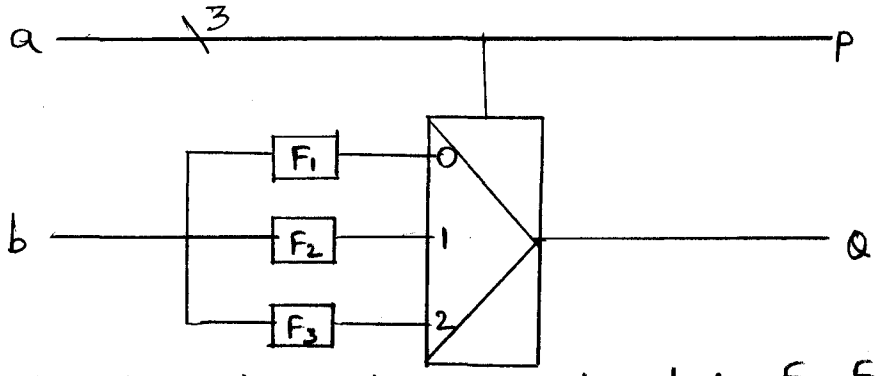
Consider a ternary toffoli gate with a, b, c as I/p bits and P, Q, R as O/p bits.



The truth table for the above ckt. is:

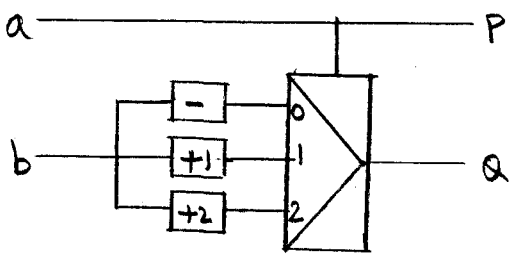
a	b	c	P	Q	R
0	0	0	0	0	0
0	0	1	0	0	1
0	0	2	0	0	2
0	1	0	0	1	0
0	1	1	0	1	1
0	1	2	0	1	2
0	2	0	0	2	0
0	2	1	0	2	1
0	2	2	0	2	2
1	0	0	1	0	0
1	0	1	1	0	1
1	0	2	1	0	2
1	1	0	1	1	1
1	1	1	1	1	2
1	1	2	1	1	0
1	2	0	1	2	2
1	2	1	1	2	0
1	2	2	1	2	1





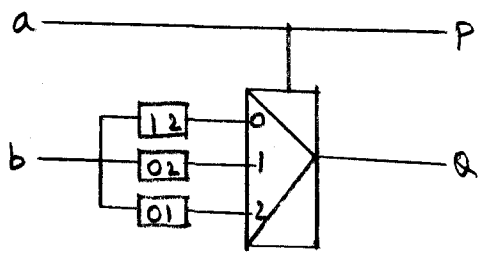
The toffoli gate with three i/p fns F_1, F_2 and F_3 which are arbitrary qubit fns. Is as shown.

Consider two of the 216 gates.



a \ b	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

a	b	P	Q
0	0	0	0
0	1	0	1
0	2	0	2
1	0	1	1
1	1	1	2
1	2	1	0
2	0	2	2
2	1	2	0
2	2	2	1



a \ b	0	1	2
0	0	2	1
1	2	1	0
2	1	0	2

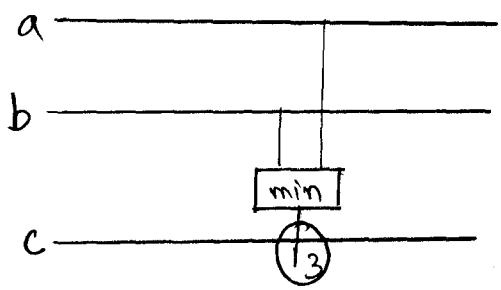
a	b	P	Q
0	0	0	0
0	1	0	2
0	2	0	1
1	0	1	2
1	1	1	1
1	2	1	0
2	0	2	1
2	1	2	0
2	2	2	2

It can be observed that the above two gates are reversible from the truth table shown.

It can be proved for all 216 gates and each of the qubit gates is reversible.

Since the ternary toffoli gate is composed of the qubit gates which are reversible, the ternary toffoli gate is reversible.

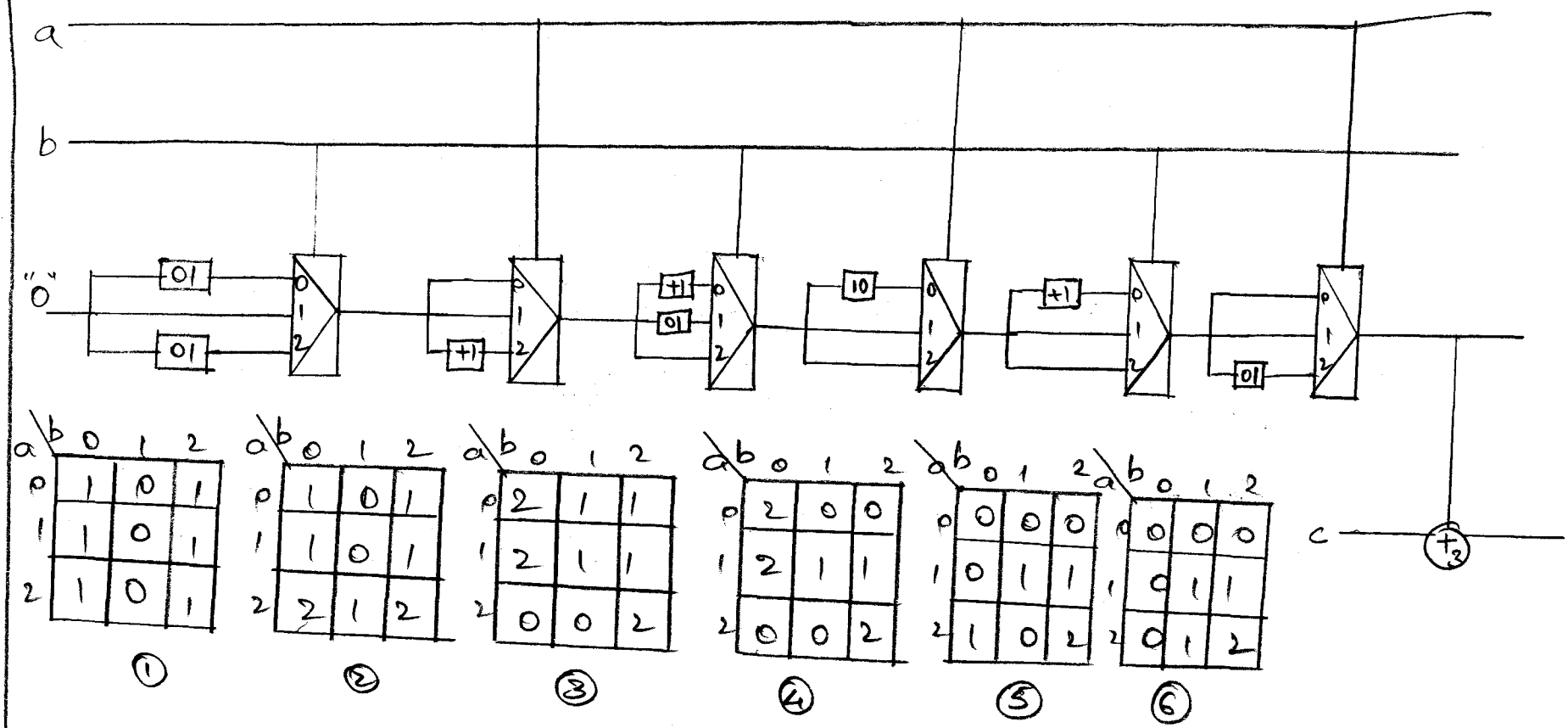
3. Design a new gate similar to ternary toffoli.



Sol: The min gate can be represented as follows:

a \ b	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

This can be realized as shown:



a \ b	0	1	2
0	1	0	1
1	1	0	1
2	1	0	1

①

a \ b	0	1	2
0	1	0	1
1	1	0	1
2	2	1	2

②

a \ b	0	1	2
0	2	1	1
1	2	1	1
2	0	0	2

③

a \ b	0	1	2
0	2	0	0
1	2	1	1
2	0	0	2

④

a \ b	0	1	2
0	0	0	0
1	0	1	1
2	1	0	2

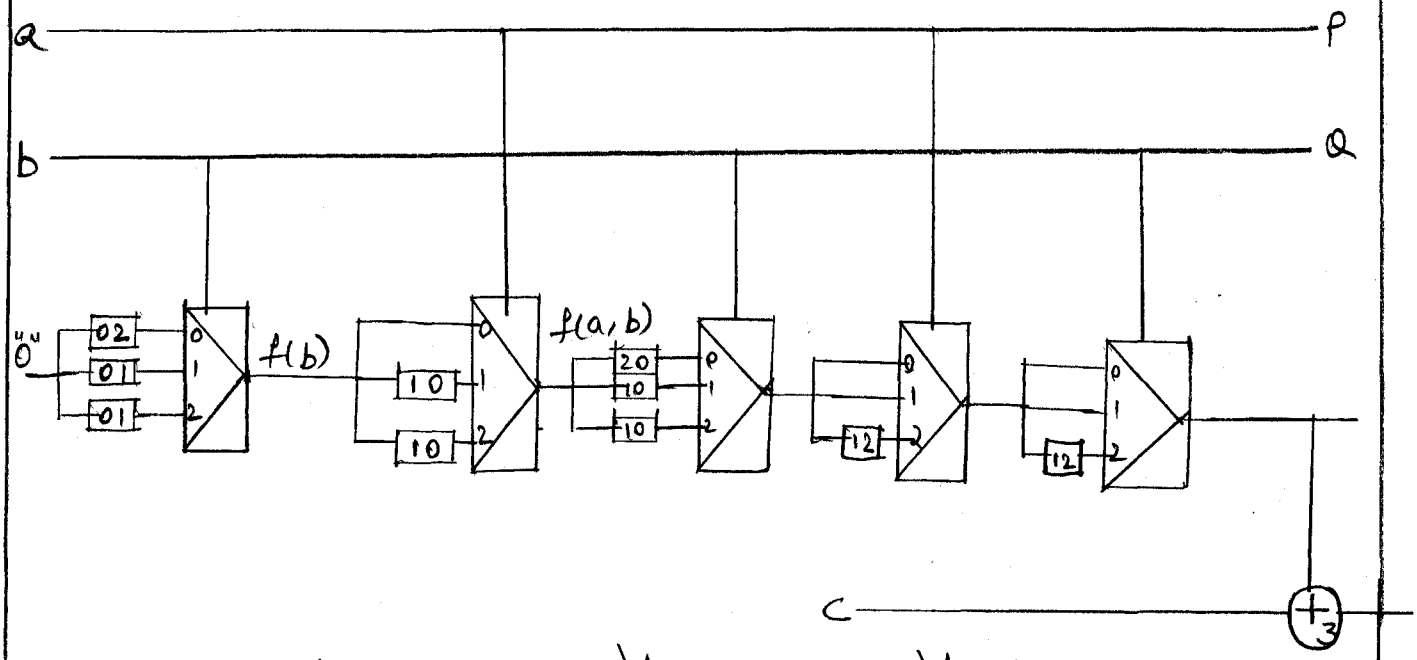
⑤

a \ b	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

⑥

4. Prove that the gate shown in the class is minimum or find better Solution.

Soln: One of the Solutions is :



a \ b	0	1	2
0	2	1	1
1	2	1	1
2	2	1	1

①

a \ b	0	1	2
0	2	1	1
1	2	0	0
2	2	0	0

②

a \ b	0	1	2
0	0	0	0
1	0	1	1
2	0	1	1

③

a \ b	0	1	2
0	0	0	0
1	0	1	1
2	0	2	2

④

a \ b	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

⑤