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Modelling of time-dependent behavior of sandstones for deep underground openings

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Abstract Time-dependent behavior of geomaterials, is of a great importance for the design of deep underground structures as for tunnels and waste disposal cavities. In order to ensure long time safety (for several centuries) of these, a good prediction of delayed strains and damage occurrence is required. For this purpose, a new viscoplastic model has been implemented in a two-dimensional explicit finite-difference program. This rheological model is defined by constants which are material dependent. For our purpose, several primary creep and relaxation laboratory tests have been performed on dry sandstone rock specimens. Primary creep tests are used to determine the model parameters, then the experimental relaxation curves are compared to the numerical program response for the same strain path. The whole study leads to confirm the relaxation curves as a well predicted behavior response of the rock, when the creep model constants are known. Finally, the delayed convergence modelling trend of an underground circular tunnel excavated in the same material is discussed.

Key words : rock; sandstone; creep; relaxation; rheological; model; time; convergence; tunnel; program; finite-difference; geomaterial

1. Introduction

Rocks exhibit delayed behavior, depending on their composition, on the temperature, as well as on the induced stress amplitude. Among these, limestones, marbles, sandstones and halite mechanical behaviors are known to be time dependent.

Many important project site works on different rock foundations have proved that creep amplitude depends on the stresses induced by the building structure, as reported earlier by B. Schneider on shales of Hendrik Verwoerd dam at 1200 m depth [Schneider, 1967]. Halite, used for underground disposal is known to behave strongly within time (fig. 1).

Tunnels bored in deep rock formations exhibit delayed convergence with more efforts on the lining, and a section reduction while excavating without supporting. Indeed, convergence measurements are a helpful monitoring for the tunnel supporting and lining [Egger, 1990].

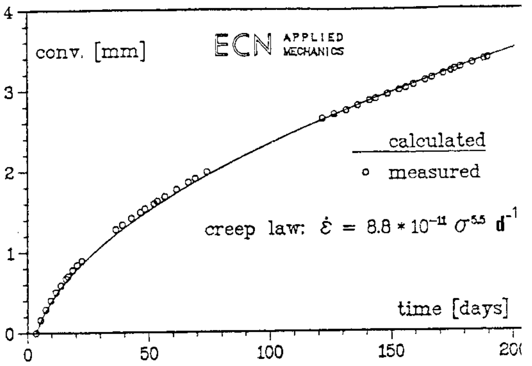

Fig. 1 : Convergence measurements in a ϕ315 mm borehole across the Asse rock salt mine, at 1000 m of depth [ROLNIK, 1984]

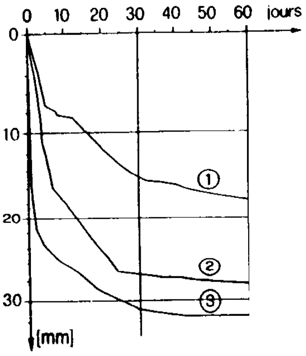

Fig.2: Convergence change in the Nordstern (Ruhr) mine, at 1100 m of depth: (1) 25 cm shotcrete thick, 5 m long bolts, (2) 15 cm, 3m, (3) as (2) and delayed lining [EGGER, 1990]

Fig. 2, reported by P. Egger, shows a 1100 m depth convergence and lining interaction on the Ruhr coal-mining works.

Dry marl of the M'jaara earth dam in northern Morocco was submitted to laboratory test under constant triaxial stress tensor. The behavior has turned out to failure after a primary creep duration (figure 3). In the Gibraltar strait fixed link, Malabata experimental gallery convergence monitoring, at 147 m depth, for more than 3 years, shows how saturated clay flysches behaves strongly with time (fig. 4).

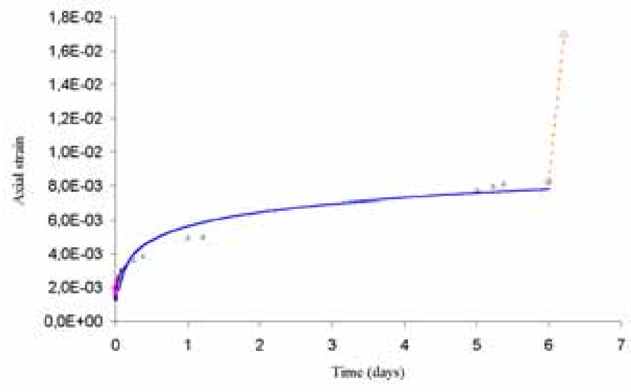

Fig.3: Triaxial creep of a dry marl specimen from M'Jaara dam foundation (σ_3=4.3 MPa σ_1=8 MPa) [EJJAAOUANI and al., 1995]

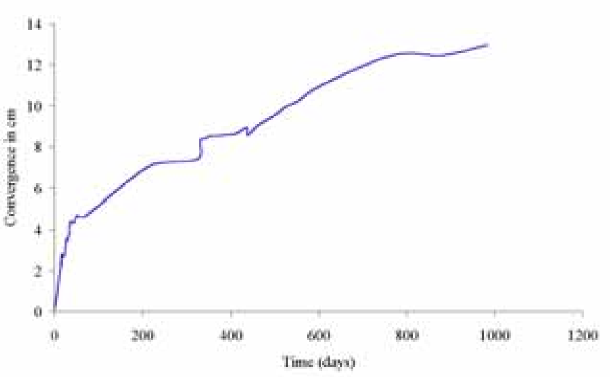

Fig.4 : Convergence in Malabata experimental gallery at 147 m depth for Gibraltar strait fixed link [Hammoud, 1998]

Here are some examples showing the time influence on the rock behavior. For engineering needs, accurate delayed modelling behavior allows sensible predicting calculations, for best economy and safety purposes while excavating or tunneling in these geomaterials.

2. Rock delayed behaviour

Viscous geomaterial rheology consists in: -a flow rule which links together stress and strain tensors, -a viscoplastic failure surface as a stress function with hardening parameters and -a viscous threshold which separates elastic from elastoviscoplastic behavior.

As a flow rule for a specific material has to fit both experimental and in situ responses, the creep and relaxation testing paths, must be well described by the former. Indeed, these experimental data are essential for the determination of the law mechanical parameters. Secondly, the validation of the law is to be brought up on different manners, on experimental quasistatic testing paths, or/and by fitting in situ monitored cases, as in situ tunnel convergence data. For the latter, the law must previously be implemented in a finite element or alternate numerical method code.

The flow rule seems to be different for soft soils and rocks. For the latter, we are considering the Norton-Lemaître law, as a particular case of the Perzyna viscoplastic formulation , as reminded herein.

The mechanical model presented is based on the Perzyna's overstress-viscoplasticity theory [Perzyna, 1966], formulated for isotropic rate-sensitive plastic materials which gives incremental constitutive equations for transient (or primary) creep phase. The basic assumption is that the strain rate can be decomposed into an elastic and inelastic (irreversible) part :

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij} \quad (1)$$

where the inelastic strain, which combines viscous and plastic effects, is given by the following relation:

$$\dot{\varepsilon}^{vp}_{ij} = \gamma \langle \Phi(F) \rangle \frac{\partial G}{\partial \sigma_{ij}} \quad (2)$$

with :

γ	the material viscosity coefficient
G	the viscoplastic potential, as a function of the stress tensor,
Φ	the flow rule defined by $:< \Phi(F) > = ½ \, [\Phi(F) + \lvert\Phi(F)\rvert]$
$\lvert \;\; \rvert$	the classical absolute value
F	the static yield function
$f(\sigma_{ij}, \varepsilon^{vp}_{kl})$	classical plastic function, assumed to be representing the von Mises criteria

The scalar function F includes the overstress concept through the classical plastic function f and the strain hardening parameter κ, which depends only on the equivalent viscoplastic strain, involving the assumption of isotropic hardening phenomena. The function F has the following form :

$$F(\sigma_{ij},\varepsilon_{kl}^{vp}) = \frac{f(\sigma_{ij},\varepsilon_{kl}^{vp})}{\kappa(\varepsilon_{vp})} - 1 \text{ with } \quad \varepsilon_{vp} = \int_0^t \left(\frac{2}{3} \dot{\varepsilon}_{ij}^{vp} \otimes \dot{\varepsilon}_{ij}^{vp} \right)^{\frac{1}{2}} d\tau \quad (3)$$

For our purposes, we assume the associatively of the behavior (e.g. $f \equiv G$). This assumption infers that no volume changes can occur while viscoplastic deformations are developing. This fact, classically assumed for the rock delayed behavior, is being confirmed experimentally for the rock tested hereafter in this paper.

In order to come out onto a practical and representative tool, Φ and the strain hardening parameter κ are introduced as proposed by Lemaitre [Lemaitre and Chaboche, 1996]:

$$\Phi(F) = (F+1)^n \quad \text{and } \kappa(\varepsilon_{vp}) = (\varepsilon_{vp})^{-m/n} \quad (4)$$

The inelastic strain rate (which is assumed to combine viscous and plastic effects) is then given by the following relation:

$$\dot{\varepsilon}_{ij}^{vp} = \frac{3}{2} A.q^{n-1}.(\varepsilon_{vp})^m s_{ij} \quad (5)$$

with: s_{ij} the deviatoric part of the stress tensor,
q the second invariant of the stress tensor,

The Norton-Lemaitre law identifies the material behavior through three parameters :

A the viscosity coefficient of the material,
n the stress exponent,
$m^* = -m/n$ the strain hardening exponent.

3. Sandstone laboratory data fitting

Norton-Lemaitre formulation leads to a power creep law, defined by convenience in a principal coordinates plan by:

$$\varepsilon_{ij}^{vp} = a I q^{\beta} t^{\alpha} \quad (6)$$

where:

I is the identity tensor
a, β and α are related to A, m and n by the following relations:

$$\alpha = \frac{1}{1-m} \qquad \beta = \frac{n}{1-m} \qquad a = \left(\frac{A}{\alpha}\right)^{\alpha}$$

3.1. Axial creep strain fitting

Several uniaxial creep tests were carried out on a homogenous dry sandstone, under various uniaxial stresses [Sahli, 1988]. Fitting the experimental measures gives up the average values of α, β and a. Since both axial and lateral strains are simultaneously monitored within time, one can easily determine how is the volume variation during creep. Results lead to conclude on a constant volumetric creep, as assumed before. Detailed results for the rock tested are given herein, in fig. 5 to 8:

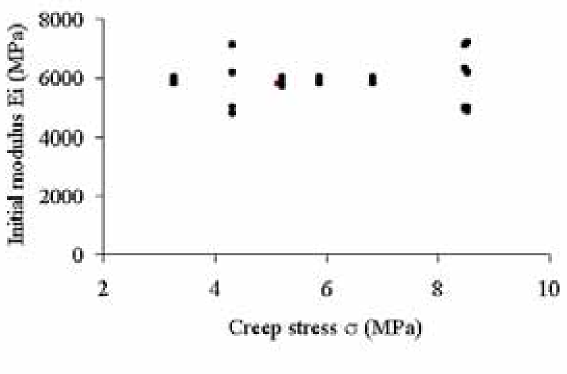

Fig.5 : Instant creep loading modulus of dry sandstone from Villarlod

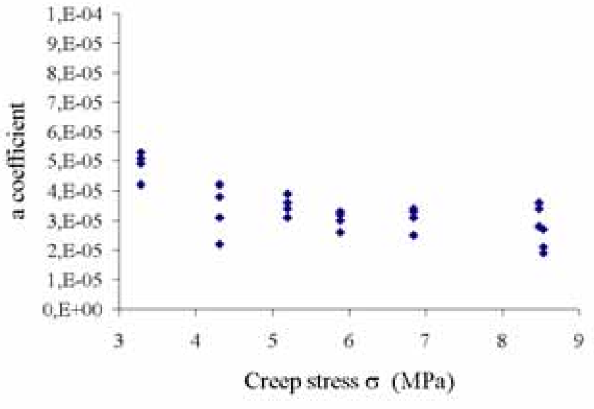

Fig.6 : Axial coefficient a [days] in Norton-Lemaitre time dependent model, for dry sandstone from Villarlod

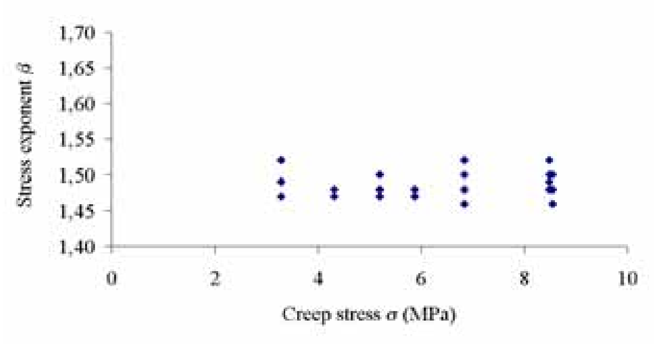

Fig.7 : Stress exponent β in Norton-Lemaitre time dependent model, for dry sandstone from Villarlod

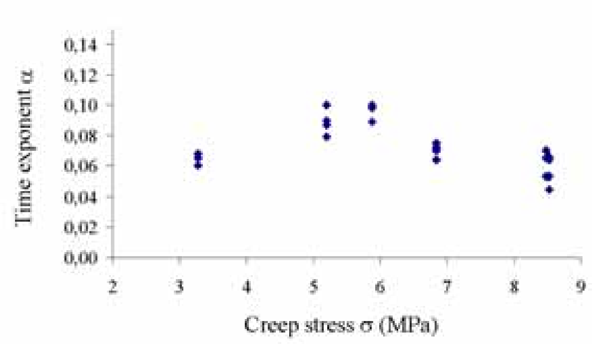

Fig.8 : Time exponent α in Norton-Lemaitre time dependent model, for dry sandstone from Villarlod

Table 1 shows how the model parameters vary around the average values:

Table 1 : Norton-Lemaitre power Law constants, given by fitting creep tests on a sandstone

	E_i [MPa]	**a** [days]	α	β	**A**[s]	**n**	**m**
Average value	5.85 10^3	3.35 10^{-5}	7.36 10^{-2}	1.49	1.38 10^{-52}	21.11	-13.20
Variation coefficient	9.6%	25.5%	21.4%	1.2%	364%	21.9%	23.4%

Hereafter are some conclusions about these experimental values:

- The initial loading modulus of all specimen vary between 4900 MPa and 7200 MPa, with a variation coefficient of 10% , due to the specimen dispersion
- Creep stress exponent β is equal to 1.5 for all specimen
- a coefficient of the time dependent model is quiet equal to 3.4 10^{-5} (for time in days) with 26% dispersion variation
- finally, the time exponent α is lightly varying with creep stress , around the average value of 0.074, as indicated fig.8, from approximately 0.05 to 0.1

3.2. Lateral creep strain fitting

The lateral creep strains were also monitored for all specimens. These data let us determine the volume variation during sandstone creep. In table 2, we are giving the global coefficient ν^*, the initial coefficient ν_{init} , and the viscoplastic coefficient ν_{vp}.

Table 2 : volumetric variation coefficients, obtained by fitting creep tests on a sandstone

	ν^*	ν_{init}	ν_{vp}
Average value	0.46	0.39	0.48
Variation coefficient	16.2 %	21.9 %	17.6 %

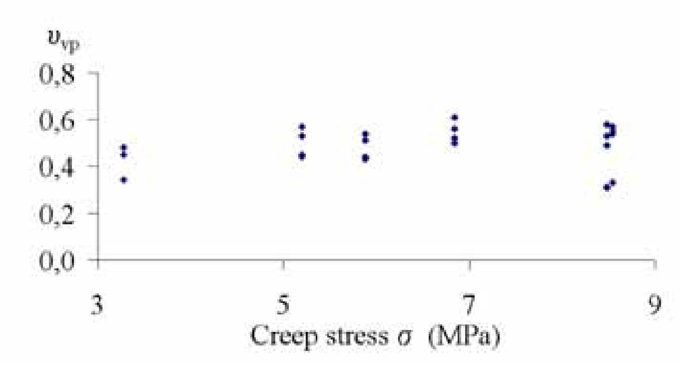

Fig.9 : viscoplastic volumetric coefficient ν_{vp}, for dry sandstone from Villarlod

Fig. 9 gives up the experimental viscoplastic ν_{vp} coefficients, obtained for different specimens tested. These values confirm the assumption of delayed behavior tested with no significant volume change, during the process.

4. Finite difference program FLAC simulation

Power time-dependent model implementation in the FLAC 3.4 numerical code, let us predicting the delayed behavior of the rock tested, along different paths as creep and relaxation loadings, with an adequate accuracy [Boidy and al.]

4.1. Uniaxial specimen creep simulation

To validate the new time-dependent model implemented in the explicit finite difference program, the creep tests are simulated for the same paths conducted in laboratory.

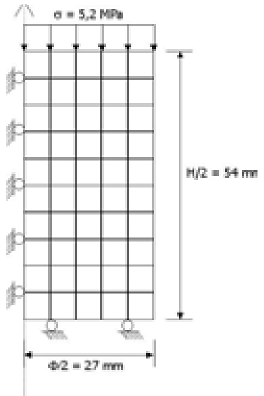

Fig.10 :Specimen mesh discretisation and boundary conditions

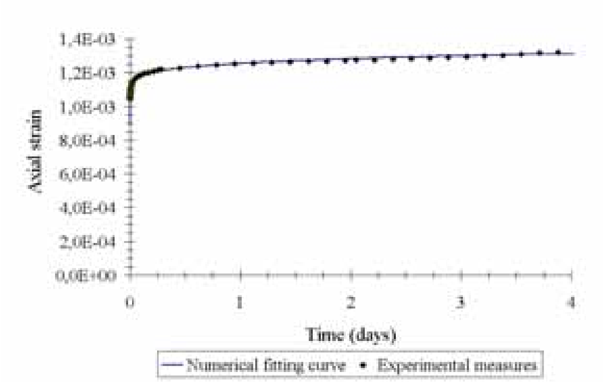

Fig.11 : creep simulating curve of specimen Nr.10

Fig. 10 gives the specimen mesh introduced for calculations (54 mm diameter, 108 mm height) and fig.11 shows the numerical fitting curve of experimental points.

4.2. Specimen relaxation simulation

Uniaxial and triaxial relaxation tests on sandstone specimens were carried out. All the specimen were loaded at constant strain rate of 1.4 10^{-3} s^{-1}, as far as the relaxation strain value is reached. As mentioned by Lemaitre and Chaboche, while the relaxation behavior and the creep one are closely dependent. Assuming the decomposition of the total strain, as written in eq. 1, the relaxation path of the specimen means the uniaxial tensor rate is given by

$$\dot{\sigma} = -\dot{\varepsilon}^{vp} \cdot E \quad (7)$$

$\dot{\varepsilon}^{vp}$ is the varying delayed strain rate given by the Norton-Lemaitre time dependent formulation. Within the relaxation path, primary viscoplastic strain rate is positive and tends to vanish after a while (for $\alpha \in]0,1]$), the stress rate is always negative and should also vanish at the same amount of time. In other words, apart from the time dependent model considered, the primary creep behavior can only generate a primary relaxation behavior and vice versa.

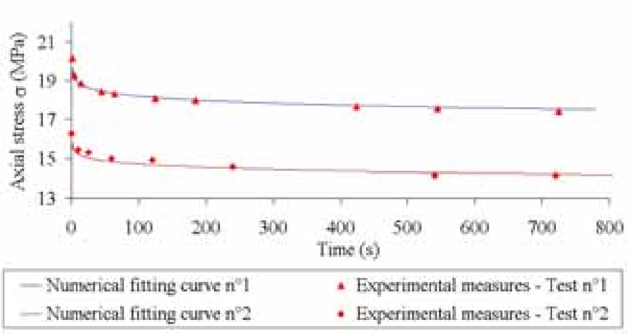

Fig.12 : uniaxial relaxation of dry sandstone specimen Nr.7 (at constant strain : Test n°1 ε_0=6.82 10^{-3}, Test n°2 ε_0=5.46 10^{-3})

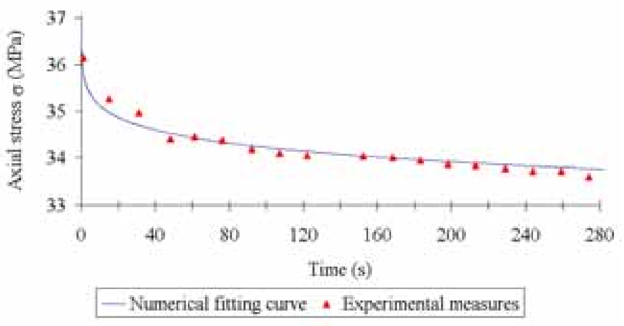

Fig.13 : triaxial relaxation of dry sandstone specimen Nr.8 (at constant confining stress σ_3=3 MPa and constant strain ε_0=6.82 10^{-3})

Fig. 12 and 13 show the accuracy of the numerical prediction obtained, comparing to experimental results.

4.3. Delayed convergence of a deep tunnel facing

We simulate the theoretical convergence as a response of a deep tunnel with 5 m diameter, excavated at 300 meters depth in an homogenous sandstone rock mass, with a 2.2 dry density, out of the water table. At depth, we assume an isotropic stress distribution, so both vertical and horizontal directions are symmetrical axes. Initial modulus is 6000 MPa, Initial expansion coefficient is 0.44. Viscous model parameters are: $a=3.4\ 10^{-5}$ [days], $\alpha=0.1$ et $\beta=1.48$.

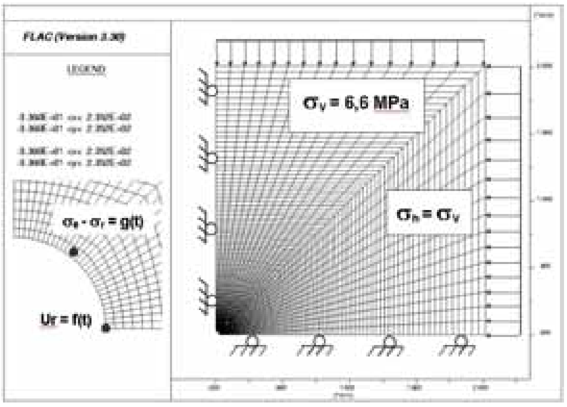

Fig.14 : Tunnel mesh discretisation and boundary conditions

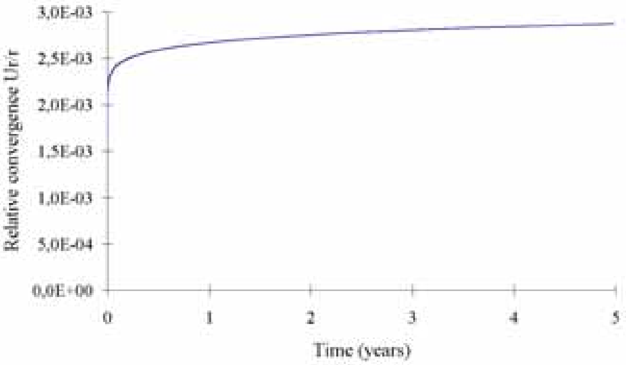

Fig.15 : Time-dependent Convergence of a deep tunnel facing in a viscous media

Fig. 14 shows the FLAC media discretisation mesh and fig.15 the theoretical convergence given by the simulation over 3 years. This means the convergence trend does not keep up with the creep law which is a power law as written on equation 6.

5. Conclusion

This paper deals with the viscous behavior of a dry homogenous sandstone. The behavior seems to fit the viscoplastic Norton-Lemaitre law. Law parameters have been determined by fitting the creep experimental curves of different specimens from the rock mass. While the time exponent of the creep law vary slightly with creep stress value, the other parameters can be considered constant with a reasonable variation coefficient due to specimen dispersion. Assuming the decomposition of the rate of global strain as a sum of the elastic and viscoplastic strain rates, delayed media behavior can then be accurately predicted. This was done here by implementing the time-dependent model behavior in a numerical finite difference program. The rock response was then obtained for the relaxation path and the delayed tunnel convergence occurring at the facing of the material opening. For the latter case (fig.15), the tunnel convergence does not follow the creep law trend.

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