Problem Sheet - 1

1. Two vector fields $\vec{a}_1$ and $\vec{a}_2$ have the same divergence and curl at every point in volume V and have the same normal component at every point on the surface S enclosing V. Prove that $\vec{a}_1$ and $\vec{a}_2$ are equal everywhere.

2. Starting from the Hermite's differential equation

$$y'' - 2xy' + 2ny = 0$$

(n = constant integer), derive by Frobenius method the standard form of Hermite polynomial $H_n(x)$. Hence prove that

$$H_n'(x) = 2nH_{n-1}(x)$$

3. State Gauss' divergence theorem. Hence prove that for two scalar functions u and v

$$\int u\vec{\nabla}v.\vec{dS} = \int u\vec{\nabla}.(\vec{\nabla}v)dV + \int \vec{\nabla}u.\vec{\nabla}v dV$$

where the volume V is enclosed by the surface S.

4. By the method of separation of variables, reduce Laplace's equation in rectangular Cartesian coordinate system to ordinary differential equations.

5. Explain the terms reversible process and quasistatic process. The equation of state of an ideal elastic system is

$$F = CT\Big(\frac{L}{L_0} - \frac{L_0^2}{L^2}\Big)$$

where C is a constant and L_0(the value of L at zero tension F) is a function of temperature T only. Show that the isothermal Young's modulus is given by

$$Y = \frac{F}{A} + 3CT\frac{L_0^2}{AL^2}$$