Mathl Comput. Modelling, Vol. 12, No. 8, pp. 935-938, 1989
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0895-7177/89 $3.00 + 0.00

MATHEMATICAL INTERPOLATION IN PARALLAX-BAR HEIGHTING

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(Received September 1988; accepted for publication October 1988)

Communicated by E. Y. Rodin

Abstract—Different interpolation algorithms are compared for accuracy in correcting crude heights obtained from parallax-bar measurements. The conventional correction equation is programmed and weighted against polynomial and Shepard's interpolations. The results show that Shepard's interpolation method is superior to both the conventional and the polynomial solutions. These results may serve a useful purpose in application areas in which parallax-bar heighting is still being used.

INTRODUCTION

The determination of object height using x-parallax measurement on a stereopair of aerial photographs is one of the basic practices in photogrammetry. Although this method of object height determination is not widely used nowadays in topographic science, it still finds a good deal of practice in other application fields where stringent height accuracy figures are not sought. Areas such as geology, forestry, geography and reconnaissance surveys are examples in which parallax-bar heighting is often used to determine object heights. The basic formula of the parallax-height relation is applicable only to vertical aerial photography. However, it is quite common to have tilted aerial photography in practice. The correction of the tilt effect on the determined height has been treated thoroughly by Thompson [1, 2] and Corne [3] who developed a mathematical model, based on the deformation arising due to effects of tilts w and ϕ about the x- and y-axes respectively to determine the corrected heights of points in the stereomodel.

THE PARALLAX-HEIGHT RELATIONSHIP

The determination of heights from changes in the x-parallax measured on two overlapping photographs forming a stereomodel starts with the well-known parallax-height formula:

$$\Delta h_{AB} = \frac{\Delta P_{AB}(H - h_A)}{P_A + \Delta P_{AB}}. \tag{1}$$

In this equation, A is a reference point of known height h_A and the height h_B of point B is to be determined; Δh_{AB} is the difference in height between points A and B; H is the flying height above datum; P_A is the absolute parallax of point A and ΔP_{AB} is the difference in parallax between points A and B.

Equation (1) is valid for vertical photography only; and when tilts are present, which is usually the case, the height determined is called *crude height*; and in this case a correction dh should be applied to give the *corrected height*. This correction is usually determined by using the familiar equation:

$$dh = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 \tag{2}$$

which is based on *first order* approximations for tilts w and ϕ about the x- and y-axes respectively. In equation (2), x and y are the coordinates of the points in the photo coordinate system and $a_i(i = 1, \ldots, 4)$ are coefficients defining the correction surface. They are determined by solving five simultaneous equations using at least five points on the model with known ground elevation values (the so-called height control points).

In some application fields, it is necessary to achieve a certain accuracy; though such an accuracy will always be far below that required for topographic mapping and contour drawing. Geographic surveys, landslide studies, drainage patterns and preliminary site investigations are examples of such cases (e.g. see [4, 5]). In such cases, it will be necessary to perform further corrections to the crude heights in order to achieve the required accuracy.

In this study, an attempt is made to incorporate higher order correction terms to equation (2) in order to judge the significnce of these additional parameters in obtaining more accurate height values from parallax-bar measurements. In addition, an interpolation method developed by Shepard [6] will also be tested for the purpose.

The interpolation polynomial used instead of equation (2) has the following form:

$$dh = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 x^2 y + a_6 y^2 + a_7 y^2 x + a_8 x^2 y^2 \tag{3}$$

where x, y and a_i are as before.

It is clear that a minimum of nine control points is required to solve for the unknown parameters a_i in equation (3). The terms of equation (3) are then truncated one by one, proceeding from right to left; and the computer program is re-run to evaluate the heights of the check points.

Shepard's interpolation method is the solution for a problem of fitting a surface to given data. In general, if D is a domain in the (x, y)-plane and F is a real-valued function defined on D, and if we are given values $F_i - F(x_i, y_i)$ of F at some set of points (x_i, y_i) located in D $(i = 1, 2, \dots, N)$; then the problem is to find a function f defined on D which reasonably approximates F. Now if ρ is the usual distance metric in the plane; and given a point (x, y); and $r_i = \rho[(x, y), (X_i, Y_i)]$ for $i = 1, 2, \dots, N$ and for $0 < \mu < 1$, then Shepard's interpolation for the stated problem is:

$$f(x, y) = \left(\sum_{i=1}^{N} \frac{F_i}{r_i^{\mu}}\right) \Big/ \left(\sum_{i=1}^{N} \frac{1}{r_i^{\mu}}\right) \quad \text{when } r_i = 0 \text{ for all } i \quad \text{and} \quad = F_i \quad \text{when } r_i \neq 0.$$

The value of $f(x, y)$ at check points (or non-data points) is obtained as a weighted average of all the data values, where the ith measurement is weighted according to the distance of (x, y) from point (x_i, y_i). According to Schumaker [7], μ must be chosen < 1 in order to avoid cusps at the control points.

MEASUREMENTS AND TESTS

In order to compare polynomial and Shepard's interpolation algorithms with the conventional procedure of equation (2) for correcting heights from tilt effects, a stereomodel covering a rather hilly area in western Sudan was made available by Sudan Survey Department. The photography used in the experiment was at a scale of 1:40,000 and was acquired with a Wild RC-10 camera of 152 mm focal length. Because of lack of sufficient control for the higher order solutions of equation (3), some data points were derived photogrammetrically.

The model was viewed under the mirror stereoscope and the parallax bar was used to read parallax readings. A FORTRAN computer program was written in order to effect the computation of the transformation parameters and the height values.

RESULTS OF THE TESTS

The parallax bar readings and the elevation of one point were used to compute the crude heights for all points in the model. The control point distribution used in the model is shown in Fig. 1. Table 1 shows the crude height errors, the errors in height after the conventional solution and the errors in height after applying Shepard's and polynomial solutions.

Table 2 summarizes the overall accuracy for different solutions of the model, the standard deviation of the mean is given in metres and as a ‰ of the flying height.

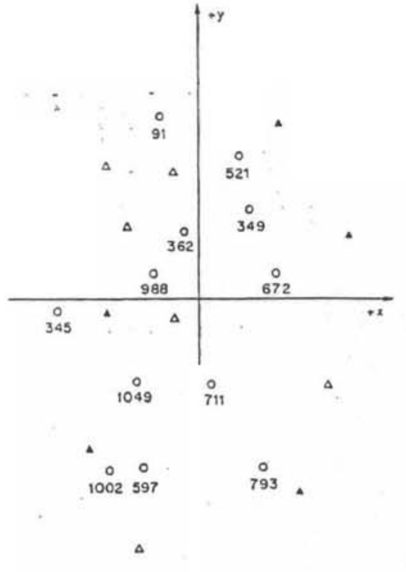

Fig. 1. Control distribution of the experiment: ▲, control point used with all solutions; △, additional control point used with the polynomial solution; ○, check point.

ANALYSIS OF RESULTS

Inspection of Tables 1 and 2 shows that Shepard's interpolation method gave height discrepancy values of the same order of those given by the higher order polynomial solutions and far better results than those given by the conventional solution ($\delta = 1.4$ m for Shepard's solution as compared to $\delta = \pm 3.15$ m for the conventional solution). For Shepard's solution, the maximum errors are always at points in the far edges of the model. Hence if the solution is applied to points inside the model better results can be obtained. The advantages of Shepard's interpolation solution for photogrammetric applications are therefore efficiency and inexpensiveness since less control points (or data points) are required and hence less computer effort and time are involved.

The problem stated in this paper is applicable to many photogrammetric systems where the heights determined suffer from systematic errors that can be corrected by the same approach. This is now being investigated and the results will be the subject of a separate paper.

Table 1. Errors in heights after various solutions (at the check points)

Point number	Initial error of crude height (m)	Error in height after conventional solution (m)	Error after the nine-term polynomial solution (m)	Error after eight-term polynomial solution (m)	Error after seven-term solution (m)	Error after six-term solution (m)	Error after Shepard's interpolation (m)
345	7.16	7.02	4.12	3.98	5.16	6.22	3.19
1049	11.21	7.19	2.42	2.12	3.01	4.59	−2.31
91	7.98	−6.13	−2.98	−1.78	−3.29	−5.12	−1.92
521	16.28	3.83	1.89	2.54	4.14	7.34	0.84
349	11.78	5.91	2.56	1.89	0.12	4.01	−1.03
362	6.42	−3.10	3.98	−4.34	−2.11	−0.01	+1.32
672	7.01	2.11	0.91	1.01	0.99	0.78	0.01
988	16.03	10.59	4.12	5.32	6.71	7.98	5.20
711	11.54	11.36	4.92	4.11	5.11	5.09	−2.08
793	9.65	6.12	2.67	2.35	3.43	3.62	3.21
597	11.25	−3.21	−1.77	−3.03	−1.20	−3.01	−1.11
1002	0.43	−1.29	0.78	1.12	3.92	4.12	0.98

Table 2. Errors in height as standard deviation and ‰ H

Type of solution	RMSE δ (m)	Error as ‰ H
Nine-term polynomial	2.15	8.6
Eight-term polynomial	1.37	5.5
Seven-term polynomial	1.91	7.6
Six-term polynomial	1.46	5.8
Conventional solution	3.15	12.6
Shepard's interpolation	1.40	5.6

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