

Problem Sheet - 4

1. Show that a Fourier transform of a Gaussian is also a Gaussian curve considering the following form of the Gaussian

$$g(k) = e^{-\alpha(k-k_0)^2}$$

Show that the product of the exponential widths obtained is a constant. (Note that exponential width refers to the value of the function when it has decreased to $1/e$ times its maximum value.) Can you tell the physical implication of this relation and the field of physics where it plays a very vital role.

2. Prove Schwarz inequality. Show that the necessary and sufficient condition for the existence of derivative of a complex function is the Cauchy Riemann equations.
3. Write down Cauchy Riemann equations in polar co-ordinates. Where is the following function

$$f(z) = |z|^4$$

differentiable, if at all it is.

4. State and explain Fermat's principle. Establish the reflection formula for concave surface reflector from this principle.
5. Find by matrix method the relation

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{r}$$

in case of refraction at the curved surface where the symbols have their usual meaning.

6. A concave lens is placed at a distance of 25cm in front of a concave mirror of focal length 20cm. It is found that an object placed at a distance 45cm in front of the lens coincides with its own inverted image formed by the combination. Using refraction matrix find the focal length of the lens.
7. Give the theory of Newton's rings to determine the wavelength of monochromatic light and show how from their study the refractive index of a liquid can be determined.
8. Obtain an expression for the resolving power of a plane diffraction grating and hence give an estimate of the number of lines required to resolve D_1 and D_2 lines of Na in the 2nd order.