King Mongkut's University of Technology Thonburi

FDE618: Transport Phenomena in Food Processing
ChE610: Fundamentals of Transport Phenomena

Chapter 1
Review of Basic Concepts and Mathematical Tools

Before deriving a set of governing equations, which results from invoking the laws of conservation of momentum, energy and mass, it is necessary to discuss certain preliminary topics. The first topic of discussion is the way in which the conservation equations are derived. An issue regarding the choice of reference frame to be employed (Eulerian versus Lagrangian) is discussed next. A theorem, which relates derivatives in the Lagrangian framework to derivatives in the Eulerian framework, is then discussed. Some other tools required in the study of transport phenomena are also mentioned.

Note that the problems considered in these notes are:

- Flow problems: laminar, incompressible flows of Newtonian fluids
- Heat/Mass Transfer problems: conduction (diffusion) and convection

Radiation heat transfer, turbulent transport processes as well as non-Newtonian fluids are not covered in these notes.

1.1 Concept of Continuum

There are basically two ways of deriving the equations which govern the transport phenomena – statistical and continuum approaches. The statistical method approaches the question from the molecular point of view. It treats the fluid as consisting of molecules whose behavior is governed by the laws of dynamics. The macroscopic phenomena are assumed to arise from the molecular action of the molecules and the theory attempts to predict the macroscopic behavior using the laws of mechanics and probability theory. Although the technique is rather elegant and may be used to treat gas flows in situations where the continuum concept is not valid, it is incomplete for dense gases and liquids.

The alternative method, which is used to derive the governing conservation equations, uses the continuum concept. In this technique individual molecules are ignored and it is assumed that the fluid is a continuous matter (no holes). At each point of this continuous matter there is a unique value of field variables, e.g., velocity, pressure, temperature. This continuous matter obeys the conservation laws of momentum, energy and mass, which give rise to a set of partial differential equations governing the field variables. The solution of these equations then defines the variation of each field variable with space and time.

The continuum approach requires that the mean free path of the molecules be very small compared with the smallest physical length of the flow field since only in this way can meaningful averages over the molecules at a point be made and the molecular structure of the fluid be ignored. If this condition is satisfied there is no distinction between light gases, dense gases or even liquids – the results apply well to all fluids.

The continuum concept is valid as long as there exists a volume size that is much larger than the volume occupied by a single molecule of the fluid but is much smaller than the smallest length scale of the problem. A criterion commonly used to evaluate the validity of the continuum concept is based on the Knudsen number (Kn):

$$\text{Kn} = \text{mean free path of molecules/characteristic length of flow} \qquad (1.1A)$$

The criterion for the validity of the continuum approach is:

When Kn < 0.01: continuum approach is valid
When Kn > 0.1: must use statistical approach

At intermediate values of Knudsen number it is sometimes possible to use continuum equations with modified boundary conditions involving a relaxation of the no slip condition.

Since the vast majority of phenomena encountered in engineering applications fall within the continuum domain, this technique will be used throughout these notes.

1.2 Eulerian and Lagrangian Coordinates

Two basic coordinate systems are available: Eulerian (fixed) and Lagrangian (moving) coordinates.

In the Eulerian framework the independent variables are the spatial coordinates x, y, z, and time t. Attention is focused on the fluid which passes through a control volume which is fixed in space. The fluid inside the control volume at any instant in time consists of different fluid particles.

In the Lagrangian approach, on the other hand, attention is focused on a particular mass of fluid as it flows. In this frame x, y, z, and t are no longer independent variables, since if it is known that our portion of fluid passed through the coordinates x_0, y_0, z_0 at some time t_0, then its position at some later time may be calculated if the velocity components are known. The independent variables in the Lagrangian system are then x_0, y_0, z_0 and t, where x_0, y_0, z_0 are the coordinates at which a specified fluid element passed through at time t_0.

Originally, the fundamental principles that provide the basis of all classical mechanics were formulated to describe the motion of a rigid body or particle, i.e., a closed system. Examples of this are the law of conservation of mass, which indicates that the mass of the body cannot change with time; and Newton’s second law of motion, which states that the rate of change of the momentum of the body is equal to the force. These two laws can be expressed as follows:

Conservation of mass:

$$\frac{dm}{dt} = 0 \qquad (1.2A)$$

Newton’s second law of motion:

$$m\frac{d\vec{v}}{dt} = \vec{F} \qquad (1.2B)$$

This approach is based on the Lagrangian system, which is not convenient for analyzing a problem in continuum mechanics, i.e., in the study of deformable materials (like fluids). For fluids the focus of interest is generally a fixed region of space through which the fluid flows rather than a particular body of fluids. In transport processes the interests are the values of field variables at various points in space. For example, it is the function *P(x,y,z,t)* that is of interest rather than the pressure of a particular fluid particle as a function of time.[1] This is the Eulerian description of the problem.

In order to derive the governing conservation equations it is necessary to transform the basic physical principles of mechanics from the Lagrangian to the Eulerian point of view. The tools that are needed to accomplish this task are: the material derivative operator and the Reynolds Transport Theorem.

1.3 Material (Substantial) Derivative

Let α be any field variable (e.g., temperature, velocity), i.e.,

$$\alpha = \alpha(x, y, z, t) \tag{1.3A}$$

It is desirable to derive an expression that relates the rate of change of α with time for a particular fluid element that happens to be at the point *x*, *y*, *z* at the time *t*. Of course, it is possible to simply define α_0 to be the value of α for this fluid particle and the desired rate of change is simply the derivative $d\alpha_0/dt$. However, this does not really meet the present needs of representing this quantity in terms not of the variable referring to a particular element but in terms of an operator involving the field variable. Note that this is not simply the partial time derivative of α with respect to time. In other words,

$$\frac{d\alpha_o}{dt} \neq \frac{\partial \alpha(x, y, z, t)}{\partial t} \tag{1.3B}$$

For example, let α be the velocity and consider a steady flow in a converging channel. The velocity at every point is constant in time so the partial time derivative of *v* with respect to time at every point is zero. However, every fluid particle is being accelerated so that the derivative of the velocity of a fluid element with respect to space is not zero.

To derive the definition of the operator that, when acting on $\alpha = \alpha(x, y, z, t)$ gives $d\alpha_0/dt$ the following approach is taken. Consider an increment of time *dt* and write the expression for the change in α_0 of a particular (moving) particle during this time.

$$d\alpha_o = \frac{\partial \alpha}{\partial t} dt + \frac{\partial \alpha}{\partial x} dx + \frac{\partial \alpha}{\partial y} dy + \frac{\partial \alpha}{\partial z} dz \tag{1.3C}$$

where *dx*, *dy* and *dz* are the changes in the position coordinates of the fluid element during the time *dt* and the partial derivatives are evaluated at the point in space occupied by the particle at the time *t*. Dividing both sides of the equation with *dt* yields:

---

[1] The situation is quite different in the case of multiphase flows, e.g., gas-particle flow, where tracking of individual particles in order to learn of their instantaneous properties (e.g., velocity, temperature, etc.) are sometimes needed.

$$\frac{d\alpha_o}{dt} = \frac{\partial \alpha}{\partial t} + \frac{\partial \alpha}{\partial x}\frac{dx}{dt} + \frac{\partial \alpha}{\partial y}\frac{dy}{dt} + \frac{\partial \alpha}{\partial z}\frac{dz}{dt} \tag{1.3D}$$

But when $dt \to 0$, $dx/dt = v_x$, $dy/dt = v_y$ and $dz/dt = v_z$. Thus,

$$\frac{d\alpha_o}{dt} = \frac{\partial \alpha}{\partial t} + v_x \frac{\partial \alpha}{\partial x} + v_y \frac{\partial \alpha}{\partial y} + v_z \frac{\partial \alpha}{\partial z} \tag{1.3E}$$

The quantity on the right hand side is the so-called material or substantial derivative (also the Eulerian time derivative). As a convenience in the writing of this derivative a symbol that operates on a field variable to give the material derivatives is defined as follows:

$$\frac{D\alpha}{Dt} \equiv \frac{\partial \alpha}{\partial t} + (\vec{v}\cdot\nabla)\alpha = \frac{\partial \alpha}{\partial t} + v_x \frac{\partial \alpha}{\partial x} + v_y \frac{\partial \alpha}{\partial y} + v_z \frac{\partial \alpha}{\partial z} \tag{1.3F}$$

Thus,

$$\frac{d\alpha_o}{dt} = \frac{D\alpha(x,y,z,t)}{Dt} \tag{1.3G}$$

If it is assumed that, for example, α_o is the velocity of the fluid particle, the material derivative is simply the acceleration of the particle. Note that there can be acceleration, even when the flow is steady. This is possible since the convective acceleration terms in the material derivative can be non-zero. An example of this situation is the steady flow in a converging channel mentioned earlier.

1.4 Reynolds Transport Theorem (RTT)

This theorem provides a relationship between the rate of change of an integral over a moving system consisting of particular fluid elements and operators involving an integral over a fixed volume of space. It thus allows an expression of a time derivative following a fluid body (Lagrangian frame) in terms of field variables described in the Eulerian frame.

Let α be any field variable, i.e., a function of *x*, *y*, *z* and *t*. Consider the volume integral of α over the volume of space $V_s(t)$ that is occupied by a particular collection of fluid elements called the system at time *t*. Let the particular fixed region of space occupied by these fluid elements at time *t* be called V_c, the control volume (when does $V_s(t)$ equal to V_c?)

It is desirable to evaluate the derivative with respect to time of the integral of α over $V_s(t)$:

$$\frac{d}{dt}\int_{V_s(t)} \alpha(x,y,z,t)dV \tag{1.4A}$$

Because the volume of space over which the integral is to be evaluated is a function of time it is not possible to simply move the derivative inside the integral and replace $V_s(t)$ with V_c. Remember that the objective of this assignment is to find a representation of the rate of change of the integral that refers only to the control volume and not the system.

To accomplish this goal first make use of the definition of the derivative to expand (1.4A) as follows:

$$\frac{d}{dt}\int_{V_s(t)}\alpha(t)dV = \lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left[\int_{V_s(t+\delta t)}\alpha(t+\delta t)dV - \int_{V_s(t)}\alpha(t)dV\right]\right\} \tag{1.4B}$$

Some of the independent variables are not written out for the sake of brevity.

Now add and subtract the following quantity to the right hand side of (1.4B) inside the limit:

$$\frac{1}{\delta t}\int_{V_s(t)}\alpha(t+\delta t)dV \tag{1.4C}$$

The right hand side of (1.4B) now becomes:

$$\lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left[\int_{V_s(t+\delta t)}\alpha(t+\delta t)dV - \int_{V_s(t)}\alpha(t+\delta t)dV\right] + \frac{1}{\delta t}\left[\int_{V_s(t)}\alpha(t+\delta t)dV - \int_{V_s(t)}\alpha(t)dV\right]\right\} \tag{1.4D}$$

Note that the limits for the second group of two integrals are the same and that when the limit is taken this group is simply the integral of the partial time derivative of $\alpha(t)$:

$$\lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left[\int_{V_s(t)}\alpha(t+\delta t)dV - \int_{V_s(t)}\alpha(t)dV\right]\right\} = \int_{V_s(t)}\lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left[\alpha(t+\delta t)-\alpha(t)\right]\right\}dV = \int_{V_c}\left(\frac{\partial\alpha}{\alpha t}\right)dV$$

(1.4E)[2]

Now consider the first pair of integrals in the limit shown in (1.4D). Note that the integrand is the same in each but that the volumes of integration are different. Therefore, the difference between the two integrals is simply the integral over the difference in volumes:

$$\int_{V_s(t+\delta t)}\alpha(t+\delta t)dV - \int_{V_s(t)}\alpha(t+\delta t)dV = \int_{V_s(t+\delta t)-V_s(t)}\alpha(t+\delta t)dV \tag{1.4F}$$

This incremental volume, shown in Figure 1.1, can be related to the velocity $\vec{v}$ and the outer directed unit normal vector $\vec{n}$ at the same point as follows. The component of $\vec{v}$ in the direction of $\vec{n}$ is $(\vec{n}\cdot\vec{v})$. The distance between the surfaces of the two volumes is thus $(\vec{n}\cdot\vec{v})\delta t$. If the incremental area of the surface at the same point is *dA*, then the incremental volume is:

$$dV = (\vec{n}\cdot\vec{v})\delta t dA \tag{1.4G}$$

[2] Why can we replace $V_s(t)$ with V_c in (1.4E)?

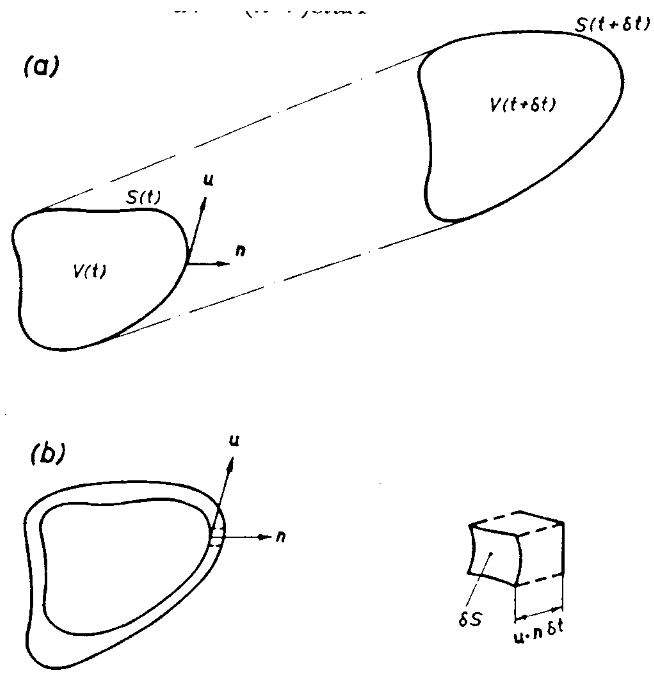

Figure 1.1. (a) Arbitrarily shaped system at times *t* and *t*+*δt*. (b) Superposition of the control volume at these times showing an element *δV* of the volume change

Thus, the volume integral on the right hand side of (1.4F) can be replaced by a surface integral over the surface of the system:

$$\int_{V_s(t+\delta t)-V_s(t)} \alpha(t+\delta t)dV = \int_{A_s(t)} \alpha(t+\delta t)(\vec{n}\cdot\vec{v})\delta t dA \qquad (1.4H)$$

Now dividing by δt and taking the limit:

$$\lim_{\delta t\to 0}\left[\frac{1}{\delta t}\int_{A(t)} \alpha(t+\delta t)(\vec{n}\cdot\vec{v})\delta t dA\right] = \int_{A_c} \alpha(t)(\vec{n}\cdot\vec{v})dA \qquad (1.4I)$$

This can be converted to a volume integral by the use of Gauss's theorem:

$$\int_{A_c} \alpha(t)(\vec{n}\cdot\vec{v})dA = \int_{V_c} \nabla\cdot(\alpha\vec{v})dV \qquad (1.4J)$$

Now it is possible to sum the terms given on the right hand side of (1.4E) and (1.4J) to yield the final form of equation (1.4B):

$$\frac{d}{dt}\int_{V_s(t)} \alpha(t)dV = \int_{V_c}\left[\frac{\partial\alpha}{\partial t} + \nabla\cdot(\alpha\vec{v})\right]dV \qquad (1.4K)$$

Equation (1.4K) is the Reynolds Transport Theorem (RTT).

1.5 Index Notation

In the study of transport phenomena it is often convenient to use index notation to refer to coordinates and components of vectors and tensors in a Cartesian three-space. For example, the position vector, $\vec{r}$, can be written in terms of coordinates x_i as:

$$\vec{r} = x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3 \qquad (1.5A)$$

where $\vec{e}_i$ are unit vectors. The dummy index i can take on any one of the values 1,2 and 3, and its use indicates a typical component.

Einstein summation convention, by which when an index is repeated in a single term it is to be summed over the three possible values (1, 2 and 3), will also be used here. Thus, equation (1.5A) can be written as:

$$\vec{r} = x_i \vec{e}_i \text{ (or } \sum_{i=1,2,3} x_i \vec{e}_i \text{)} \qquad (1.5B)$$

This notation convention can be used to write the definition of the material derivative as:

$$\frac{D\alpha}{Dt} \equiv \frac{\partial \alpha}{\partial t} + v_i \frac{\partial \alpha}{\alpha x_i} \qquad (1.5C)$$

The RTT can also be written with the use of index notation and the summation convention as:

$$\frac{d}{dt} \int_{V_s(t)} \alpha(t) dV = \int_{V_c} \left[\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x_i}(\alpha v_i) \right] dV \qquad (1.5C)$$

Note that the appearance of the index i twice in the last term on the right implies a summation of three terms with i set equal to 1,2 and 3.

1.6 Stress Tensor

The concept of shear and normal stresses is important in the derivation of the governing conservation equations and will be reviewed only briefly in this section.

Consider a cube of fluid as shown in Figure 1.2. Stresses are denoted by σ_{ij}. The first subscript denotes the face on which the stress acts and the second subscript denotes the direction of the stress on the positive face of the cube. On the negative face the stresses are equal in magnitude but opposite in direction to those on the positive face (the face is indicated as the plane perpendicular to the axis of the subscript, e.g., the 1 face is perpendicular to the x or x_1 axis). The idea of positive and negative faces is indicated also in Figure 1.2.

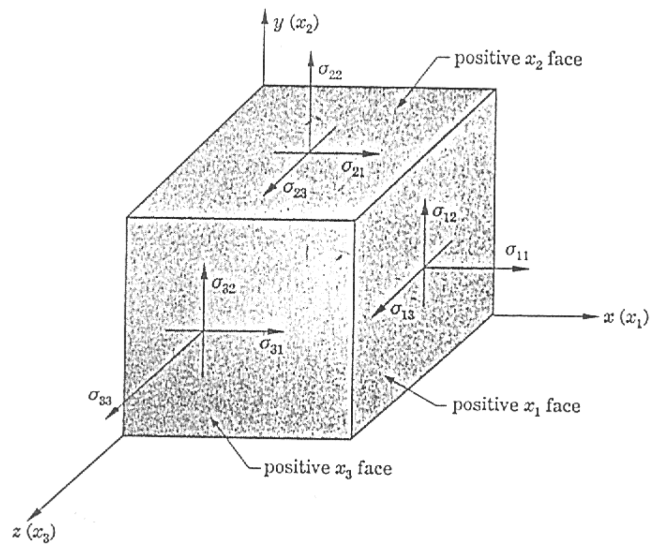

Figure 1.2. Stresses at a point in space. The positive faces are shown – the opposite ones are the negative faces.

The principle of conservation of angular momentum also requires that the stress tensor be symmetric.

1.7 The System and the Laws of Thermodynamics

To discuss about the laws of thermodynamics it is more convenient to employ the concept of the system. The examined thermodynamic system is a small volume element whose boundary moves with the velocity of the fluid $\vec{v}$. Since the system boundary moves with the fluid velocity no mass crosses its surface and the system is "closed" in the usual thermodynamic sense. The first law of thermodynamics for a closed system is:

{Change in total energy of system} = {Heat added to system} – {Work done by system} (1.7A)

Considering the change to occur in a small interval of time δt (thus dividing the above equation by δt) and assuming local equilibrium[3] it is possible to write the above equation as a rate equation:

{Rate of change of total energy of system} = {Rate of heat addition to system} –
{Rate at which work is done by system} (1.7B)

A. The total energy is the sum of the kinetic and internal energies. Per unit mass the internal energy is u and the kinetic energy is $v^2/2$ where $v^2 = \vec{v} \cdot \vec{v}$. The left hand side of (1.7B) then becomes:

[3] The local equilibrium assumption states that all equilibrium thermodynamic properties (e.g., internal energy, enthalpy, chemical potential) are assumed to exist and to be the same functions of local state variables as they are at equilibrium.

$$\{\text{Rate of change of total energy of system}\} = \frac{D}{Dt}\left[\int_{V_s} \rho\left(u + \frac{v^2}{2}\right)dV\right] \qquad (1.7C)$$

Using the RTT:

$$\frac{D}{Dt}\left[\int_{V_s} \rho\left(u + \frac{v^2}{2}\right)dV\right] = \int_{V_c} \rho\left[\frac{D}{Dt}\left(u + \frac{v^2}{2}\right)\right]dV \qquad (1.7D)$$

B. The rate at which heat enters the system is written in terms of the heat flux vector $\vec{q}$. Heat enters the system by crossing the system surface area A_s:

$$\{\text{Rate of heat addition to system}\} = \int_{A_s} \vec{n} \cdot (-\vec{q})dA \qquad (1.7E)$$

The quantity $\vec{n}$ is the outwardly directed unit normal vector to the surface of the system. The negative sign is needed to obtain the heat entering the system. Application of Gauss's theorem to (1.7E) yields:

$$\int_{A_s} \vec{n} \cdot (-\vec{q})dA = -\int_{V_s} \nabla \cdot \vec{q}\, dV \qquad (1.7F)$$

C. The rate at which work is done is the dot product of a force and a velocity. Two types of forces act on the system: body forces and surface (contact) forces. Body forces are those proportional to either the volume or mass of the fluid and comprise of forces involving action at a distance, e.g., gravitational, electrostatic, centrifugal, and coriolis force. Surface forces are, on the other hand, those acting on an element of fluid through its bounding surfaces and describe the influence of the surrounding fluid on the fluid element under consideration. Surface forces per unit area are called stress.

Using the symbol $\vec{G}$ for the body force per unit mass:

$$\{\text{Rate of work done by system}\} = \int_{V_s} \rho \vec{G} \cdot \vec{v}\, dV \qquad (1.7G)$$

The force on the surface of the system is $(\vec{n} \cdot \underline{\sigma})dA$. The rate at which work is done by the system is:

$$\{\text{Rate of work done by system}\} = \int_{A_s} (\vec{n} \cdot \underline{\sigma}) \cdot \vec{v}\, dA \qquad (1.7H)$$

where $\underline{\sigma}$ is the stress tensor. Using Gauss's theorem and the equality involving the gradient operator, symmetrical tensor and a vector (not proved here):

$$\int_{A_s} (\vec{n} \cdot \underline{\sigma}) \cdot \vec{v}\, dA = \int_{V_s} (\vec{v} \cdot \nabla \cdot \underline{\sigma} + \underline{\sigma} : \nabla \vec{v}) dV \qquad (1.7I)$$

The enthalpy per unit mass, h, is related to the internal energy per unit mass by:

$$h = u + \frac{p}{\rho} = u + p\tilde{v} \qquad (1.7J)$$

or,

$$dh = du + \frac{dp}{\rho} - \frac{p}{\rho^2} d\rho \qquad (1.7K)$$

Temperature can be introduced through:

$$dh = c_p dT + \left[\tilde{v} - T \left(\frac{\partial \tilde{v}}{\partial T} \right)_p \right] dp \qquad (1.7L)$$

and defining the thermal expansion coefficient, β, as:

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right) \qquad (1.7M)$$

Equation (1.7L) can thus be written as:

$$dh = c_p dT + \left[\frac{1}{\rho} - T \left(\frac{\partial \frac{1}{\rho}}{\partial T} \right)_p \right] dp \qquad (1.7N)$$

1.8 Constitutive Equations

To relate the fluxes (momentum, heat and mass fluxes) to macroscopic variables of interest in transport processes appropriate constitutive equations are needed. They are discussed in turn in the following subsections.

1.8.1 Newton's Law of Viscosity

Consider a fluid contained between two large parallel plates of area A, which are everywhere separated by a very small distance Y (why Y has to be very small?) Assuming that the system is initially at rest but that at time t = 0 the lower plate is set in motion in the x direction at a constant velocity V. As time passes the fluid gains momentum and finally the steady state velocity profile shown in Figure 1.3 is established.

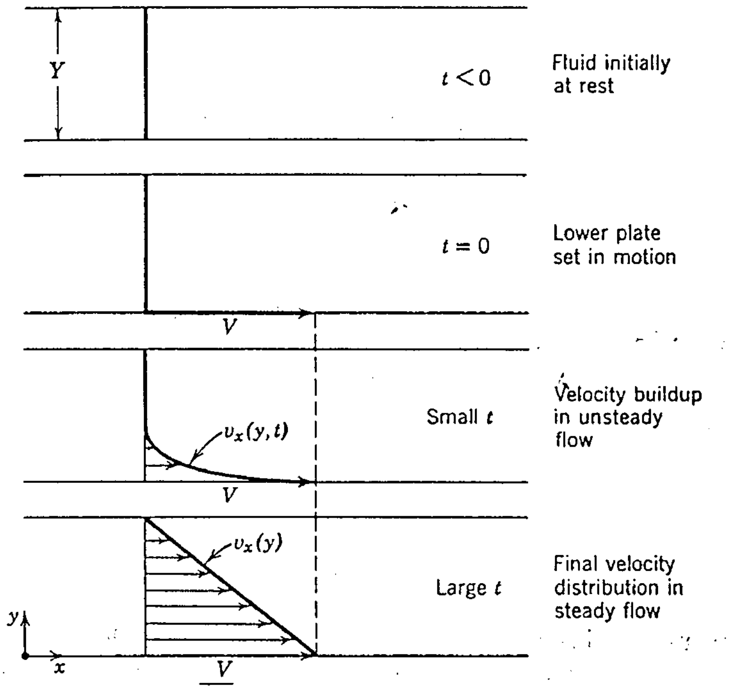

Figure 1.3. Buildup to steady laminar velocity profile for the plane Couette flow.

When this final state of steady motion has been attained a constant force *F* is required to maintain the motion of a lower plate. This force may be expressed as follows:

$$\frac{F}{A} = \mu \frac{V}{Y} \tag{1.8A}$$

That is the force per unit area is proportional to the velocity decrease in the distance *Y*. The constant of proportionality μ is called the viscosity of the fluid. We can write (1.8A) using tensor notation as:

$$\tau_{yx} = -\mu \frac{\partial v_x}{\partial y} \tag{1.8B}$$

This states that the shear force per unit area (shear stress) is proportional to the negative of the local velocity gradient. This is known as the Newton's law of viscosity. Fluids that behave in this fashion are termed Newtonian fluids. In some cases it may be useful to have a symbol to represent the viscosity divided by the density of the fluid:[4]

$$\nu = \frac{\mu}{\rho} \tag{1.8C}$$

[4] This is especially useful when analogies between momentum, energy and mass transport are to be made.

in which ν is called the kinematic viscosity. Newton's law of viscosity in various coordinate systems is summarized in Bird et al. (1960, pp. 88-90) or Bird et al. (2002, pp. 843-844).

1.8.2 Fourier's Law of Heat Conduction

An analogy between momentum and energy transport yields an equation of the similar form for heat conduction:

$$q_y = -k \frac{\partial T}{\partial y} \tag{1.8D}$$

This equation is the one-dimensional form of Fourier's law of heat conduction valid when $T = T(y)$. It states that the heat flux by conduction is proportional to the temperature gradient. This equation is valid for the heat conduction process in solids, liquids, and gases.

In an isotropic medium it is possible to write (1.8D) as:

$$\vec{q} = -k \nabla T \tag{1.8E}$$

This is the three-dimensional form of the Fourier's law. It states that the heat flux vector $\vec{q}$ is proportional to the temperature gradient ∇T and is oppositely directed. Thus in an isotropic medium heat flows by conduction is in the direction of steepest temperature descent. For some materials, e.g., fibrous food materials, this situation may not be valid. In those cases anisotropic thermal conductivity values must be used. Equation (1.8E) must then be rewritten as:

$$\vec{q} = -\underline{k} \nabla T \tag{1.8F}$$

where $\underline{k}$ is the thermal conductivity tensor. In addition to the thermal conductivity a quantity known as the thermal diffusivity, α, is widely used in the heat transfer literature:

$$\alpha = \frac{k}{\rho c_p} \tag{1.8G}$$

where c_p is the specific heat capacity at constant pressure of the material.

As noted earlier conduction is the only mode of heat transfer considered in these notes. Fourier's law of heat conduction in various coordinate systems is listed is also listed in Bird et al. (1960, p. 317) or Bird et al. (2002, p. 845).

1.8.3 Fick's Law of Mass Diffusion

In equation (1.8B) the viscosity μ is defined as the proportionality factor between momentum flux and velocity gradient. In equation (1.8D) the thermal conductivity k is defined as the proportionality factor between heat flux and temperature gradient. Now it is possible to define the mass diffusivity D_{AB} in a binary system in an analogous fashion:

$$\vec{J}_A = -D_{AB} \nabla C_A \qquad (1.8H)$$

This is Fick's first law of diffusion written in terms of the molar flux $\vec{J}_A$. This equation states that species A diffuses in the direction of decreasing concentration of A just as heat flows by conduction in the direction of decreasing temperature.

A number of other mathematically equivalent statements of Fick's first law have appeared in the literature and are summarized, for example, in Bird et al. (1960, p. 502) or Bird et al. (2002, p. 537). Fick's first law of binary diffusion in various coordinate systems is also summarized in Bird et al. (2002, p. 846).

It is interesting to note that by using kinematic viscosity, thermal diffusivity and mass diffusivity the three constitutive equations have surprisingly similar form. The student is encouraged to verify this similarity as an assignment.

References

1. Bird, R.B., Stewart, W.E., Lightfoot, E.N., 1960, **Transport Phenomena**, Wiley, New York.
2. Bird, R.B., Stewart, W.E., Lightfoot, E.N., 2002, **Transport Phenomena**, **2nd Edition**, Wiley, New York.

Suggested Readings

1. Bejan, A., 1984, **Convection Heat Transfer**, Wiley, New York.
2. Bejan, A., 1995, **Convection Heat Transfer, 2nd Edition**, Wiley, New York.
3. Currie, I.G., 1993, **Fundamental Mechanics of Fluids, 2nd Edition**, McGraw-Hill, New York.
4. Hughes, W.F., Brighton, J.A., 1999, **Schaum's Outline of Theory and Problems of Fluid Dynamics**, McGraw-Hill, New York.
5. Kakac, S., Yener, Y., 1995, **Convective Heat Transfer, 2nd Edition**, CRC Press, Boca Raton.
6. Panton, R.L., 1995, **Incompressible Flow, 2nd Edition**, Wiley, New York.