# Problem Sheet - 2

1. State Fourier's theorem. Is this theorem applicable at the point of discontinuity of a function? Expand the function

$$f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ x, & 0 \leq x < \pi \end{cases}$$

$$f(x + 2\pi) = f(x)$$

in a Fourier series. From this expansion, show that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

2. Find the eigen values and a set of orthonormal eigen vectors of the following matrix

$$\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$$

3. Show how the second law of thermodynamics leads to the concept of entropy. Prove that for irreversible change, the net change in entropy of the universe will be positive.

4. Given that

$$du = xdy - (2y + x^3 e^x)dx$$

solve the equation when du=0. Is du a perfect differential?

5. If $\vec{r}$ be the coordinate vector, prove that for a central force $\vec{F} = f(r)\vec{r}$, $\vec{\nabla}$ x $\vec{F}$ =0.

6. Show that for an odd function, the Fourier series reduces to the Fourier sine series.

7. Find the eigen values and eigen vectors of the following matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Is this matrix unitary?