C.A. Rubtsov

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In the book the common principles of a reflexive method $\omega-$reflections of mathematical objects and represented exposition of mathematical objects of a new nature are given. Alongside with brief performance infinite of a spectrum of new numbers (in particular, $\Delta-$ and $\triangle-$ numbers, "located" for $(-\infty)$), the $\omega-$images of integro-differential objects are adduce. The concept of formation local and global $\omega-$spectra of various objects is generated.

The book is calculated on a wide circle of the readers interested by mathematics. In her a problematics of reconstruction of foundations of this science from a position of the theory of objects availabl . is explained, are indicated mark of new creative solutions in various sections of mathematics. The book opens an immense field for independent searching and research.

The reviewers: prof. M.F. Caliagin
(МВОКУ, Moscow)
prof. V.G. Sichenko
(БГПУ, Belgorod)

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“I would like to convert your attention to a history of mastering by the mathematical fact. It always history of *slow* and long collective work…”

(From the letters H. Lebesgue).

THE FOREWORD

Almost all material was written by me as separate fragments still in 1987-1988 years. under an impression of the several read books till mathematician and papers from an *enormous* collective transactions under a title “the mathematical encyclopedia”. In further, gaining a speciality on a computer science and more and more taking a great interest her, I did not discover time for systematization of the obtained new mathematical facts and seriously was not engaged in development of primary reasons… Nevertheless, some fragments have directed on publication. I count as the debt to *thank* the experts (as a rule, scientists with the world name), rendering to me moral support by the kind letters). The professor John Ewing (Editor-elect "The American Mathematical Monthly", Indiana University, USA) marked (July 19, 1991): “*…we find your papers quite interesting…*”. Him has joined (February 27, 1992) prof. J.H.Sampson (Editor "American Journal of Mathematics", Johns Hopkins University, USA): *“… the referee found interesting results in your manuscript… ”* and offered to continue work. Large desire to understand all a material in whole have expressed the scientists: prof. E.M. Moskal (Univ. of Waterloo, Canada), prof. M. Mignotte (Univ. L.Pasteur, France), prof. V.S. Varadarajan (Univ. of California, USA), prof. Franco Tricerri (Italia), prof. H. Wilf (Univ. of Pensylvania, USA), prof. Takasi Nagahara (Okayama University, Japan) and others.

From the Russian scientists most carefully material studied prof. A.V. Cherniavski, prof. N.D. Vorobiev, prof. M.F. Caliagin, prof. V.G. Sichenko. The last two are the official reviewers of the given monography. A part of the notes, made by them, the author has taken into account at final preparation of the manuscript for printing.

In 1993 I have visited Institute of mathematics in Strasbourg by the invitation of the director prof. M. Mignotte, and per 1994 was invited to an International Congress of mathematicians (ICM 94), which passed in Zurich (Switzerland). Has received the personal invitations from the President of a Congress of the professor H. Carnal - Univ. Of Berne and term of Organizational Committee of a Congress of the professor Erwin Bolthausen - Univ. Of Zurich. Certainly, all this has sped up writing the monography. I shall notice, that already after the publication in 1989 in a magazine “The cybernetics” of the first fragment has arisen necessity of more detailed illumination of a material.

At first was decided to write the manuscript in conventional mathematical style, by explaining a material from *more mature* positions, carefully by processing it. However, soon it has become obvious that to be guided it is necessary on a wide circle of the readers interested by mathematics, as *in the book the foundations of mathematics are affected*. The account should be availablis and nonstandart. It has updated the purpose of the book: *to wake in the reader desire of creativity*, for what it was required each chapter to complete by problematic abstract. And whether it is unimportant the reader will engage in serious scientific work on this or that problem or simple will execute amateurish exercise till mathematician. Principal: *process of creativity undoubtedly will bring sufficing…*

Concerning a content, I shall notice, that the spectrum of problems, affected in the book, is rather extensive: since an axiomatics of numbers of a new nature (Δ-numbers) and finishing ω-reflections of the whole sections of mathematics. Naturally, are possible both local, and global objections and variances with a position of the author. However, “The true – daughter of time” (“Temporis filia veritas ”) is known.

Constantin Rubtsov.

In L. Euler in the letter to d'Alembert stated, that $\ln n$ has indefinitely many values, which everyone are complex numbers, behind an elimination of a case, when $n > 0$, then one of values is real. But the d'Alembert up to an extremity of the life (he, as well as L. Euler, has died in 1783) has not agreed with this statement, assume, that $\ln(-1) = 0$

$$\left(-1 = \frac{1}{-1} \Rightarrow \ln(-1) = \ln(+1) - \ln(-1) \Rightarrow \right.$$
$$\left. \Rightarrow 2 \cdot \ln(-1) = \ln(+1) = 0 \Rightarrow \ln(-1) = 0\right).$$

This dispute is considered completed for the benefit of the Euler. However, …

… In an outcome of a research of an operation “easier” than addition the extension of a field $\mathbf{R}$ real numbers [51, 52, 55] is obtained. Actually, realizing reflection of a field $\mathbf{R}$ concerning “improper” number $(-\infty)$ (negative “actual” infinity), the reflexive image of a field $\mathbf{R}$, designated as Δ_0 – field of numbers of a new nature (Δ-numbers)[1] is generated. In the book the brief exposition of a set Δ-numbers is given and is shown, that $k^{\Delta a} = -\left(k^a\right)$, where $(a, k) \in \mathbf{R}$, $k \neq 1$, and $\ln(-1) = \Delta 0$ and $2 \cdot \ln(-1) = \ln(+1) = 0$. (To d'Alembert, to some extent, was right!) …

By entering representation about ω-spaces and ω-reflections of mathematical objects, the author has received a lot of mathematical objects of a new nature (MONN). In the book *some* outcomes of this research are reduced only. In a fundamental principle it there was an attempt to synthesize methods of *object-oriented programming* and *analogies*. In the total the common scheme of exposition of new objects is generated. And, as a *individual* example of this scheme, in the book the separate fragments of modelling MONN are represented.

Research and the development MONN is realized at three adjacent levels:

a) immediate study MONN for interior development of mathematics and rationalization of a solution some it of the tasks;

b) support of the first level with the purposes of the extension of areas of

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[1] The supporters only of “potential” infinity should recollect, what still Newton did not recognize negative numbers and, naturally, did not consider zero as number, as, till it to the estimation, “number: a ratio of one magnitude to other that sorts accepted for unit”, and Leibnitz supposed, that “the ratios, in which previous or succeedent less than zero are not true. ”

In the present book $(-\infty)$ is introduced as a intersection of sets Δ_0 and $\mathbf{R}$:

$$\Delta_0 \cap \mathbf{R} = \{-\infty\}.$$

admissible values of functions;

c) research of possibilities of a new method called reflexive in a plane of deriving of concrete practical outcomes by modernizing of known methods of searching of optimum solutions.

From MONN in work the mathematical spaces, operation, number, functions, derivatives, integrals and some other are circumscribed. All of them have grown out of *elementary* ω-reflections with an exponential function of connection.

CHAPTER 1. SPACES AND OPERATIONS

§ 1.1. Introductory notes

Mathematical space in the given work is understood as all set of mathematical objects, connections and ratios between them. For example, if ω_0 is a well-known mathematical space, $\omega_0 = \bigcup_{k=1}^{n} \Im_k$, where $\Im_k$ – various sections of mathematics containing, in turn, final or infinite a set of mathematical objects. Not only separate objects, ratio between them and different blocks[2] from objects and ratios as the formulas, theorems, mathematical receptions, but also *all* in the whole *mathematical space* ω_0 (or other space $\omega_j = \bigcup_{p=1}^{m} \Im_p$), consisting from these objects, ratios and blocks is considered too as mathematical object, i.e. $\omega_0 = \left\{ \mathbf{R}, \mathbf{C}, \ldots, +, -, \times, \div, \ldots, \mathbf{U}, \frac{\partial^n U}{\partial x_i^{\,n}}, \int_r U\, dx_i, \ldots \right\}$, where $\mathbf{R}, \mathbf{C}$ – sets of real, complex and other numbers; $\mathbf{U}, \frac{\partial^n U}{\partial x_i^{\,n}}, \int_r U\, dx_i, \ldots$ – accordingly set of every possible functions (U), derivatives $\left(\frac{\partial^n U}{\partial x_i^{\,n}}\right)$, integrals $\int_r U\, dx_i, \ldots$ and other mathematical objects, ratios and blocks, *is global mathematical object*. It is natural, that, in this case, it is expedient to investigate a spectrum of enclosures of sets of objects and to describe their hierarchy. However, with the purpose of simplification of an account in work this problem is not considered. At last, elementary are taken only from all ratios, i.e. the algebraic operations, operands and which outcomes are included into the same set. Some algebraic operations of a new nature as components of infinite of a set of algebraic operations are considered below. Examples such are δ-operations forming

[2] As the block we shall name logically connected final (compact set) or infinite (area) a set of plants and ratios.

infinite a set Δ_n – of operations easier of addition, and also set of reflexive operations generated on a set of functions, underlying ω-reflection. In the present work two reflexive operations obtained because of an exponential function ω-reflection are investigated only.

§ 1.2 Concepts ω-reflections

The world, enclosing us, consists of *objects* described by *properties*, which allow to distinguish one object from others.

The properties of objects can be subdivided on two aspects:

1) *Interior* - connected with a structure of object;

2) *Exterior* - difiniendums a structure of other object, which constituent is given.

In global understanding of the essence of object of its property too are elementary objects.

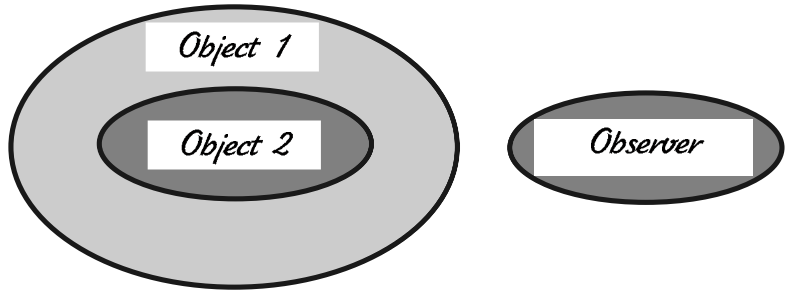
In the real world we deal with objects having infinite number of properties. For work with these objects it is enough to be limited only to dominating properties, i.e. what interest us or render influence to properties, interesting for us. The passage from object with infinite number of properties to object with a final package of properties is thus ensured. Let's mark, that *such replacement is not necessary means approximation: it is possible to realize* ***absolutely exact****, replacement, operating with a finite number of properties*.

Let's explain earlier told in following simples theorems[3]. For the greater clearness of the definitions the concept of the *observer* is introduced. The observer is a "abstract" object (more often in it of a role the man appears), observable (researched) not included in a property. The observer perceives observable object, but any influence to its properties does not render.

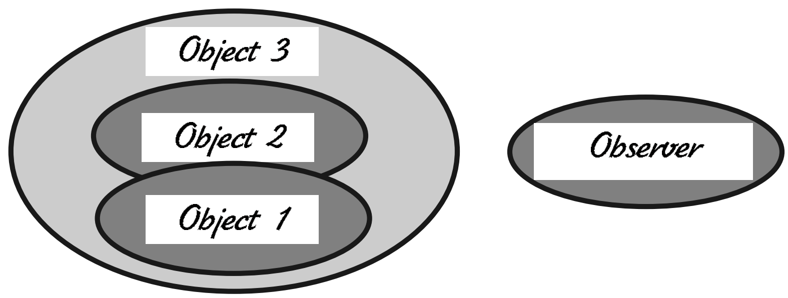
The theorem 1. If two objects have a finite number of identical interior and exterior properties, these objects are not than other, as the same object.

The theorem 2. If one object comprises another, the observer interested by interior properties of the second (firmware) object, can not pay attention to existence of the first object.

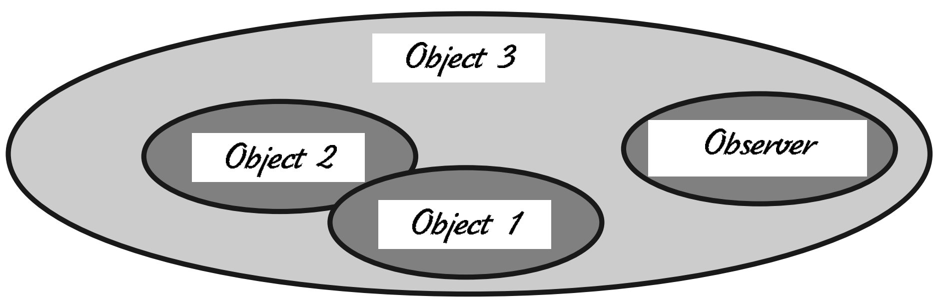
[3] The proofs of these isolated theorems in the present book are absent. It is supposed to reduce them in the *common theory of objects*. The indexing of the introductory theorems does not correspond to uniform indexing of the theorems of the book.



The theorem 3. If there are two objects with identical interior properties located inside third, they have various exterior properties. (If the exterior properties coincide, according to the theorem 1 it is the same object).



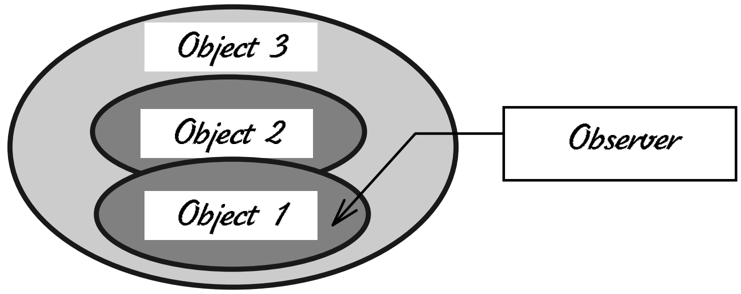
From reduced clearly, that the observer will not remark a difference between objects 1 and 2, as it is interested in their interior properties.



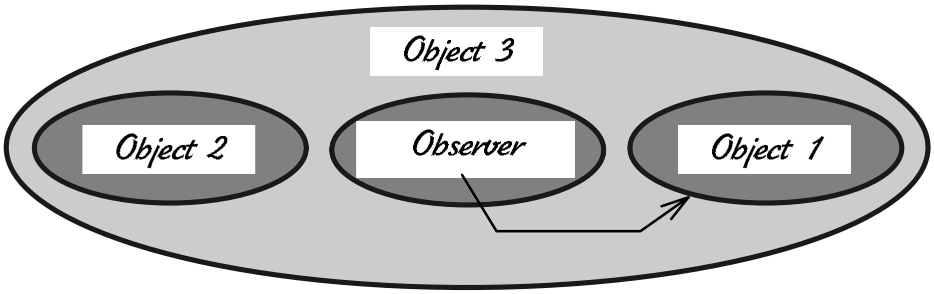
The observer, being interested interior properties of objects 1 and 2, observes also exterior properties of these objects concerning object 3, it will see distinction (on exterior properties) between them.

The interesting effect can be received if to observe object 2 of object 1, i.e. to reflection object 2 in object 1 in view of differences in exterior properties.

The theorem 4. If the objects 1 and 2 have identical interior properties, but various exterior, concerning the observer from object of 1 interior property of objects 1 and 2 are various.



The theorem 5. For observation of object 2 of object 1 with identical interior properties, taking into account differences in exterior properties concerning object 3, it is enough to know difference of one interior property of object 1 from a similar observable property of object 2. All remaining displayed interior properties can be received from this difference.



Here it is necessary to notice restriction on the theorem 5. *Completely to reflection object 2 in object 1 it is possible only in the event that all reflections interior properties of object 2 are available in interior properties of object 1.* In remaining cases we can speak about reflection of *separate* properties.

The theorem 6. If two objects have identical interior properties, but various exterior and at reflection of the second object in first any mirror property of object 2 coincides with any property of object 1, also remaining mirrors properties are available in object 1.

The above mentioned theorems were by a basis at shaping theoretical basis ω-reflection. Let's make some available explanations. *The method ω-reflections of any objects (material and abstract) is a* ***system*** *of operations of* ***separation*** *of properties (functions) of objects and* ***transformation*** *of these objects in an outcome of a modification of these properties (functions).*

Orb of application of the above-stated method are all *global* space (material and abstract), understood as infinite a set of objects and connections between them.

Global assigning of a method ω-reflection this selective separation and modification of functions describing any object. Therefore the identification of object and it transformation is realized. The method ω-reflections is a most known mode of interaction of “observer” and “object”.

Passing to the mathematical interpretation of a method ω-reflections, we shall formulate dominating postulates of the *mathematical concept* ω-*reflections*:

– in similar on a structure abstract mathematical ω-spaces the reflections of objects are admissible one-to-one;

– the operation of reflection from one space in another reduces in transformation of object in the correspondence with some function of connection;

– exist infinite spectra ω-images of mathematical objects.

Let's explain it on an actual material.

Let all known mathematical objects (number, functions, functionals, derivatives, integrals etc.) belong to space ω_0. As both objects and connections between them it is possible to arrange on appropriate levels (for example, operation of multiplication to count the additions are higher an order, and the exponentation accordingly is higher than multiplication), that, using conditionality and relativity of any hierarchy, instead of ω_0 we shall enter concept of some abstract space ω_1. It is identical ω_0 on an *interior* structure, i.e. “observer” were in ω_1, will not remark any differences from ω_0. However, all objects and connections between them are transformed at reflection them from ω_1 in ω_0, that underlines distinction of these spaces, despite of *interior* analogy. Let *function of connections* (f_c) between ω_1 and ω_0 is exponential $f_c = k^x$ $(k > 1)$. Any number a $(a \in \mathbf{R})$ from ω_1 will be reflection in ω_0 as k^a, i.e.

$a \setminus \omega_1 \to \omega_0 \setminus k^a$, where k – *factor of image.*

Image in ω_0 operation of addition $(+)$ will be multiplication:

$$(a+b) \setminus \omega_1 \to \omega_0 \setminus k^{a+b} = k^a \cdot k^b,$$

where k^a and k^b – images in ω_0 numbers a and b, noted in ω_1.
The multiplication is transformed to new operation "$\odot$", which we shall name as reflexive multiplication:

$$a \cdot b \setminus \omega_1 \to \omega_0 \setminus k^{a \cdot b} = \left(k^a\right)^{\log_k k^b} = k^a \odot k^b.$$

Similarly, $(a-b) \setminus \omega_1 \to \omega_0 \setminus k^{a-b} = k^a / k^b$,

$$\frac{a}{b}\setminus\omega_1\to\omega_0\setminus k^{a/b}=\left(k^a\right)^{1/\log_k k^b}=k^a\triangle k^b,$$

where $\triangle$ – *reflexive division.*

The exponentation too turns to new operation $a^{\to b}=\underbrace{a\odot a\odot\ldots\odot a}_{(\log_k b)\in\mathbf{Z}}$. An image in ω_0 taking the logarithm in ω_1 we shall designate $\mathrm{ilog}_a b$. Introducing labels $(:a)=1/a$ and $\triangle a=k\triangle a$, $(k,a)\in\mathbf{R}$, $k^a\in\mathbf{Z}$ and $k\neq 1$, we shall receive the identified label of numbers on ω-factor. Really, let in ω_1 the negative number $-a$, $(a\in\mathbf{R}_+)$ that is given

$$-a\setminus\omega_1\to\omega_0\setminus k^{-a}=\frac{1}{k^a}=:k^a\in\mathbf{R}_f,$$

where $\mathbf{R}_f$ – set of fractional numbers of a type $1/b\ (:b)$.

$$\left(:k^a\right)\setminus\omega_1\to\omega_0\setminus k^{1/k^a}=\triangle\left(k^{k^a}\right).$$

At shaping space ω_1 image in ω_0 object from ω_1 are noted with feature from below: $\overset{1}{\underline{a}}$ or it is simplis $\underline{a}$ $\left(\overset{1}{\underline{a}}\equiv\underline{a}\equiv k^a\right)$. Then, obviously, $\underline{a}\cdot\underline{b}=(\underline{a+b})$, and at $a<b$ $\underline{a}/\underline{b}=(\underline{a-b})=\left(\underline{-(b-a)}\right)=:(\underline{b-a})$, $\underline{a}^{\log\underline{b}}=\underline{a}\cdot\underline{b}=\underline{a}\odot\underline{b}=\underline{b}\odot\underline{a}$.

The theorem 1.1 *Operations "$\odot$" is algebraic operation possessing properties alternatively, commutability and associativity on a set* $\mathbf{R}_+$. [*)]

Proof. The fact of a membership of operation "$\odot$" to algebraic is quite obvious, as any operands (x,y) and outcome $\left(x^{\log y}\right)$ enter into the same set $(\mathbf{M})$, i.e. $\left(x,y,x^{\log y}\right)\in\mathbf{M}$. (For Example, if $x>0$, and $y<0$, how it will be clear from

[*)] It is uneasy to prove the theorem 1.1. for $(x,\ y,\ z)\in\ \mathbf{R}$.

chapter 2, $\log y = \Delta \log|y|$, and $x^{\Delta \log|y|} = -x^{\log|y|} \in \mathbf{R}_{-}$, i.e. $\left(x, y, x^{\log y}\right) \in \mathbf{R}$.

a). We shall prove alternatively of operation "$\odot$":

Let $x, y \in \mathbf{R}_{+}$, i.e. $x > 0$ and $y > 0$. Then

$$(x \odot y) \odot y = \left(x^{\log_k y}\right) \odot y = \left(x^{\log_k y}\right)^{\log_k y} = x^{\log^2 k(y)}$$

On the other hand, $x \odot (y \odot y) = x \odot \left(y^{\log_k y}\right) = x^{\log_k y^{\log_k y}} =$

$= x^{\log^2 k(y)}$, i.e. $(x \odot y) \odot y = x \odot (y \odot y)$, that proves *right* alternatively of operation "$\odot$".

$(x \odot x) \odot y = \left(x^{\log_k x}\right) \odot y = \left(x^{\log_k x}\right)^{\log_k y} = x^{(\log_k x) \cdot (\log_k y)}$. On the other hand, $x \odot (x \odot y) = x \odot \left(x^{\log_k y}\right) = x^{\log_k x^{\log_k y}} = x^{(\log_k y) \cdot (\log_k x)}$, i.e. $(x \odot x) \odot y = x \odot (x \odot y)$, that proves *left* alternatively of operation "$\odot$".

b). We shall prove a commutability of operation "$\odot$": $x \odot y = x^{\log_k y}$, $y \odot x = y^{\log_k x}$. After a taking the logarithm of these expressions, we shall receive:

$$\log_k (x \odot y) = (\log_k y) \cdot (\log_k x),$$
$$\log_k (y \odot x) = (\log_k x) \cdot (\log_k y),$$

i.e. $\log_k (x \odot y) = \log_k (y \odot x)$. Whence $x \odot y = y \odot x$, as was to be shown.

c). We shall prove an associativity of operation "$\odot$" at $(x, y, z) \in \mathbf{R}_{+}$:

$(x \odot y) \odot z = \left(x^{\log_k y}\right) \odot z = \left(x^{\log_k y}\right)^{\log_k z} = x^{(\log_k y) \cdot (\log_k z)}$, and

$x \odot (y \odot z) = x \odot \left(y^{\log_k z}\right) = x^{\log_k y^{\log_k z}} = x^{(\log_k z) \cdot (\log_k y)}$, i.e. $(x \odot y) \odot z = x \odot (y \odot z)$, that proves an associativity of operation "$\odot$".

In summary we shall remark, that by analogy with ω_1 it is possible to generate any space ω_i. Moreover, thus it is possible to use other function of connection. However, in the given monography, as already it was specified, the exponential function of connection was applied more often.

§1.3 Relations because of operations ⦿ and ◬

Let's reduce some relations, which follow from a sense of operations ⦿ and ◬: $(:a)⦿(:b) = a⦿b$ $(a>1, b>1)$ by analogy with $(-a)\cdot(-b) = a\cdot b$ $(a>0, b>0)$, $a⦿k = a$, $a^{\to k} = a$, $a^{\to 1} = k$, $a⦿1 = 1$, $a⦿0 = 0\,(a\neq 0)$, $a^{\to b_1}⦿a^{\to b_2} = a^{\to(b_1\cdot b_2)}$, $a^{\to b_1}◬a^{\to b_2} = a^{\to\left(\frac{b_1}{b_2}\right)}$, $\left(a^{\to k_1}\right)^{\to k_2} = a^{\to(k_1⦿k_2)}$, $\sqrt[\to b]{a} = a^{\to(◬b)}$, $\sqrt[\to b]{a^{\to b}} = a$, $a^{\to(k_1◬k_2)} = \sqrt[\to k_2]{a^{\to k_1}}$, $\operatorname{ilog}_a a^{\to b} = b$, $\operatorname{ilog}_a b = ◬\operatorname{ilog}_b a$, $\operatorname{ilog}_a(k_1⦿k_2) = \operatorname{ilog}_a k_1\cdot\operatorname{ilog}_a k_2$, $\operatorname{ilog}_a(k_1◬k_2) = \dfrac{\operatorname{ilog}_a k_1}{\operatorname{ilog}_a k_2}$, $\operatorname{ilog}_a b^{\to k} = k⦿\operatorname{ilog}_a b$, $\operatorname{ilog}(b)◬\operatorname{ilog}\left(a^{\to k}\right) = ◬k⦿\operatorname{ilog}_a b$, $\operatorname{ilog}_a b = \operatorname{ilog}_c b◬\operatorname{ilog}_c a$, $a⦿b^c = a^c⦿b$, $a⦿(b\cdot c) = (a⦿b)\cdot(a⦿c)$, $a⦿\left(\dfrac{b}{c}\right) = \dfrac{a⦿b}{a⦿c}$, $(a\cdot c)◬b = (a◬b)\cdot(c◬b)$, $a◬b^c = \sqrt[c]{a}◬b$ etc.[4]

Let's reduce, part of proofs.

1. We shall prove a property $\left(a^{\to k_1}\right)^{\to k_2} = a^{\to(k_1⦿k_2)}$: $\left(a^{\to k_1}\right)^{\to k_2} =$

$$= \underbrace{(a⦿\ldots⦿a)}_{\log_k k_1}⦿\underbrace{(a⦿\ldots⦿a)}_{\log_k k_1}⦿\ldots⦿\underbrace{(a⦿\ldots⦿a)}_{\log_k k_1} = \underbrace{(a⦿a⦿\ldots⦿a)}_{\log_k k_1\cdot\log_k k_2},$$

$\log_k k_1\cdot\log_k k_2 = \log_k k_1^{\log_k(k_2)} = \log_k(k_1⦿k_2)$, i.e.

$$\left(a^{\to k_1}\right)^{\to k_2} = \underbrace{a^{\to k_1}⦿a^{\to k_1}⦿\ldots⦿a^{\to k_1}}_{\log_k k_2} = a^{\to(k_1⦿k_2)}.$$

[4] It is uneasy to notice similarity of operations ⦿ multiplication, ◬ and division if to realize a replacement $\{+\,;-\}\to\{\,\cdot\,;\div\,\}$.

At a proof the definition of operation $a^{\to b}$ is used only (look "Labels").

2. We shall discover $\sqrt[\to b]{a}$, by designating $\sqrt[\to b]{a} = a^{\to x}$. Then $\left(a^{\to x}\right)^{\to b} = a$, i.e. $\left(a^{\to x}\right)^{\to b} = a^{\to (x \odot b)}$. Let $(x \odot b) = k$. Whence $x = k \triangle b = \triangle b$. So,

$$\sqrt[\to b]{a} = a^{\to (\triangle b)}.$$

3. For installation of equality $\sqrt[\to b]{a^{\to b}} = a$ we shall designate $\sqrt[\to b]{a^{\to b}} = a^{\to x} \Rightarrow \left(a^{\to x}\right)^{\to b} = a^{\to (x \odot b)} = a^{\to b}$, i.e. $(x \odot b) = b$. In an outcome, $x^{\log_k b} = b \Rightarrow x = k$ and $a^{\to x} = a^{\to k} = \underbrace{a \odot a \odot \ldots \odot a}_{\log_k k = 1} = a$.

4. We shall prove, that $a^{\to (k_1 \triangle k_2)} = \sqrt[\to k_2]{a^{\to k_1}}$. Let's note a right member as $\sqrt[\to k_2]{a^{\to k_1}} = a^{\to x}$. Whence $\left(a^{\to x}\right)^{\to k_2} = a^{\to k_1} \Rightarrow$ $\Rightarrow a^{\to (x \odot k_2)} = a^{\to k_1}$, i.e. $x \odot k_2 = k_1$. Have received $x = k_1 \triangle k_2$, as was to be shown.

5. We shall prove, that $\mathrm{ilog}_a k_1 \cdot \mathrm{ilog}_a k_2 = \mathrm{ilog}_a (k_1 \odot k_2)$. As

$$a^{\to b_1} \odot a^{\to b_2} = \underbrace{(a \odot a \odot \ldots \odot a)}_{\log_k b_1} \odot \underbrace{(a \odot a \odot \ldots \odot a)}_{\log_k b_2} =$$

$$= \underbrace{a \odot a \odot \ldots \odot a}_{\log_k b_1 + \log_k b_2 = \log_k (b_1 \cdot b_2)} = a^{\to (b_1 \cdot b_2)}.$$

Whence $b_1 \cdot b_2 = \mathrm{ilog}_a \left(a^{\to b_1} \odot a^{\to b_2}\right)$. Let's designate $a^{\to b_1} = k_1$, $a^{\to b_2} = k_2$, i.e. $b_1 = \mathrm{ilog}_a k_1$, $b_2 = \mathrm{ilog}_a k_2$ and $\mathrm{ilog}_a k_1 \cdot \mathrm{ilog}_a k_2 =$

$= \mathrm{ilog}_a(k_1 \odot k_2)$, as was to be shown.

The note. Similarly it is possible to prove relations $a^{\to b_1} \vartriangle a^{\to b_2} = a^{\to (b_1/b_2)}$ and $\mathrm{ilog}_a(k_1 \vartriangle k_2) = \dfrac{\mathrm{ilog}_a k_1}{\mathrm{ilog}_a k_2}$.

6. The equality $\mathrm{ilog}_a b^{\to p} = \mathrm{ilog}_a \underbrace{(b \odot b \odot \ldots \odot b)}_{\log_k(p)}$ reduces in a proof of the important property $\mathrm{ilog}_a b^{\to p} = p \odot \mathrm{ilog}_a b$. Really,

$$\mathrm{ilog}_a \underbrace{(b \odot b \odot \ldots \odot b)}_{\log_k(p)} = \underbrace{(\mathrm{ilog}_a b)\cdot(\mathrm{ilog}_a b)\cdot \ldots \cdot(\mathrm{ilog}_a b)}_{\log_k(p)} =$$

$$= (\mathrm{ilog}_a b)^{\log_k(p)} = (\mathrm{ilog}_a b) \odot p = p \odot \mathrm{ilog}_a b.$$

7. We shall transform $a^{\to(\mathrm{ilog}_a b)} = a^{\to(k \vartriangle \mathrm{ilog}_a b)}$ $\Rightarrow$ $b^{\to(\mathrm{ilog}_b a)} = a$.

8. We shall prove, that $\mathrm{ilog}_a b = \mathrm{ilog}_c b \vartriangle \mathrm{ilog}_c a$. As $b =$ $= a^{\to(\mathrm{ilog}_a b)} = \left(c^{\to(\mathrm{ilog}_c a)}\right)^{\to(\mathrm{ilog}_a b)} = c^{\to(\mathrm{ilog}_c a \odot \mathrm{ilog}_a b)}$, $\mathrm{ilog}_c a \odot$ $\odot\, \mathrm{ilog}_a b = \mathrm{ilog}_c b$, i.e. $\mathrm{ilog}_c b \vartriangle \mathrm{ilog}_c a = \mathrm{ilog}_a b$, as was to be shown.

9. Extremely simpl to install equalities $a \odot b^c = a^c \odot b$, $a \odot (b \cdot c) = (a \odot b)\cdot(a \odot c)$, $(a \cdot c) \vartriangle b = (a \vartriangle b)\cdot(c \vartriangle b)$, $a \vartriangle b^c = \sqrt[c]{a} \vartriangle b$:

a) $a \odot b^c = a^{\log_k(b^c)} = (a^c)^{\log_k b} = (a^c) \odot b$;

b) $a \odot (b \cdot c) = a^{\log_k(b \cdot c)} = a^{\log_k b + \log_k c} = a^{\log_k b} \cdot a^{\log_k c} =$ $= (a \odot b)\cdot(a \odot c)$. (Is similar, $a \odot \left(\dfrac{b}{c}\right) = \dfrac{(a \odot b)}{(a \odot c)}$);

c) $(a\cdot c)\triangle b=(a\cdot c)^{1/\log_k b}=a^{1/\log_k b}\cdot c^{1/\log_k b}=(a\triangle b)\cdot(c\triangle b)$;

d) $a\triangle b^c=a^{1/\log_k\left(b^c\right)}=\left(a^{1/c}\right)^{1/\log_k b}=\sqrt[c]{a\triangle b}$ etc.

10. We shall prove, that $\operatorname{ilog}_{a\to p}(b)=\triangle p\odot\operatorname{ilog}_a b$.

Let's transform a right member of equality:

$$\triangle p\odot\operatorname{ilog}_a b=\triangle p\triangle\operatorname{ilog}_b a=\triangle\left(p\odot\operatorname{ilog}_b a\right)=\triangle\left(\operatorname{ilog}_b^{a\to p}\right)=\operatorname{ilog}_{a\to p}(b),$$

as was to be shown.

§1.4 ω-reflections of numbers and functions

Let's consider some aspects ω-reflections of numbers and functions. Let's give the following introductory definitions:

The definition 1.1. The function $y={}^{x}k$ $\left(k\in\mathbf{R},\mathbf{k}\neq\mathbf{1},x\in\mathbf{Z},x>0\right)$[5] is named *superexponential*, if ${}^{x}k=\left.k^{k^{\cdot^{\cdot^{\cdot^{k}}}}}\right\}x$. (By analogy with exponential $y=k^x=\underbrace{k\cdot k\cdot\ldots\cdot k}_{x}$), where $x=\mathrm{var}$, $k=const$.

The definition 1.2. The function $y=\operatorname{slog}_k x$ $\left(k\in\mathbf{R},k\neq1\right)$ is named *superlogarithmic* or *superlogarithmic*, if she is of an inverse superexponential function, i.e. from $y=\operatorname{slog}_k x$ follows $x={}^{y}k$. According to the classical definition this function can accept only whole values $(y\in\mathbf{Z})$.

Note 1. The definition 1.2 seemed contradicts the known theorem: “If the function f is defined, is continuous and will increase (decreases) on gap I, set it values is some gap J, on which there is a function g, the *inverse* function f (is designated $g=f^{-1}$) and possessing properties: g is continuous, will in-

[5] Though in the reduced classical definition the restriction on x $\left(x\in\mathbf{Z},\ x>0\right)$ is entered, we in further shall use quite often extended range of definition of function $y={}^{x}k$, namely $x\in\mathbf{R}$. It is connected to the new nonstandart approach and objects of a new nature, which analysis in the book is absent.

crease (or decreases) on gap J in an assotiation from increase (decrease) function f on gap I."

Actually, if to expand a range of definition of an superexponential function and to suppose $x \in \mathbf{Q}$ ($q \in \mathbf{Q}$, $q = \frac{a}{b}$, where $a, b \in \mathbf{Z}$), the existence $y = \mathrm{slog}_k x$ does not contradict this theorem. Moreover, the further extension of ranges of definition of *superexponential* and *superlogarithmic* functions is obviously possible to us in connection with the extension of a field of real numbers, about what will be told in chapter 2.

Note 2. In the definitions 1.1 and 1.2 the superfunctions of the *first* order are represented. Obviously, there are superfunctions *higher* is ordinal. For example, *superexponential second order* $y = {}^{\underline{x}}k = \left.{k^{\cdot^{\cdot^{\cdot^{k}}}}}^{k}\right\}x$ *superlogarithmic second order* $y = \mathrm{sslog}_k x$.

From $y = \mathrm{sslog}_k x$ follows $x = y\left\{{k^{\cdot^{\cdot^{\cdot^{k}}}}}^{k}\right. = {}^{\underline{y}}k$ etc.

The definition 1.3. The function $y = {}^{n}x$ $(n \in \mathbf{Z})$ is named *superdegree*, if from $y = {}^{n}x$ follows $y = \left.x^{x^{\cdot^{\cdot^{\cdot^{x}}}}}\right\}n$, where $n = const$, $x = \mathrm{var}$.

For example, $y = {}^{3}x = x^{x^{x}}$.

The definition 1.4. The function $y = {}^{n}\!\!\sqrt{x}$ $(n \in \mathbf{Z})$ is named as *the superradical n of a degree*, if she is of inverse superdegree function, i.e. from $y = {}^{n}\!\!\sqrt{x}$ follows $x = {}^{n}y$.

Expanding areas of admissible values of *superdegree* function and *superradical* according to a note 1 ($n \in \mathbf{Q}$, $n \in \mathbf{R}$, $n \in \mathbf{R}_0$ etc.), it is possible to receive more global representation about a behaviour of the above-stated functions.

It is uneasy to notice, that the *basic superlogarithmic identity* is fair:

$$ {}^{\mathrm{slog}_k a}k \equiv \left(\mathrm{slog}_k a\right) * k \equiv a \qquad (1.2)$$

Really, let $\left(\mathrm{slog}_k a\right) * k = x$. Then from the definition 1.2. it is possible to note $\mathrm{slog}_k a = \mathrm{slog}_k x$. Whence $x = a$. Besides it is useful to specify, that

$$^{x}k = k^{\left(^{x-1}k\right)} = k^{\left(^{-1+x}k\right)} = k^{k^{\left(^{-2+x}k\right)}} \quad \text{etc..} \quad \left(^{1+x}k = k^{\left(^{x}k\right)}\right);$$

$$^{\mathrm{slog}_k a}k = k^{\left(^{(-1+\mathrm{slog}_k a)}k\right)} \Rightarrow a = k^{\left(^{(-1+\mathrm{slog}_k a)}k\right)} \Rightarrow \log_k a = {}^{(-1+\mathrm{slog}_k a)}k\,.$$

In further, we shall take advantage of the obvious formulas:

$$^{x}k = k^{\left(^{-1+x}k\right)},\ \left(1+\mathrm{slog}_k a\right) * k = k^{a}, \quad (1.3)$$

$$\left(2+\mathrm{slog}_k a\right) * k = \left(\left(1+\mathrm{slog}_k a\right) * k\right) * k = k^{k^{a}},$$

$$\left(-1+\mathrm{slog}_k a\right) * k = \log_k a\,.$$

Not stopping on a research of various regularities originating in connection with introduction of functions of a new nature (1.1; 1.2; 1.3; 1.4), we shall mark as an example some of most elementary:

$$\log^{b}_{\surd\hat{J}\bar{a}} = \surd\hat{J}\bar{a}\cdot\log_a b;\ \boxed{^{\infty}(:a) = :\left(\surd\hat{J}\bar{a}\right)}\ ^{6}$$

$$\log_a^{\surd\hat{J}\bar{b}} = \frac{1}{\surd\hat{J}\bar{b}}\cdot\log_a b;\ \left(^{\infty}\surd\hat{J}\,\overline{:\left(\surd\hat{J}\bar{a}\right) = :a}\right);$$

$$\log^{b}_{{}^{3}\surd\hat{J}\bar{a}} = {}^{2}\left({}^{3}\surd\hat{J}\bar{a}\right)\cdot\log_a b; \quad (1.4)$$

$$\log^{b}_{{}^{n}\surd\hat{J}\bar{a}} = {}^{n-1}\left({}^{n}\surd\hat{J}\bar{a}\right)\cdot\log_a b;$$

$$^{n}\left({}^{n}\surd\hat{J}\bar{a}\right) = a;\ \mathrm{slog}^{a}_{\left({}^{n}\surd\hat{J}\bar{a}\right)} = n \text{ and others.}$$

The theorem 1.2. *The image of number* a $(a \in \mathbf{R})$ *at reflection* $\backslash\omega_i \to \omega_0\backslash$ *with factor of reflection* k $(k \in \mathbf{R})$ *is defined under the formula* ${}^{(i+\mathrm{slog}_k a)}k$, i.e.

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[6] Let's remind to the reader, that $\surd\hat{J}\bar{a}$ – it is the superradical (instead of radical!).

$$a \equiv \overset{i}{a} \setminus \omega_i \to \omega_0 \setminus {}^{(i+\mathrm{slog}_k a)}k \equiv \left(i + \mathrm{slog}_k a\right) * k \qquad (1.5)$$

Proof. Applying a method of a mathematical induction, we shall consider a series of special cases with factor of image k:

a) Let $i = 0$. Then $a \equiv \overset{0}{a} \setminus \omega_0 \to \omega_0 \setminus a \equiv \left(0 + \mathrm{slog}_k a\right) * k$ (under the formula 1.2 $\left(\mathrm{slog}_k a\right) * k \equiv a$).

b) At $i = 1$ we shall receive $a \equiv \overset{1}{a} \setminus \omega_1 \to \omega_0 \setminus k^a \equiv k^{\left(\mathrm{slog}_k a\right) * k}$ (under the formula 1.2). Whence $k^a \equiv {}^{(1+\mathrm{slog}_k a)}k$ (as $\left(1 + \mathrm{slog}_k a\right) * k =$ $= k^{\left(\mathrm{slog}_k a\right) * k} = k^a$ under the formula 1.3).

c) At $i = 2$ accordingly under the formulas 1.3 we have:

$$\left(1 + \mathrm{slog}_k a\right) * k = k^{\left(1+\mathrm{slog}_k a\right) * k} = k^{k^{\left(\left(\mathrm{slog}_k a\right) * k\right)}} = k^{k^a}.$$

Really, $a \equiv \overset{2}{a} \setminus \omega_2 \to \omega_0 \setminus k^{k^a}$.

d) Let $i = -1$. Then

$$a \equiv \overset{-1}{a} \setminus \omega_{-1} \to \omega_0 \setminus \log_k a.$$

It is uneasy for understanding if to reflection $\log_k a$ from space ω_1 in space on an order below, i.e. in ω_0: $\log_k a \setminus \omega_1 \to \omega_0 \setminus k^{\log_k a} = a$. By replacing indexes of spaces[7], we shall receive:

$\log_k a \setminus \omega_0 \to \omega_{-1} \setminus a$. Whence $a \setminus \omega_{-1} \to \omega_0 \setminus \log_k a$.

On the other hand, by designating $\left(-1 + \mathrm{slog}_k a\right) * k = x$, we shall discover $\mathrm{slog}_k x = -1 + \mathrm{slog}_k a$ or $1 + \mathrm{slog}_k x = \mathrm{slog}_k a$. In an outcome,

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[7] It is possible to show, that the reflection $\setminus \omega_{i+1} \to \omega_i \setminus$ is equivalent to reflection $\setminus \omega_i \to \omega_{i-1} \setminus$ in connection with primary exposition ω – spaces and ω – reflection, i.e.

$$\setminus \omega_{i+1} \to \omega_i \setminus \equiv \setminus \omega_i \to \omega_{i-1} \setminus \text{ etc..}$$

$a=\left(1+\operatorname{slog}_k x\right)*k$ or $k^{\log_k a}=k^{\left(\operatorname{slog}_k x\right)*k}=k^x$, i.e. $x=\log_k a$. Is obtained

$$a\setminus\omega_{-1}\to\omega_0\setminus\left(-1+\operatorname{slog}_k a\right)*k.$$

Let's pass from special cases to common. Let equality 1.5 correctly, i.e.

$$a\equiv\overset{i}{a}\setminus\omega_i\to\omega_0\setminus\left(i+\operatorname{slog}_k a\right)*k.$$

Let's prove, that in this case correctly equality

$$\overset{i+1}{a}\setminus\omega_{i+1}\to\omega_0\setminus\left(1+i+\operatorname{slog}_k a\right)*k$$

As, $\left(1+\operatorname{slog}_k a\right)*k=k^a$, $\operatorname{slog}_k\left(k^a\right)=1+\operatorname{slog}_k a$. Let's image number a from ω_{i+1} In space ω_i:

$$a\setminus\omega_{i+1}\to\omega_i\setminus k^a$$

At last, we shall image number k^a From ω_i In ω_0:

$$k^a\setminus\omega_i\to\omega_0\setminus\left(i+\operatorname{slog}_k k^a\right)*k\text{, i.e.}$$

$$k^a\setminus\omega_i\to\omega_0\setminus\left(1+i+\operatorname{slog}_k a\right)*k.$$

So, $a\setminus\omega_{i+1}\to\omega_0\setminus\left(1+i+\operatorname{slog}_k a\right)*k$, as was to be shown. Therefore, the formula 1.5 is correct.

The lemma 1.1. *Functions* $f\left(x_1,x_2,\dots,x_n\right)$, *being in the object of space* ω_1, *at image in space* ω_0 *will be transformed to function* $k^{f\left(\log_k^{x1},\log_k^{x2},\dots,\log_k^{xn}\right)}$ $\left(k\in\mathbf{R},k\neq 1\right)$, *i.e.*

$$\boxed{f\left(x_1,x_2,\dots,x_n\right)\setminus\omega_1\to\omega_0\setminus k^{f\left(\log_k^{x1},\log_k^{x2},\dots,\log_k^{xn}\right)}}\text{, (1.6)}$$

if k^x- function of connection between spaces ω_1 and ω_0.

Proof. Let ω_1 is connected with ω_0 function $f_i = \mathrm{F}(x) = k^x$ $(k \in \mathbf{R}, k \neq 1)$, where k – factor of image from ω_1 in ω_0. Then number a $(a \in \omega_1)$ will be imaged in ω_0 as $\mathrm{F}(a) = k^a$ $\left(a \setminus \omega_1 \to \omega_0 \setminus k^a\right)$, and the function $f(x)$ by virtue of identity of a structure[8] ω_1 and ω_0 Is transformed in $\mathrm{F}\left(f\left(\mathrm{F}^{-1}(x)\right)\right)$, where $\mathrm{F}^{-1}(x)$ – function inverse $\mathrm{F}(x)$. Really, in ω_0 the numbers have other scale on a comparison with similar numbers in space ω_1 and consequently at passage $\setminus \omega_1 \to \omega_0 \setminus$ the functions $f(x)$ are necessary are to changed by argument, taking into account a new scale, i.e.

$$f(x) \setminus \omega_1 \to \omega_0 \setminus \mathrm{F}\left(f\left(\mathrm{F}^{-1}(x)\right)\right), \text{ or}$$

$$f(x_1, x_2, \cdots, x_n) \setminus \omega_1 \to \omega_0 \setminus k^{f\left(\log_k^{x1}, \log_k^{x2}, \ldots, \log_k^{xn}\right)},$$

as was to be shown.

Corollary. The function $f(x)$ at reflection from some space ω_j in any space ω_i is transformed in $\mathrm{F}\left(f\left(\mathrm{F}^{-1}(x)\right)\right)$, i.e.

$$f(x) \setminus \omega_j \to \omega_i \setminus \mathrm{F}\left(f\left(\mathrm{F}^{-1}(x)\right)\right), \quad (1.7)$$

where $\mathrm{F}(x)$ – function connecting of number of spaces ω_j and ω_i. If number $a \in \omega_j$,

$$a \setminus \omega_j \to \omega_i \setminus \mathrm{F}(a).$$

The theorem 1.3. ***Functions*** $f(x_1, x_2, \cdots, x_n)$***, being in the object of any space*** ω_i***, at image in space*** ω_0 ***will be transformed to function***

[8] Identity of a structure of spaces ω_1 and ω_0 Is understood inspatial as analogy of mathematical objects and connections between them. All numbers, functions, integro-differential objects, theorem etc. inside space ω_1 same, as in ω_0. The modification of objects is observed only at passage them from ω_1 in ω_0 (or from ω_0 in ω_1).

$$\left(i+\operatorname{slog}_k f\left(\left(\left(-i+\operatorname{slog}_k x_1\right)*k\right),\left(\left(-i+\operatorname{slog}_k x_2\right)*k\right), \ldots,\left(\left(-i+\operatorname{slog}_k x_n\right)*k\right)\right)\right)*k,$$

if k^x ***– function of connection between spaces*** ω_i ***and*** ω_0.

The note. It is reduced variant of an *existence* theorem and *uniqueness* one-time $\omega-$reflection.

Proof. Let's prove the theorem by a method of a mathematical induction.

a). Let $i=0$. Then

$$f\left(x_1, x_2, \ldots, x_n\right) \backslash \omega_0 \rightarrow \omega_0 \backslash \mathrm{F}_0=\left(\operatorname{slog}_k f\left(\left(\left(\operatorname{slog}_k x_1\right)*k\right),\right.\right.$$

$$\left.\left.\left(\left(\operatorname{slog}_k x_2\right)*k\right), \ldots,\left(\left(\operatorname{slog}_k x_n\right)*k\right)\right)\right)*k.$$

As $\left(\operatorname{slog}_k x_j\right)*k=x_j$ (according to the formula 1.2), $\mathrm{F}_0=$ $=\left(\operatorname{slog}_k f\left(x_1, x_2, \ldots, x_n\right)\right)*k=f\left(x_1, x_2, \ldots, x_n\right)$. Really, imagining function f from ω_0 in that the space ω_0, we shall receive the same function f.

b). Let $i=1$. Then $f\left(x_1, x_2, \ldots, x_n\right) \backslash \omega_1 \rightarrow \omega_0 \backslash \mathrm{F}_1=(1+$ $\left.+\operatorname{slog}_k f\left(\left(\left(-1+\operatorname{slog}_k x_1\right)*k\right),\left(\left(-1+\operatorname{slog}_k x_2\right)*k\right), \ldots,\left(\left(-1+\operatorname{slog}_k x_n\right)*k\right)\right)\right)*k$.

Let's designate expression $\left(-1+\operatorname{slog}_k x_j\right)*k$ for a_j:

$a_j=\left(-1+\operatorname{slog}_k x_j\right)*k$, $\operatorname{slog}_k a_j=-1+\operatorname{slog}_k x_j \Rightarrow \operatorname{slog}_k x_j=$ $=1+\operatorname{slog}_k a_j \Rightarrow x_j=\left(1+\operatorname{slog}_k a_j\right)*k=k^{\left(\left(\operatorname{slog}_k a_j\right)*k\right)}=k^{a_j}$ (as according to the formula 1.2 $\operatorname{slog}_k a_j * k={}^{\left(\operatorname{slog}_k a_j\right)}k=a_j$). Whence $\log_k x_j=a_j$, i.e. $\mathrm{F}_1=\left(1+\operatorname{slog}_k f\left(\log_k x_1, \log_k x_2, \ldots, \log_k x_n\right)\right)*k$.

By designating $\operatorname{slog}_k f\left(\log_k x_1, \log_k x_2, \ldots, \log_k x_n\right)=S$, we shall receive: $\mathrm{F}_1=(1+S)*k=k^{(S*k)}$. From this expression follows:

$$F_1 = k^{f\left(\log_k x_1, \log_k x_2, \dots, \log_k x_n\right)}$$

$$f\left(x_1, x_2, \cdots, x_n\right) \setminus \omega_1 \to \omega_0 \setminus k^{f\left(\log_k x_1, \log_k x_2, \dots, \log_k x_n\right)},$$

that coincides with an outcome of a lemma 1.1 (with the formula 1.6).

Is admissible, that the formula $F_0 = f\left(x_1, x_2, \cdots, x_n\right) \setminus \omega_i \to \omega_0 \setminus F_i = \left(i + \mathrm{slog}_k f\left(\left(\left(-i + \mathrm{slog}_k x_1\right) * k\right), \left(\left(-i + \mathrm{slog}_k x_2\right) * k\right), \dots, \left(\left(-i + \mathrm{slog}_k x_n\right) * k\right)\right)\right) * k$ is correct. Let's prove, that the following image of function f In this case will be fulfilled:

$$f\left(x_1, x_2, \cdots, x_n\right) \setminus \omega_{i+1} \to \omega_0 \setminus F_{i+1} = \left(1 + i + \mathrm{slog}_k f\left(\left(\left(-1 - i + \mathrm{slog}_k x_1\right) * k\right), \left(\left(-1 - i + \mathrm{slog}_k x_2\right) * k\right), \dots, \left(\left(-1 - i + \mathrm{slog}_k x_n\right) * k\right)\right)\right) * k$$

By designating expression $\left(-1 - i + \mathrm{slog}_k x_j\right) * k$ for b_j:

$$b_j = \left(-1 - i + \mathrm{slog}_k x_j\right) * k, \qquad \mathrm{slog}_k b_j = -1 - i + \mathrm{slog}_k x_j \Rightarrow$$

$$\Rightarrow 1 + i + \mathrm{slog}_k b_j = \mathrm{slog}_k x_j \qquad \Rightarrow \qquad x_j = \left(1 + i + \mathrm{slog}_k b_j\right) * k =$$

$= k^{\left(i + \mathrm{slog}_k b_j\right) * k} \Rightarrow \log_k x_j = \left(i + \mathrm{slog}_k b_j\right) * k$ and $i + \mathrm{slog}_k b_j = \mathrm{slog}_k \log_k x_j$. Whence $b_j = \left(-i + \mathrm{slog}_k \log_k x_j\right) * k$, i.e.

$$F_{i+1} = \left(1 + i + \mathrm{slog}_k f\left(b_1, b_2, \dots, b_n\right)\right) * k =$$

$$= k^{\left(i + \mathrm{slog}_k f\left(b_1, b_2, \dots, b_n\right)\right) * k} = k^{F_i\left(\log_k x_1, \log_k x_2, \dots, \log_k x_n\right)}.$$

The obtained outcome testifies (according to a lemma 1.1 and it to a corollary) that has a place image of function F_i:

$$F_i\left(x_1, x_2, \cdots, x_n\right) \setminus \omega_1 \to \omega_0 \setminus k^{F_i\left(\log_k x_1, \log_k x_2, \dots, \log_k x_n\right)}.$$

By virtue of identity of a structure ω-spaces the last image is equivalent to the following:

$$F_i(x_1, x_2, \cdots, x_n) \backslash \omega_0 \to \omega_{-1} \backslash k^{F_i(\log_k x_1, \log_k x_2, \ldots, \log_k x_n)}.$$

Then $f(x_1, x_2, \cdots, x_n) \backslash \omega_i \to \omega_0 \backslash F_i(x_1, x_2, \cdots, x_n)$,

and $f(x_1, x_2, \cdots, x_n) \backslash \omega_i \to \omega_{-1} \backslash k^{F_i(\log_k x_1, \log_k x_2, \ldots, \log_k x_n)}$.

It is obvious, that the image $\backslash \omega_{i+1} \to \omega_i \backslash$ is similar to image $\backslash \omega_i \to \omega_{i-1} \backslash$ in connection with the above-stated identity ω-spaces (and reference 8), i.e.

$$f(x_1, x_2, \cdots, x_n) \ \backslash \omega_i \to \omega_0 \backslash \ F_i(x_1, x_2, \cdots, x_n) \ \backslash \omega_0 \to$$

$$\to \omega_{-1} \backslash \quad k^{F_i(\log_k x_1, \log_k x_2, \ldots, \log_k x_n)} \Rightarrow$$

$$\Rightarrow \quad f(x_1, x_2, \cdots, x_n) \backslash \omega_i \to \omega_{-1} \backslash k^{F_i(\log_k x_1, \log_k x_2, \ldots, \log_k x_n)}.$$

This expression can be copied, by replacing image $\backslash \omega_i \to \omega_{-1} \backslash$ on identical $\backslash \omega_{i+1} \to \omega_0 \backslash$:

$$f(x_1, x_2, \cdots, x_n) \backslash \omega_{i+1} \to \omega_0 \backslash k^{F_i(\log_k x_1, \log_k x_2, \ldots, \log_k x_n)},$$

i.e. $f(x_1, x_2, \cdots, x_n) \ \backslash \omega_{i+1} \to \omega_0 \backslash \ F_{i+1} = k^{F_i(\log_k x_1, \log_k x_2, \ldots, \log_k x_n)} =$

$= \left(1+i+\mathrm{slog}_k f\left(b_1, b_2, \ldots, b_j, \ldots, b_n\right)\right) * k$,

where $b_j = \left(-i + \mathrm{slog}_k \log_k x_j\right) * k$, as was to be shown.

The theorem is proved.

In summary we shall remark, that, if function of connection between spaces ω_i and ω_j $f_c = k^x$, the function $y = \log_k x$ is invariant of rather these spaces. Really,

$$\log_k x \backslash \omega_i \to \omega_j \backslash k^{\log_k \log_k x} = \log_k x,$$

i.e. $\mathrm{i}\log_k x \equiv \log_k x$.

The properties of function $y = \log_k x$ are saved at image $\backslash \omega_i \to \omega_j \backslash$. For example, let $\omega_i = \omega_{-1}$, and $\omega_j = \omega_0$. Then

$$\frac{\log_k \alpha}{\log_k \beta} \backslash \omega_{-1} \to \omega_0 \backslash \log_k \left(\frac{\log_k k^{\alpha}}{\log_k k^{\beta}} \right) = \log_k \alpha - \log_k \beta .$$

If $\omega_i = \omega_1$, and $\omega_j = \omega_0$,:

$$\frac{\log_k \alpha}{\log_k \beta} \backslash \omega_1 \to \omega_0 \backslash k^{\frac{\log_k \log_k \alpha}{\log_k \log_k \beta}} = (\log_k \alpha) \triangle (\log_k \beta) \text{ and т.д..}$$

So, the ratio $\frac{\log_k \alpha}{\log_k \beta}$ at ω-reflection is saved under condition of a replacement of operation of division on appropriate ω-image of this operation.

§1.5 Problems of a common subject ω-reflections

Summarizing above-stated, we shall mark some problems originating at a realization ω-reflections:

1. Searching common regularities ω-reflections of objects for various functions of connection. Exposition of precise restrictions superimposed on function of connection between ω-spaces.

2. Reduction of examples ω-reflections with other functions of connection (in the present work is represented, as one exponential function of connection k^x, was specified, only).

3. Definition of quotients and universal criterions describing ω-transformation of the graphs of functions. For example, installation of *factor ω-modification* of the elements of length of an arc (in case of function of one variable), square (for function two variable) and hypersquare (for function three and more variables) at $\backslash \omega_i \to \omega_j \backslash$, where $i, j \in \mathbf{Z}$ and $i \neq j$.

4. Deepening of development ω-images because of exponential functions k^x. In particular, research of function $y = {}^{x}k$ and $y = \mathrm{slog}_k x$. Searching of regularities at an operation with superdegrees and superlogs.

Certainly, other problems are possible also, at study of a common problem about ω-images of functions.

CHAPTER 2. THE EXTENSION OF A FIELD OF REAL NUMBERS [50; 52; 55; 56]

§ 2.1. Common notes

Analyzing properties of well-known algebraic operations on N, Z, Q, R - sets natural, whole, rational and real numbers accordingly and considering these operations as ingredients common infinite of a set of operations, the series of the formulas, invariant rather these operations is uneasy to receive. Partially they are reduced in a table 1. Invariant concerning operation of the formula have an independent value, as are regularities in a set of operations, which are mathematical objects and create specific infinite a set. Besides these formulas are carriers of applied orientation. In the present work not accent attention on it, as an invariance - subject of a special research. Let's mark only that fact, what exactly the essence of this research has allowed to open operations of a type " $\circ$ " and "Δ". Last place in a basis of representation about Δ-numbers, which set $\mathbf{R}_\Delta$ has given premises for shaping concept of the extension of a field of real numbers.

§ 2.2 Invariant formulas

We shall reduce examples of the invariant formulas.

2.2.1 $(a,b)\in\mathbf{R}$, $i\in\{1,2,3\}$.

$$
{}^{n}_{2}\Re^{x}_{2} \approx {}^{n-1}_{2}\Re_{2}^{\left({}^{n-2}_{1}\Re_{a}^{\left({}^{n-1}_{3}\Re^{x}_{a}\right)}\right)} \tag{2.1}
$$

The formula (2.1) is a compact entry of a method of iterations (for $n=3$, $i=2$: $\sqrt{c}\approx\frac{c/a+a}{2}$; for $n=4$, $i=2$: ${}^{2}\bar{\jmath}c\approx\sqrt{a\cdot\log_a c}$,

${}^{3}\hat{\surd}\bar{c} \approx \sqrt[3]{a^2 \cdot \log_a \log_a c}$ etc., where $c > 0$, $a \neq \{0,1\}$, a – first approximate value of the superradical).

Note. At an analytical operation with the superradicals it is expedient to use obvious regularities of a type:

$$\frac{\log b}{\log {}^{2}\hat{\surd}\bar{a}} = {}^{2}\hat{\surd}\bar{a} \cdot \log_a b, \quad \frac{\log {}^{2}\hat{\surd}\bar{b}}{\log \mathrm{a}} = \frac{1}{{}^{2}\hat{\surd}\bar{b}} \cdot \log_a b \text{ etc.}$$

2.2.2. $(a,b) \in \mathbf{R}$, $a \neq \{0,1\}$, $n \in \mathbf{N}$, $n = 1$ at $a < b$:

$$ {}^{n}_{1}\mathfrak{R}_a^{\left(k + {}^{n}_{3}\mathfrak{R}_a^{b}\right)} = {}^{n-1}_{1}\mathfrak{R}_a^{{}_{k}b}, \tag{2.2}$$

where k – number of repeated operations $(k \in \mathbf{Z})$.

Example 2.1. a) $b/a = (-1) + \frac{a+b}{a}$, $\log_a b = (-1) + \log_a (a \cdot b)$, $\operatorname{slog}_a b = (-1) + \operatorname{slog}_a a^b$; б) $b/a = 1 + \frac{b-a}{a}$, $\log_a b = 1 + \log_a (b/a)$, $\operatorname{slog}_a b = 1 + \operatorname{slog}_a \log_a b$ etc.

Because of above-stated it is possible to place a series of useful relations. In particular,

$$\lim_{p \to +\infty} {}^{p}\left(\sqrt[e]{e}\right) = e.$$

2.2.3. $a \in \mathbf{R}$, $a \neq \{0,1\}$, $n \in \mathbf{Z}$:

$$ {}^{n}_{2}\mathfrak{R}_a^{a} = {}^{n+1}_{2}\mathfrak{R}_\infty^{a} \tag{2.3}$$

Example 2.2. $\frac{x}{+\infty} = x - x$, $\quad \sqrt[\infty]{x} = \frac{x}{x}$, $\quad {}^{\infty}\hat{\surd}\bar{x} = \sqrt[x]{x}$ etc.

2.2.4.
$$ {}^{n}_{1}\mathfrak{R}_2^{2} = 4. \tag{2.4}$$

2.2.5. $${}_{2}^{3}\Re_{c}^{\left({}_{1}^{3}\Re_{a}^{b}\right)} = {}_{1}^{3}\Re_{a}^{\left({}_{2}^{2}\Re_{c}^{b}\right)} \qquad (2.5)$$

For $n=-1$ the formulas are fair:

$${}_{1}^{-1}\Re_{a}^{a-1^{a}} = a+1, \quad {}_{1}^{-1}\Re_{a}^{a^{a}} = a+2, \quad {}_{1}^{-1}\Re_{a}^{n^{a}} = n+1, \quad (n>a).$$

Because of them, as it was specified, the new operations (in particular, operation " $\circ$ " and inverse to her "Δ") are defined.

Definition 2.1. The operation of an order zero " $\circ$ " names such operation $(n=0)$ between operands a and b, that from $a\circ b=c$ follows $c=a+1$ at $a>b$, $c=a+2=b+2$ at $a=b$ and $c=b+1$ at $a<b$, $a\circ(-\infty)=a$ and $(-\infty)\circ b=b$. This operation is commutative $(a\circ b=b\circ a)$.

Definition 2.2. The solutions of an equation $x\circ a=(-\infty)$ $(a\in \mathbf{R})$ are named Δ-numbers (by analogy $x+a=0$, $a>0$ follows $x=(-a)$ - negative numbers; $x\cdot a=1$, $a\in \mathbf{N}$, $a\neq 1$ follows $x=(:a)$ – fractional).

Exists infinite a spectrum Δ_n-numbers, an origin and which hierarchy easier additions are connected to operations at observance of a duality principle and condition

$$\Delta_n \cap \Delta_{n+1} = \{v_n\},\ v_n = {}_{3}^{n}\Re_{a}^{a}.$$

§ 2.3 Axiomaticses Δ-numbers

We shall consider some axioms Δ-numbers. Thus alongside with well-known arithmetical operations the operations " $\circ$ " and "Δ" are taken into account. Actually, represented axioms is a system of axioms of real numbers interpreted by existence of a set Δ-numbers (Δ_0) and new operations " $\circ$ " and "Δ". Therefore is lower the attempt of generalization of sets Δ-numbers (Δ_0) and real numbers $\mathbf{R}$, i.e. shaping of a set $\mathbf{R}_0 = \Delta_0 \cup \mathbf{R} \equiv \mathbf{R}_\Delta \cup \mathbf{R}$ is made.

1. For anyone's $\Delta a, \Delta b \in \Delta_0$ the relation of the order is defined: $\Delta b > (\Delta a)$ at $b < a$, $\Delta b = (\Delta a)$ at $b = a$ and $\Delta b < (\Delta a)$ At $b > a$, and, if $\Delta a < (\Delta b)$ and $\Delta b < (\Delta c)$, $\Delta a < (\Delta c)$ (transitivity order), where $(a, b, c) \in \mathbf{R}$.

2. For anyone's $\Delta a, \Delta b \in \Delta_0$ the unique number $\Delta c = \Delta a \circ (\Delta b)$, $\Delta c \in \Delta_0$, and $\Delta c = \Delta(a \circ b)$ is defined.

The axiom 1 agrees the definitions is correct for $a, b \in \mathbf{R}_0$.

Note 1. Let's explain, that immediately from the definition of operation " $\circ$ " follows

$$\Delta a \circ (\Delta b) = \begin{cases} \Delta b + 1, & \Delta b < \Delta a, \quad b > a \\ \Delta b + 2, & \Delta b = \Delta a, \quad b = a \\ \Delta a + 1, & \Delta a < \Delta b, \quad b < a \end{cases}$$

3. For anyone's $a, b \in \mathbf{R}_0$ has a place a relation $a \circ b = b \circ a$ (commutability of zero operation).

4. An axiom of signs. For anyone $a \in \mathbf{R}_0$ has a place of an identity $\Delta\Delta \equiv \circ$ $(\Delta(\Delta a) \equiv a)$, $\Delta\circ \equiv \Delta$ and $\circ\Delta \equiv \Delta$ $(\Delta(\circ a) \equiv \Delta a$, $\circ(\Delta a) \equiv \Delta a)$, $\circ\circ \equiv \circ$ $(\circ(\circ a) \equiv a)$. Similarly, $-(-a) = a$, $-(+a) = -a$ etc.

5. For anyone's $a, b \in \mathbf{R}_0$ The unique(sole) number $a + b$, called by a sum of numbers a and b is defined. For anyone's $a, b \in \mathbf{R}$ has a place equality $\Delta a + \Delta b = \Delta b + \Delta a$ (commutability of addition Δ-numbers), and also $\Delta a + (\Delta b + \Delta c) = (\Delta a + \Delta b) + \Delta c$ (associativity of addition Δ-numbers).

Note 2. For anyone's $a, b \in \mathbf{R}$ $\Delta a + \Delta b = a + b$ $\Delta a + b = = \Delta(a + b) = a + (\Delta b)$ (rule of signs), $\Delta a + (\Delta b \circ (\Delta c)) = a + (b \circ c) = = \Delta a + \Delta b \circ (\Delta a + \Delta c) = a + b \circ a + c$ (distributivity of addition Δ-numbers concerning operation of an order zero) etc.

The *rule of signs* " $\circ$ " and " Δ " :

$$\boxed{\begin{array}{c} \circ + \circ = \circ \\ \Delta + \circ = \Delta \\ \circ + \Delta = \Delta \\ \Delta + \Delta = \circ \end{array}}$$

similar $(+)\cdot(+)=(+)$, $(-)\cdot(+)=(-)$, $(+)\cdot(-)=(-)$, $(-)\cdot(-)=(+)$, correctly is real: $\Delta b+(\Delta c)=((-\infty)\Delta b)+(\Delta c)=$ $=(-\infty)+(\Delta c)\Delta(\circ b)+\Delta c=(\Delta(-\infty))\Delta(\Delta(b+c))=(-\infty)\Delta(\Delta(b+c))=$ $=(\Delta(\Delta(b+c)))=b+c=\circ(b+c)$, i.e. $\Delta+\Delta=\circ$, and $\Delta b+\circ c=((-\infty)\Delta b)+$ $+c=(-\infty)+c\Delta b+c=(-\infty)\Delta b+c=\Delta(b+c)$, i.e. $\Delta+\circ=\Delta$ or $\Delta a+\Delta b=\circ(a+b)=a+b=b+a=\circ(b+a)=\Delta b+\Delta a$ (is comparable: $(-a)\cdot(-b)=+(a\cdot b)=a\cdot b=b\cdot a=+(b\cdot a)=(-b)\cdot(-a)$), $\Delta a+(\Delta b+$ $+\Delta c)=\Delta a+\circ(b+c)=\Delta(a+b+c)=\circ(a+b)+\Delta c=(\Delta a+\Delta b)+\Delta c$; $\Delta a+(\Delta b\circ\Delta c)=\Delta a+\Delta(b\circ c)=a+(b\circ c)$ (is comparable: $(-a)\cdot((-b)+(-c))=$ $=(-a)\cdot(-(b+c))=a\cdot(b+c)$) etc. It is necessary to pay attention to the elementary regularities in a hierarchy of operations. For example, $\Delta a+\Delta b=$ $=a+b$, $(-a)\cdot(-b)=a\cdot b$, $(:a)\odot(:b)=a\odot b$, $(\dot{\vartriangle}a)\boxtimes(\dot{\vartriangle}b)=a\boxtimes b$ etc.[9]

On a rule of signs it is possible to note, that $\Delta b+1=\Delta b+(\circ 1)=$ $=\Delta(b+1)$ etc., i.e. $\Delta a\circ(\Delta b)=\Delta(a\circ b)$, that confirms an axiom 2.

Note 3.

a) $\forall\,(a,b,c)\in\mathbf{R}_0$ $(a\circ b)-c=a-c\circ b-c$;

b) $\forall\,(a,b)\in\mathbf{R}_0$ $(a\circ b)-a=(b\circ 2\cdot a)-a$;

c) $\forall\,(a,b,c,d)\in\mathbf{R}_0$ $(a\circ b)+(c\circ d)=(a+c\circ a+d)\circ$ $\circ(b+c\circ b+d)$;

6. For anyone's $a,b\in\mathbf{R}_0$ has a place a relation $a+b=b+a$ (commutability of addition).

7. For anyone's $a,b,c\in\mathbf{R}_0$ has a place a relation $a+(b+c)=$ $=(a+b)+c$ (associativity of addition).

8. There is a number $\Delta 0\in\Delta_0$ such, that $\Delta 0+\Delta a=a$ $(a\in\mathbf{R})$ and for anyone $\Delta a\in\Delta_0$, $\Delta a\neq\Delta 0$ there is $b\in\mathbf{R}$ such, that $\Delta a+b=\Delta 0$, and $b=(-a)$.

9. For anyone's $a,b,c\in\mathbf{R}_0$ has a place a relation $a+(b\circ c)=$ $=a+b\circ a+c$ (distributivity of addition concerning operation of an order zero).

10. For anyone's $\alpha,\beta\in\mathbf{R}$ and $\Delta a,\Delta b\in\Delta_0$ the following relations are fair:

[9] The proof is reduced in the theorem 2.11.

$$\alpha + \Delta a = \Delta(\alpha + a) \in \Delta_0;$$
$$(\alpha \circ \beta) + \Delta a = \alpha + \Delta a \circ \beta + \Delta a;$$
$$\alpha + (\Delta a \circ (\Delta b)) = \alpha + \Delta a \circ \alpha + \Delta b;$$
$$\alpha + (\beta + \Delta a) = (\alpha + \beta) + \Delta a;\ 0 + \Delta a = \Delta a;$$
$$\alpha + (\Delta a + (\Delta b)) = (\alpha + (\Delta a)) + (\Delta b) = \Delta a + (\alpha + \Delta b)$$

Note 4. The axiom 10 will be coordinated with the previous axioms. Really: $(\alpha \circ \beta) + \Delta a = \beta + 1 + \Delta a = \Delta(\beta + 1 + a)$ at $\beta > \alpha$, $\alpha + \Delta a \circ \beta + \Delta a = \Delta(\alpha + a) \circ (\Delta(\beta + a)) = \Delta(\beta + a) + 1 = \Delta(\beta + 1 + a)$, at $\beta > a$; $\alpha + (\Delta a \circ (\Delta b)) = \alpha + \Delta b + 1 = \Delta(\alpha + 1 + b)$ at $\Delta b < \Delta a$; $\alpha + (\Delta a) \circ \alpha + \Delta b = \alpha + \Delta b + 1 = \Delta(\alpha + 1 + b)$ At $\Delta b < \Delta a$; $\alpha + (\beta + \Delta a) = = \alpha + \Delta(\beta + a) = \Delta(\alpha + \beta + a) = (\alpha + \beta) + \Delta a$; $0 + \Delta a = \Delta(0 + a) = \Delta a$; $\alpha + (\Delta a + (\Delta b)) = \alpha + (a + b)$; $(\alpha + (\Delta a)) + (\Delta b) = \Delta(\alpha + a) + \Delta b = = \alpha + a + b$; $\Delta a + (\alpha + \Delta b) = \Delta a + \Delta(\alpha + b) = a + \alpha + b$.

Note 5. If by two binary operations to consider operation of an order zero and addition (i.e. to reduce a conventional order of operations of addition and multiplication on 1), the set Δ_0 is a *ring* in the new interpretation last. Moreover, Δ_0 is *algebra* above a field of real numbers $\mathbf{R}$ in view of lowering about operations accordingly on 1. Δ_0 – the ring is associative-commutative. Certainly, it is expedient to not separate Δ_0 from $\mathbf{R}$, as it and is made in the present text $(\mathbf{R}_0 = \Delta_0 \cup \mathbf{R})$, and to consider Δ*-numbers* as *real*, but possessing the specifity (as fractional or negative in relation to whole and positive).

Note 6. Concerning "zero" for Δ-numbers it is possible to note: $\Delta a + 0 = \Delta a$ (compare $a + 0 = a$), $(-\infty) \circ a = a$, i.e. at $a \in \mathbf{R}_0$ the relations $a + 0 = a$ and $(-\infty) \circ a = a$ are fair.

11. For anyone's $\Delta a, \Delta b \in \Delta_0$ the unique number $\Delta a \cdot \Delta b$, called by a product of numbers Δa and Δb, $\Delta a \cdot \Delta b \neq \Delta b \cdot \Delta a$ is defined. And, $(\Delta a) \cdot b = a \cdot b$ $(b \in \mathbf{N}_2)$, $(\Delta a) \cdot b = \Delta(a \cdot b) = \Delta(a \cdot b)$ $(\Delta a) \cdot b = \Delta(a \cdot b)$ $(b \in \mathbf{N}_1)$, $a \cdot (\Delta b) = \Delta(a \cdot b)$, $\Delta a \cdot \Delta b = \Delta((\Delta a) \cdot b)$,

$$\sum_{i=1}^{n} (\Delta a) \cdot C_i = \Delta a \cdot \sum_{i=1}^{n} C_i,\ \sum_{i=1}^{n} C_i \cdot (\Delta a) = a \cdot \sum_{i=1}^{n} (\Delta C_i).$$

Note 7. The apparent commutability of multiplication Δ-numbers is false:

$$\Delta a\cdot\Delta b=\Delta(a\cdot\Delta b)=\{\Delta(a\cdot b);a\cdot b\},$$
$$\Delta b\cdot\Delta a=\Delta(\Delta b\cdot a)=\{\Delta(b\cdot a);b\cdot a\}.$$

The reason of a paradox that in the first case an outcome depends on parity of number b, and in the second case - from parity of number a.

12. For anyone $\Delta a\in\Delta_0$ there is unit $(\Delta a\cdot 1=\Delta a)$, i.e. for any number $a\in\mathbf{R}_0$ there is unit $(a\cdot 1=a)$.

13. For anyone $a\in\mathbf{R}_0$, $a\neq\{0;\Delta 0\}$ there is $b\in\mathbf{R}_0$ such, that $a\cdot b=1$.

14. For anyone's $a,b,c\in\mathbf{R}_0$ has a place a relation $a\cdot(b\cdot c)=(a\cdot b)\cdot c$ (associativity of multiplication).

Note 8. Let's remark, that

$$(\Delta a\cdot\Delta b)\cdot\Delta c=(\Delta(\Delta a\cdot b))\cdot\Delta c=\begin{cases}\Delta(a\cdot b)\cdot\Delta c & \text{for } b\in N_2\\ (a\cdot b)\cdot\Delta c & \text{for } b\in N_1\end{cases}=$$

$$=\begin{cases}\Delta(\Delta(a\cdot b)\cdot c) & \text{for } b\in N_2\\ \Delta(a\cdot b\cdot c) & \text{for } b\in N_1\end{cases}=\begin{cases}\Delta(a\cdot b\cdot c) & \text{for } b\in N_2,\quad c\in N_2\\ a\cdot b\cdot c & \text{for } b\in N_2,\quad c\in N_1\\ \Delta(a\cdot b\cdot c) & \text{for } b\in N_1\end{cases};$$

$$\Delta a\cdot(\Delta b\cdot\Delta c)=\Delta a\cdot(\Delta(\Delta b\cdot c))=\begin{cases}\Delta a\cdot(\Delta(b\cdot c)) & \text{for } \mathrm{c}\in N_2\\ \Delta a\cdot(b\cdot c) & \text{for } \mathrm{c}\in N_1\end{cases}=$$

$$=\begin{cases}\Delta(\Delta a\cdot(b\cdot c)), & c\in N_2\\ a\cdot b\cdot c, & c\in N_1\quad b\in N_2\\ \Delta(a\cdot b\cdot c), & c\in N_1,\quad b\in N_1\end{cases}=\begin{cases}\Delta(a\cdot b\cdot c), & c\in N_2\text{, for anyones } b,\\ a\cdot b\cdot c, & c\in N_1,\quad b\in N_2,\\ \Delta(a\cdot b\cdot c), & c\in N_1,\quad b\in N_1.\end{cases}$$

15. For $(a,b)\in\mathbf{R}$ the equalities $a^{\Delta b}=-(a^b)$, $\log(-a)\in\Delta_0$, and $\Delta\log a\in\log(-a)$, and $(\Delta 0//2)\in\log i$ take place. The formula $a^{\Delta b}=-(a^b)$ has a deep sense (by analogy $a^{-b}=:(a^b)$, where $(a,b)\in\mathbf{R}_+$.

16. For anyone's $\Delta a,\Delta b\in\Delta_0$ and $\alpha\in\mathbf{R}$ has a place a relation $\alpha+(\Delta a+\Delta b)=\alpha\cdot\Delta a+\alpha\cdot\Delta b$.

Note 9. The axiom 15 will be coordinated with the previous axioms: $\alpha\cdot(\Delta a+\Delta b)=\alpha\cdot(a+b)$, $\alpha\cdot\Delta a+\alpha\cdot\Delta b=\Delta(\alpha\cdot a)+\Delta(\alpha\cdot b)=$ $=\alpha\cdot a+\alpha\cdot b=\alpha\cdot(a+b)$.

Note 10a. Let's consider the following paradox: $6^{\Delta 2}=-36$ and $6^{\Delta 2}=(2{\cdot}3)^{\Delta 2}=2^{\Delta 2}\cdot 3^{\Delta 2}=(-4)\cdot(-9)=+36$. The essence of a paradox consists in некоммутативности of operation of multiplication on a set $\Delta-$numbers. Really, taking the logarithm, we shall receive

$$\log(2{\cdot}3)^{\Delta 2}=(\log(2\cdot 3))\cdot\Delta 2=(\log 2+\log 3)\cdot\Delta 2\neq$$

$$\neq(\log 2)\cdot\Delta 2+(\log 3)\cdot\Delta 2.$$

This expression a special case of the formula:

$$(a+b)\cdot\Delta c\neq(a\cdot\Delta c)+(b\cdot\Delta c)$$

Let's consider deriving this association for real numbers. Really, if $(a,b,c)\in\mathbf{R}$ and $(a,b,c)\in\mathbf{Z}$,

$$(a+b)\cdot c=\underbrace{(a+b)+(a+b)+\ldots+(a+b)}_{c}=a\cdot c+b\cdot c$$

and $\Delta c\cdot(a+b)=\underbrace{\Delta c+\Delta c+\ldots+\Delta c}_{a+b}=\Delta c\cdot a+\Delta c\cdot b$, $\Delta c\cdot(a+b)=$ $=\Delta c\cdot a+\Delta c\cdot b$. However, by virtue of noncommutative of multiplication (since c and $(a+b)-$number of recurrings being real numbers) $(a+b)\cdot\Delta c\neq(a\cdot\Delta c)+(b\cdot\Delta c)$.

Note 10b. Let's consider a paradox: $(\Delta a+\Delta b)\cdot\Delta c=(a+b)\cdot\Delta c=a\cdot\Delta c+b\cdot\Delta c=\Delta(a\cdot c)+\Delta(b\cdot c)=a\cdot c+b\cdot c=$ $=(a+b)\cdot c$. Whence follows $\Delta c=c$. Essence of a paradox in that, as in the note 10a.

Corollaries from axioms

1). For two numbers $a,b\in\mathbf{R}_0$ there is one number $x\in\mathbf{R}_0$ such, that $a\circ x=b$ and in case $|a-b|\neq 2$, $|a-b|>1$ number $x=b\Delta a$.

2). For operation "Δ" some axioms of operation "$\circ$" are fair. For example, $(a\Delta b)+c=(a+c)\Delta(b+c)$ etc.

So, for two numbers $a,b\in\mathbf{R}_0$ there is one number $x\in\mathbf{R}_0$ such, that $a\Delta x=b$.

Example. Let $a\Delta x=b$ and $b>a+1$. Then $x=\Delta(b-1)$, as $a\Delta(\Delta(b-1))=b$.

3). For anyone $a\in\mathbf{R}_0$ we have $a=-(-a)$. In particular, if $a=\Delta b\in\Delta_0$, $\Delta b=-(-\Delta b)=-(\Delta(-b))$.

4). For anyone's $a,b,c,d\in\mathbf{R}_0$ $b-a=d-c$ is equivalent $a+d=b+c$.

5). For two numbers $a,b\in\mathbf{R}_0$, where $a\neq\{0,\Delta 0\}$ there is unique $x\in\mathbf{R}$ such, that $a\cdot x=b$, and, in case $a\in\mathbf{N}_2$, there is an additional value $x\in\Delta_0$.

Note 11. The division Δ-numbers is carried out on the following fundamentals:

a) $\forall\, a,b\in\mathbf{R}$ $\Delta a/\Delta b=\{a/b\,;\,\Delta(a/b)\}$;

b) $\forall\, a\in\mathbf{R}$ and $b\notin\mathbf{N}_2$ $\Delta a/b=a/\Delta b=\Delta(a/b)$.

We shall prove a): $\Delta a/\Delta b=x\in\mathbf{R}\Rightarrow\Delta a=x\cdot\Delta b$. If $x\in\mathbf{R}$, $\Delta a=\Delta(x\cdot b)\rightarrow x=(a/b)\rightarrow\Delta a/\Delta b=a/b$. If $x=\Delta y\in\Delta_0$, $y\in\mathbf{R}$, $\Delta a=\Delta y\cdot\Delta b=\Delta(\Delta y\cdot b)$. The equality is feasible, if $b\in\mathbf{N}_2$; $\Delta a=\Delta(y\cdot b)\rightarrow y=a/b$.

We shall prove b): $\Delta a/b=\Delta z\rightarrow\Delta a=\Delta z\cdot b=\Delta(\Delta z\cdot b)\rightarrow z=a/b$, if $b\in\mathbf{N}_1$, i.e. $\Delta a/b=\Delta(a/b)$. Similarly $a/\Delta b=\Delta z\rightarrow a=$ $=\Delta z\cdot\Delta b=\Delta(\Delta z\cdot b)=\Delta(\Delta(z\cdot b))=z\cdot b$, at $b\in\mathbf{N}_1$, i.e. $a/\Delta b=\Delta(a/b)$.

6). For $a\in\mathbf{R}_0$ we have $\dfrac{1}{1/a}=a$.

Note 12. Let $a=\Delta b\in\Delta_0$. Then $1/\Delta b=\Delta(1/b)$ for $b\notin\mathbf{N}_2$, $1/1/\Delta b=\Delta x\;\Rightarrow\;1=\Delta x\cdot(1/\Delta b)=\Delta x\cdot\Delta(1/b)=\Delta(\Delta x\cdot 1/b)\rightarrow$ $\rightarrow\;\Delta x\cdot 1/b=\Delta 1$, i.e. $\Delta x=\Delta 1\cdot b=\Delta b$ for $b\in\mathbf{N}_1$, i.e. $1/1/\Delta b=\Delta b$.

7). For anyone's $\Delta a,\Delta b\in\Delta_0$ and $\Delta c\in\Delta_0\setminus\{\Delta 0,\mathbf{N}_2\}$

$$\frac{\Delta a}{\Delta c}+\frac{\Delta b}{\Delta c}=\frac{a+b}{c}.$$

Note 13. a). $\Delta a/\Delta c+\Delta b/\Delta c=\{a/c;\Delta(a/c)\}+\{b/c;\ \Delta(b/c)\}=(a+b)/c$, since $\Delta(a/c)+\Delta(b/c)=a/c+b/c$.

If $\Delta a,\Delta b\in\Delta_0$, and $c\in\mathbf{R}$, $\Delta a/c+\Delta b/c=(a+b)/c$ only at $c\neq\mathbf{N}_2$, since $\Delta a/c+\Delta b/c=\Delta(a/c)+\Delta(b/c)=(a+b)/c$. Is similar, if $a,b\in\mathbf{R}$, and $\Delta c\in\Delta_0\setminus\{\Delta 0,\mathbf{N}_2\}$, $\Delta a/c+\Delta b/c=(a+b)/c$.

8). For anyone $a\in\mathbf{R}_0$ the equality $-a=(-1)\cdot a$ is fair.

9). For anyone's $a,b\in\mathbf{R}_0$ the equality $-(a\cdot b)=(-a\cdot b)$ is fulfilled.

Note 14. Let $\Delta a,\,\Delta b\in\Delta_0$. Then $-(\Delta a\cdot\Delta b)=$
$=-(\Delta(\Delta a\cdot b))=\Delta(-(\Delta a)\cdot b)$, $-(\Delta a)\cdot\Delta b=\Delta(-a)\cdot\Delta b=\Delta(\Delta(-a)\cdot b)=$
$=\Delta(-(\Delta a)\cdot b)$.

10). For $a,b,c\in\mathbf{R}_0$ all axioms of the order and corollary from them, formulated for $a,b,c\in\mathbf{R}$ are fulfilled.

11). A principle of a continuity Dedekind. Let set $\mathbf{R}_0$ is divided into two classes K_1 and K_2 so, that: a) classes K_1 and K_2 are not empty; b) each number $a\in\mathbf{R}_0$ concerns only to one class; c) from a condition $a\in K_1$ and $b<a$ follows, that $b\in K_1$. Then there is a sole number $s\in\mathbf{R}_0$ such, that all numbers $a\in\mathbf{R}_0$, satisfying to an inequality $a=a'<s$, belong K_1, and all numbers satisfying to an inequality $a=a''>s$, belong to a class K_2. The number s is named as a cut of a set $\mathbf{R}_0$.

Note 15. In the whole set $\mathbf{R}_0$ has an algebraic structure of a **field** (**commutative skew field**) adjusted for specifity of axioms.

Note 16. In summary we shall remark, that a series of sentences referred to axioms and corollaries them, are proved from the appropriate definitions and other axioms.

§ 2.4 Exponentations Δ-numbers

It is uneasy to place regularities at a realization of exponentation. Let's consider the elementary case, when a and $b-$whole positive numbers. Then a^{-b} - fractional, $a^{\Delta b}$ - negative number etc., i.e. the outcome of exponentation is number on a class above, than exponent. The more detailed information can be received, analyzing situations with various classes of numbers a and b:

a). $b \in \mathbf{N}_2$ $(\Delta a)^b = \begin{cases} \Delta\left(a^b\right), a \in \mathbf{N}_2 \\ a^b, a \in \mathbf{N}_1 \end{cases}$, $b \in \mathbf{N}_1$ $(\Delta a)^b = \Delta\left(a^b\right)$,

i.e. $(\Delta a)^b = \Delta\left(a^b\right)$,

except for a case, when $a \in \mathbf{N}_1$ and $b \in \mathbf{N}_2$. Then $(\Delta a)^b = a^b$.

b). $$(\Delta a)^{-b} = \begin{cases} a^{-b}, \text{if } (a,b) \in \mathbf{N}_2 \\ \left\{ \frac{1}{a^b}, \Delta\left(\frac{1}{a^b}\right) \right\} \end{cases}$$

For negative numbers $(a < 0)$ the equalities are fair:

$$(-a)^{-b} = (-1)^b \cdot a = \begin{cases} a^{-b}, & b \in \mathbf{N}_2 \\ -(a)^{-b}, & b \in \mathbf{N}_1 \end{cases};$$

$$(-a)^{\Delta b} = (-1) \cdot a^b \in \mathbf{R};$$

$$(-a)^{\Delta b} = (-1)^b \cdot a^b, \ a^b \in \mathbf{N}_2 \text{ etc.}$$

Not stopping on a research various cases[10], numbers, connected to exponentation, we shall make such conclusion: the negative number in a degree Δ-number is a real number, and positive in a degree Δ-number is a negative number (by analogy $(:a)^{-b} = a^b \in \mathbf{Z}$, and $a^{-b} = \left(:a^b\right) \in \mathbf{R}_f$, i.e. outcome fractional number, if $(a,b) \in \mathbf{Z}$).

The full research of the extension of a field of real numbers goes out for frameworks of the present book. However, above mentioned axiomatics Δ-numbers and their properties allow to generate representation about this extension.

[10] Such research is submitted to the reader to do independently: it simply and it is enough fascinating, as contains a series of interesting conclusions and paradoxical outcomes (last arise in case of violation of rules of multiplication $\Delta -$numbers and inaccurate reversion with them).

We shall be limited to two small examples, which will help, to some extent to understand a reality Δ-numbers.

Example 1. To find solutions of an equation

$$9\cdot 8^x - 18\cdot 4^x - 2\cdot 2^x + 4 = 0, \qquad (1)$$

If $x \in \mathbf{R}_0 = \mathbf{R} \cup \mathbf{R}_\Delta$.

Solution. a). Let $x \in \mathbf{R}$, i.e. x – real number. Then $x = \{-1{,}0849626; 1\}$.

b). Let $x \in \mathbf{R}_\Delta$, i.e. $x - \Delta$-number. Then, by designating $y = 2^x$ $(y < 0$, since $x \in \mathbf{R}_\Delta)$, from (1) we shall receive

$$9\cdot y^3 + 18\cdot y^2 - 2\cdot y + 4 = 0, \qquad (2)$$

Really, if $x \in \mathbf{R}_\Delta$, $4^x = 2^{2x} = \left(-2^{x\cdot 2}\right) = -\left(2^x\right)^2 = -y^2$, $8^x = 2^{3x} = 2^{x\cdot 3} = y^3$.

Solving the equation (2), we shall discover $y = -2{,}1936608$ (radical $y = 0{,}0968304 \pm 0{,}4397678\cdot i$ does not satisfy to a condition $y < 0$). Whence, $x = \log_2(-1) + \log_2 2{,}1936608 = \Delta 0 + 1{,}1333404 = \Delta 1{,}1333404$.

So, $x = \{-1{,}0849626; 1; \Delta 1{,}1333404\}$.

By check it is easy to be convinced, that all radicals satisfy to the equation (1).

Example 2a. $(-2)^3 = 2^{(\log_2(-2))\cdot 3} = 2^{((\log_2(-1))+1)\cdot 3} = 2^{(\Delta 0+1)\cdot 3} = 2^{(\Delta 1)\cdot 3} = 2^{\Delta 3} = -8$.

Example 2б. $\sqrt{4} = 4^{1//2} = 2^{2\cdot(1//2)} = 2^{2//2} = 2^{\{\Delta 1, \circ 1\}} = 2^{\{\Delta 1, 1\}} = \{-2; 2\}$, since $2^{\circ 1} \equiv 2$, and $2^{\Delta 1} = -2$. (We shall remark, that $a^{b//c} = \sqrt[c]{a^b}$, as if $\sqrt[c]{a^b} = x$, $a^b = x^c \Rightarrow \log_a a^b = \log_a x^c \Rightarrow b = (\log_a x)\cdot c \Rightarrow \log_a x = b//c \Rightarrow x = a^{b//c}$; see § 5.2).

§ 2.5. Examples of proofs in ω-symbolics

In the present paragraph the examples of some proofs which have been carried spent in ω-symbolics, i.e. proofs of lemmas and theorems connected with ω-reflections of operations and numbers are considered. It is necessary to notice, that the package, offered to the reader, of proofs is a not full cycle of statements on the given problem. Moreover, part of reduced lemmas are not used in the theorems, that, naturally, creates a field for searching and analysis, which, in case of desire, can independent lead the reader.

It would be necessary especially to underline, that is *primary Δ-numbers are obtained from operation "$\circ$" (operation easier than addition): from $x \circ a = -\infty$ $(a \in \mathbf{R})$ follows $x = (-\infty)\Delta a \equiv \Delta a$ (by analogy, $0 - a = -a$ and $1/a = 1{:}a = {:}a$). Thus the* ***reflexivity*** *of image of numbers is observed both at algebraic, and at geometric expression.*

On the other hand, the set $\mathbf{R}_\Delta$ Δ-numbers can be obtained for one act ω-reflection. Really, as $({:}a)\backslash\omega_0 \to \omega_1\backslash\log({:}a) = (-\log a) \in \mathbf{R}_-$ at $a > 1$ (the number a does not enter into a set $\mathbf{R}_f$, $a \notin \mathbf{R}_f$), $(-a)\backslash\omega_0 \to$ $\to \omega_1\backslash\log(-a) = (\Delta\log a) \in \mathbf{R}_\Delta$ ($a \notin \mathbf{R}_-$, the number a does not enter into a set $\mathbf{R}_-$ negative numbers). Thus, all field $\mathbf{R}_\Delta$ Δ-numbers can be obtained from a set $\mathbf{R}_-$ negative numbers for one act of image $\backslash\omega_0 \to \omega_1\backslash$.

$$\lim_{|a|\to\infty} \log -|a| = \lim_{|a|\to\infty} \Delta\log|a| = \Delta\infty,$$

$$\lim_{|a|\to 0} \log -|a| = \lim_{|a|\to 0} \Delta\log|a| = \Delta(-\infty) = -\infty,$$

a). If $|a| > 1$, ω-image of number $-|a|$, hits in an interval $]\Delta 0; \Delta\infty[$ $(-1\backslash\omega_0 \to \omega_1\backslash\log(-1) = \Delta 0,$ $-|a|\backslash\omega_0 \to$ $\to \omega_1\backslash\log(-|a|) = \Delta\log|a| \in]\Delta 0; \Delta\infty[\ )$, i.e. $\lim_{|a|\to\infty} \log(-|a|) = \Delta\infty$, $\lim_{|a|\to 1} \log(-|a|) = \Delta 0$.

b). If $0 < |a| < 1$, an outcome ω-reflection $\Delta\log|a| \in \in]-\infty; \Delta 0[$, i.e.

$$-(:|b|)\setminus\omega_0\to\omega_1\setminus\log(-(:|b|))=\Delta\log(:|b|)=\Delta(-\log|b|),$$

where $|b|=\dfrac{1}{|a|}$, $|b|>1$, and $\lim\limits_{|b|\to\infty}\Delta(-\log(|b|))=\Delta(-\infty)\equiv-\infty$

and $\lim\limits_{|b|\to 1}\Delta(-\log|b|)=\Delta(-0)\equiv\Delta 0$.

In this case, the reflexivity of image is breaked, and the upgraded numerical axes looks so:

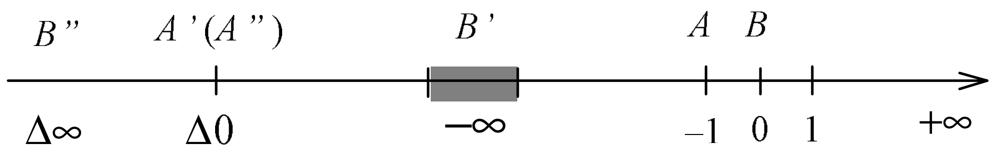

Fig. 1. An image of a upgraded numerical axes with the instruction(indication) of boundary points ω-reflection $\setminus\omega_0\to\omega_1\setminus$. The segment $|AB|$ is reverberate at $\setminus\omega_0\to\omega_1\setminus$ in an interval $A'B'$, and interval AB' – in an interval $A''B''$.

Lemma 2.1. The image of a set $\mathbf{R}$ Real numbers $\setminus\omega_i\to\omega_0\setminus$[11] is *bijection* at $k>0$, $k\neq 1$, $i\in z$.

Proof. Let any element (pre-image) $a'\in\mathbf{R}$ and $a'\in\omega_i$, is reverberate in a $\left(a'\setminus\omega_i\xrightarrow{\varphi}\omega_0\setminus a\right)$, at $\setminus\omega_i\to\omega_0\setminus$, i.e. is transformed to an image

$a=\operatorname{Im}\varphi=\{\varphi(a')|a'\in\omega_i\}$. In our case, $\{a'\}\xrightarrow{\varphi}\left\{\left(i+\operatorname{slog}_k^{a'}\right)*k\right\}$. According to the definition of function $y=\operatorname{slog}_k^x$ to each value $x>0$ there corresponds a unique value $y\in\mathbf{R}$, and at $x<0$ - the sole value $y\in\Delta_0$ (at $x=0$, $y=-\infty$), i.e. each value $a'\in\omega_i$

[11] Under ω_i and ω_0 is implied only components of these spaces as a set **R** of real numbers, i.e. $\omega_i\equiv\{a', a'\in R\}$ and set $\omega_0\equiv\{(i+\operatorname{slog}_k a')*k,\ i\in Z, k>0,\ k\neq 1\}$.

is compared one element $a = \left(\left(i + \mathrm{slog}_k^{a'}\right) * k\right) \in \omega_0$ or $a = \varphi(a') \in \omega_0$. And, on the contrary, each value $y \in R_0$ there corresponds a unique value x, equal $x = {}^y k = \left.k^{k^{\cdots^{k}}}\right\}y$, under the definition of an exponential function. Moreover, image φ and inverse to it φ^{-1} are continuous[12], i.e. are *homeomorphic*. The homeomorphism of image φ proves to be true by a series of the facts. For example, for a special case $R \setminus \omega_1 \to \omega_0 \setminus R^*$ we shall receive: if $a, b \in R$ and $a \neq b$, that $a + b \setminus \omega_1 \to \omega_0 \setminus k^a \cdot k^b = k^{a+b} \in R^*$, $\quad k \neq 1$; $\quad a \odot b = a^{\log_k b} \in R$ $a \cdot b \setminus \omega_1 \to \omega_0 \setminus k^a \odot k^b = k^{a \cdot b} \in R^*$, i.e. exists one-to-one the correspondence, at which for anyones $a, b \in R$ and for their images $k^a, k^b \in R$ to a sum $(a + b)$ and product $(a \cdot b)$ there correspond sole values $k^a \cdot k^b$ and $k^a \odot k^b$ ($\cdot, \odot -$ accordingly product and reflexive product). It proves *isomorphism* φ. The continuity of functions k^x and $\log_k^x$ proves a homeomorphism φ.

In view of above-stated φ and φ^{-1} are *epimorphic*. Thus, the given image φ represents an *injection* (as from $\varphi(a') = \varphi(a'')$ follows $a' = a''$) and *surjection*. Whence is concluded, that φ - *bijection*.

Corollary. As $\varphi -$bijection, for everyone $a' \in \omega_i$ exists $\varphi^{-1}(a')$, where $\varphi^{-1} \cdot \varphi = id_{\omega_i}$, $id_{\omega_i} : a' \to a'$.

From $\varphi^{-1}(a') = \{a' \in \omega_i | \varphi(a') = a\} \Rightarrow \varphi^{-1}(a') = \left(\left(\mathrm{slog}_k^{\varphi} - i\right) * k\right)$.

Theorem 2.1. *The system of mathematical objects* $\mathrm{O} = \langle \mathrm{O} \in R_+, \bullet, \odot, k \rangle$, *generated in an outcome of image* $\setminus \omega_1 \to \omega_0 \setminus$ *on a set* R_+ *will derivate a field at anyone* $k > 0,\ k \neq 1$.

Proof. Let objects $a, b \in R_+$. From the theorem 1.1 follows, that the operation "$\odot$" is alternative, is commutative and is associative. Let's prove a *distributivity* of operation "$\odot$" rather ordinary multiplication ("$\bullet$"):

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[12] It is not necessary to confuse a continuity of reflection and continuity of functions. It is natural, that functions $y = {}^x k$ and $y = \mathrm{slog}_k x$ in their classical definition having rupture, as $x \in \mathbf{Z}$.

$a\odot(b\cdot c)=a^{\log_k(b\cdot c)}=a^{\log_k b}\cdot a^{\log_k c}=(a\odot b)\cdot(a\odot c)$. On the other hand, $(b\cdot c)\odot a=(b\cdot c)^{\log_k a}=b^{\log_k a}\cdot c^{\log_k a}=(b\odot a)\cdot(c\odot a)=(a\odot b)\cdot(a\odot c)$.

The unit element of a system O is equal k: $a\odot k=a^{\log_k k}=a$. The inverse element a^{-1} is equal $\sqrt[\log_k a]{k}$ $\left(a^{-1}=k\Delta a\right)$. Really, $a^{-1}\odot a=k^{1/\log_k a}\odot a=$ $=\left(k^{1/\log_k a}\right)^{\log_k a}=k$. The system O is *group*, as the binary operation $(\,\cdot\,,\odot)$ is defined which to each pair of objects $a,b\in\mathrm{O}$ puts in the correspondence some outcome of operation : $a\cdot b$, $a\odot b=a^{\log_k b}$. Thus all requirements showed to group are fulfilled: $a\cdot b\in\mathrm{O}$, $a\odot b=a^{\log_k b}\in\mathrm{O}$, i.e. $\mathrm{O}-$*is closed* $\left(a>0,\ b>0\Rightarrow\Rightarrow a^{\log_k b}>0\right)$; $\mathrm{O}-$*is associative* $\left(a\odot(b\odot c)=a\odot\left(b^{\log_k c}\right)=\right.$ $\left.=a^{\log_k c\cdot\log_k b}=\left(a^{\log_k b}\right)^{\log_k c}=(a\odot b)\odot c\right)$; O contains left and right unit for everyone a from O $\left(k\odot a=k^{\log_k a}=a=a\odot k\right)$; for all $a\in\mathrm{O}$ in O there is a left and right inverse element $a^{-1}\odot a=k$ and $a\odot a^{-1}=k$, as $a^{-1}\odot a=\left(k^{1/\log_k a}\right)\odot a=\left(k^{1/\log_k a}\right)^{\log_k a}=k$, and also $a\odot a^{-1}=a\odot\left(k^{1/\log_k a}\right)=a^{1/\log_k a}=k$.

From the theorem 1.1 follows, that $\mathrm{O}-$is commutative.

The group O is commutative, additive with distributive operation "$\odot$". The group O is additive, as the operation $(\cdot)$ is defined which is an image of operation (++ at image $\backslash\omega_1\to\omega_0\backslash$, i.e. $\mathrm{O}-$*the ring,* and, is associative - commutative that an image of multiplication in a ring O is the operation "$\odot$", which is associative and is commutative.

In view of above-stated and lemma 2.1 systems $\mathrm{O}-$multiplicate group (ring with eliminated zero equal our case 1, as $a\odot 1=a^{\log_k 1}=1$).

The note 1. The deviders of zero do not exist. Really, for any elements $x,y\in R_+$ and $x\neq 1,\ y\neq 1$ $\ x\odot y\neq 1$ (“1” plays a role of zero in case of operation "$\odot$").

The set of all nonzero elements of a ring will be a *groupoid* concerning operation "$\odot$". And the ring will be a skew field, as the set will derivate it of the nonzero elements group rather "$\odot$".

The system $\mathrm{O}-$*field* (is associative - commutative a ring with the single element k, element a^{-1} and eliminated zero).

The note 2. The set $\mathbf{R}_+$ positive numbers comprises a series of classes on ω-reflections, which include such global groups as fractional numbers (R_f) and infinities a spectrum of classes of irrationals (R_i^j).

Corollary. Outcomes of image $\backslash \omega_i \to \omega_0 \backslash$ Set positive $(\mathbf{R}_+)$ Makes a *subgroup* A Group R_+ Concerning operation "$\odot$".

Proof. The operation "$\odot$" is defined on a set R_+. Let $a, b, c, d \in R_+$, $k > 0$, $k \neq 1$. Then $a \backslash \omega_i \to \omega_0 \backslash \left(\left(i + \mathrm{slog}_k^a\right) * k\right) \in A$, $b \backslash \omega_i \to \omega_0 \backslash \left(\left(i + \mathrm{slog}_k^b\right) * k\right) \in A$, $R_+ \backslash \omega_i \to \omega_0 \backslash A$, $(a \odot b) \backslash \omega_i \to$ $\to \omega_0 \backslash P = \left(\left(i + \mathrm{slog}_k^a\right) * k\right) \underline{\odot} \left(\left(i + \mathrm{slog}_k b\right) * k\right) = \left(\left(i + \mathrm{slog}_k^c\right) * k\right) \in A$, that proves a closure of a system A. ("$\underline{\odot}$" - image of operation "$\odot$" at image $\backslash \omega_i \to \omega_0 \backslash$). Let's discover C: $\left(\mathrm{slog}_k^P - i\right) * k = C \in R_+$ (at slog_k^a, $\mathrm{slog}_k^b \in R_+$). For anyone $a' \in A$ inverse element $|a'|^{-1} \in A$ $\left(|a'|^{-1} = k^{1/\log_k a'} \in A\right)$. Let's equate $k^{1/\log_k a} = \left(i + \mathrm{slog}_k^d\right) * k$. Whence $d = \left(\mathrm{slog}_k^{\left(k^{1/\log_k a}\right)} - i\right) * k$. Let's remark, that all mathematical calculations is produce in R_+, $i \in \mathbf{Z}$. So, $A \subseteq R_+$; $A -$ group rather "$\odot$"; for anyones $a, b \in A$ $a \odot b \in A$; for all $a' \in A$ there is $(a')^{-1} \in A$, i.e. $A -$ a *subgroup* of group R_+. Actually, A consists of all elements R_+ but transformed on the known law: $a \backslash \omega_i \to \omega_0 \backslash \left(\left(i + \mathrm{slog}_k^a\right) * k\right)$ that in the classical definition of function $y = \mathrm{slog}_k x$, $x \in \mathbf{Z}$, and, signifies, $a \in \mathbf{Z}$. And, to each index "i" there corresponds a unique value "k", i.e. there is a series of subgroups inserted and filled of sets R_+: $A_1 \subseteq R_+$, $A_2 \subseteq R_+$..., where $1, 2, \ldots -$ index equal i. The extension of a range of definition of function $y = \mathrm{slog}_k x$ up to $x \in \mathbf{R}$ Carries on to *full* filling of a set R_+ indicated series of subgroups.

Thus, the set R_+ can be represented by any set A_i. Last (the sets A_i) can be considered as the transformed aspects R_+, i.e. ω-reflection allows from one element (in this case sets R_+) to receive infinities a spectrum of the transformed elements identified with first (i.e. the pre-image).

Lemma 2.2. The operation "$\odot$" is *commutative* on a set $\mathbf{R}-$negative numbers.

Proof. Let $a, b \in R_-$, $k \neq 1$. Then $(-|a|)\odot(-|b|) = = (-|a|)^{\log_k(-|b|)} = (-|a|)^{\Delta\log_k|b|} = -\left((-|a|)^{\log_k|b|}\right)$; $(-|b|)\odot(-|a|) = = -\left((-|b|)^{\log_k|a|}\right)$. After a taking the logarithm of these expressions we shall receive:

$$A = \log_k\left(-\left((-|a|)^{\log_k|b|}\right)\right) = \Delta\log_k(-|a|)^{\log_k|b|} = \Delta\left(\log_k(-|a|)\times\right.$$
$$\left.\times\log_k|b|\right) = \Delta\left(\Delta\log_k|a|\cdot\log_k|b|\right);$$
$$B = \log_k\left(-\left((-|b|)^{\log_k|a|}\right)\right) = \Delta\left(\log_k(-|b|)^{\log_k|a|}\right) = \Delta\left(\Delta\log_k|b|\times\right.$$
$$\left.\times\log_k|a|\right).$$

If $\log_k^b, \log_k^a \in N_2$, that at $k > 0$ $A = B = \Delta\left(\log_k^{|a|}\cdot\log_k^{|b|}\right)$ and $(-|a|)\odot(-|b|) = (-|b|)\odot(-|a|)$. If $\log_{|k|}^{|b|}, \log_{|k|}^{|a|} \in N_1$, That $A = B = \log_{|k|}^{|a|}\times \times\log_{|k|}^{|b|}$.

If $\log_{|k|}|b|$ and $\log_{|k|}|a|$ simultaneously do not belong to a set N_2 or set N_1, the operation "$\odot$" is commutative at $k < 0$.

Really, $\Delta\log_k^{|a|} = \Delta\dfrac{1}{\log_{|a|}^k} = \Delta\left(\dfrac{1}{\Delta\log_{|a|}|k|}\right) = \dfrac{1}{\log_{|a|}|k|}$,

$\Delta\log_k^{|b|} = \dfrac{1}{\log_{|b|}|k|}$.

Substituting these values in A and B, we shall discover:

$$A = \Delta\left(\frac{1}{\log_{|a|}|k|}\cdot\log_k^{|b|}\right) = \Delta\left(\log_{|k|}|a|\cdot\left(\Delta\frac{1}{\log_{|b|}|k|}\right)\right) =$$

$$= \Delta\left(\Delta\left(\log_{|k|}|a| \cdot \log_{|k|}|b|\right)\right) = \log_{|k|}|a| \cdot \log_{|k|}|b|.$$

Similarly, $B = \Delta\left(\frac{1}{\log_{|b|}|k|} \cdot \log_k^{|a|}\right) = \Delta\left(\frac{1}{\log_{|b|}|k|} \times\right.$

$\left.\times\left(\Delta\frac{1}{\log_{|a|}|k|}\right)\right) = \log_{|k|}|b| \cdot \log_{|k|}|a|$, i.e. $A = B$ and $(-|a|)\odot(-|b|) =$

$= (-|b|)\odot(-|a|)$, as was to be shown.

In view of the theorems 1.1 and 1.2 it is possible to make a conclusion, *that the operation "$\odot$" is commutative on all a set* $\mathbf{R}$ *real numbers.*

Though the theorems 1.1, 1.2 and lemma 2.2 prove a commutability of operation "$\odot$" on $\mathbf{R}$, we shall show a commutability of this operation on a set of fractional numbers, as they will derivate independent ω- a class of numbers. Let $a, b \in \mathbf{R}$. Then

$$(:a)\odot(:b) = \left(\frac{1}{a}\right)^{\log_k\left(1/b\right)} = \left(a^{-1}\right)^{-\left(\log_k b\right)} = a^{\log_k b} = a\odot b,$$

$(:b)\odot(:a) = b\odot a = a\odot b$, as was to be shown.

Lemma 2.3. The operation "$\odot$" is *associative* on a set R_- negative numbers.

Proof. a). Let $(a,b,c) \in R_-$, $\log_k^{|c|} \in N_2$, $k \neq 1$, $k > 0$. Then

$$A = ((-|a|)\odot(-|b|))\odot(-|c|) = \left((-|a|)^{\Delta\log_k|b|}\right)\odot(-|c|) = (-|a|)^{\left(\Delta\log_k|b|\right)\times}$$

$$^{\times\left(\Delta\log_k|c|\right)} = (-|a|)^{\Delta\left(\left(\Delta\log_k|b|\right)\cdot\log_k|c|\right)} = (-|a|)^{\Delta\left(\left(\log_k|b|\right)\cdot\log_k|c|\right)}.$$

On the other hand, $B = (-|a|)\odot((-|b|)\odot(-|c|)) = (-|a|)\odot\left((-|b|)^{\Delta\log_k|c|}\right) =$

$$= (-|a|)^{\left(\Delta\log_k|b|\right)\cdot\left(\Delta\log_k|c|\right)} = (-|a|)^{\Delta\left(\left(\Delta\log_k|b|\right)\cdot\log_k|c|\right)} = (-|a|)^{\Delta\left(\log_k|b|\cdot\log_k|c|\right)},$$

i.e. $A = B$.

b). Let $\log_k^{|c|} \notin N_1$. Then

$$A = \left(-|a|\right)^{\Delta\left(\left(\Delta\log_k|b|\right)\cdot\log_k|c|\right)} = \left(-|a|\right)^{\log_k|b|\cdot\log_k|c|}$$

$B = \left(-|a|\right)^{\log_k|b|\cdot\log_k|c|}$, i.e. $A = B$, as was to be shown.

The theorems 1.1, 2.1 and lemma 2.3 allow to make a conclusion, that *the operation "$\odot$" is associative on all a field* $\mathbf{R}$ *real numbers.*

The note. The associativity of operation "$\odot$" is obvious to fractional numbers. Really,

$$\left(\frac{1}{a} \odot \frac{1}{b}\right) \odot \frac{1}{c} = a^{-\left(\log_k b \cdot \log_k c\right)};$$

$$\frac{1}{a} \odot \left(\frac{1}{b} \odot \frac{1}{c}\right) = a^{-1} \odot b^{\log_k c} = a^{-\left(\log_k b \cdot \log_k c\right)}.$$

Lemma 2.4. The operation "$\odot$" on a set $\mathbf{R}_-$ negative numbers has left and right unit, and also left and right inverse element a^{-1}.

Proof. **a**). As $k \odot \left(-|a|\right) = k^{\log_k\left(-|a|\right)} = k^{\Delta\log_k|a|} = -\left(k^{\log_k|a|}\right) = -|a|$ for anyone $k \neq 1$.

$$\left(-|a|\right) \odot k = \left(-|a|\right)^{\log_k k} = -|a|,$$

i.e. $k -$ Left and right unit.

b). We shall prove existence of a *left-inverse* of the element a^{-1}. As it we shall select $k^{1/\log_k\left(-|a|\right)}$. Then

$k^{1/\log_k\left(-|a|\right)} = k^{\Delta\left(1/\log_k|a|\right)}$ and $x = a^{-1} \odot a = \left(-|a|\right)^{-1} \odot \left(-|a|\right) =$

$$= k^{\Delta\left(1/\log_k|a|\right)} \odot \left(-|a|\right) = k^{\Delta\left(1/\log_k|a|\right)\cdot\log_k\left(-|a|\right)} =$$

$$= k^{\Delta\left(1/\log_k|a|\right)\cdot\left(\Delta\log_k|a|\right)} = k^{\Delta\left(\left(\Delta\left(1/\log_k|a|\right)\right)\cdot\log_k|a|\right)}.$$

If $\log_k|a| \in N_1$, $x = k^{\Delta\left(\Delta\left(1/\log_k|a|\right)\cdot\log_k|a|\right)} = k$, as was to be shown. If $\log_k|a| \notin N_1$, $\left(-|a|\right)^{-1} = -k^{1/\log_k\left(-|a|\right)} = -k^{\Delta\left(1/\log_k|a|\right)}$. Assume $k < 0$, we shall discover $|k|^{1/\log_k |a|} = |k|^{\Delta\left(\Delta\left(\log_{|a|}|k|\right)\right)} = |k|^{1/\log_{|k|}|a|}$ and $x = a^{-1} \odot a =$

$= |k|^{1/\log_{|k|}|a|} \odot (-|a|) = |k|^{\left(\left(1/\log_{|k|}|a|\right)\Delta \log_k |a|\right)} = -|k|^{\left(\left(\log_{|a|}|k|\right)\cdot\left(1/\log_{|a|}|k|\right)\right)} = k$,

as was to be shown.

c). We shall prove equality $(-|a|) \odot (-|a|)^{-1} = k$, i.e. existence of a *right* inverse of the element:

$$(-|a|) \odot (-|a|)^{-1} = (-|a|) \odot k^{1/\log_k(-|a|)} = (-|a|)^{\log_k k^{1/\log_k(-|a|)}} =$$

$$= (-|a|)^{1/\log_k(-|a|)} = (-|a|)^{\Delta\left(1/\log_k |a|\right)} = -\left((-|a|)^{1/\log_k |a|}\right).$$

Let $x = -(-|a|)^{1/\log_k |a|}$. Then

$$\log_k^x = \log_k\left(-(-|a|)^{1/\log_k |a|}\right) = \Delta \log_k (-|a|)^{1/\log_k |a|} = \Delta\left(\log_k (-|a|) \times \right.$$

$$\left. \times \frac{1}{\log_k |a|}\right) = \Delta\left(\frac{\Delta \log_k |a|}{\log_k |a|}\right) = \frac{\log_k |a|}{\log_k |a|} = 1$$, if $\log_{|a|}^k \in N_1$ at $k > 0$. In this case $x = k$, as was to be shown.

If $\log_{|a|}^k \notin N_1$, $k < 0$ and $\log_k^x = \Delta\left(\frac{1}{\log_{|a|}|k|} \cdot \left(\Delta \log_{|a|}|k|\right)\right) =$

$= \Delta\left(\Delta \frac{\log_{|a|}|k|}{\log_{|a|}|k|}\right) = 1$, i.e. $x = k$, as was to be shown.

Separately we shall consider appropriate operations on a set $R_f \quad (a, k \in R_+)$:

$$k \odot (:a) = k^{\log_k (:a)} = (:a);\ (:a) \odot k = (:a)^{\log_k k} = (:a).$$

$$(:a)^{-1} \odot (:a) = k^{1/\log_k (:a)} \odot (:a) = k^{-(1/\log_k a) \cdot \log_k a^{-1}} = k\,;$$

$$(:a) \odot (:a)^{-1} = (:a) \odot k^{1/\log_k (:a)} = \left(a^{-1}\right)^{\log_k k^{-(1/\log_k a)}} =$$

$$= a^{1/\log_k a} = k\,.$$

So, *the operation "$\odot$" has left and right units, and also left and right inverse element a^{-1} on all a set* $\mathbf{R}$ *real numbers.*

Lemma 2.5. The operation "$\odot$" is *distributive* concerning multiplication on a set R_- negative numbers.

Proof. Let $k > 0$, $k \neq 1$ then $A = (-|a|) \odot ((-|b|) \cdot (-|c|)) = = (-|a|)^{\log_k ((-|b|) \cdot (-|c|))} = (-|a|)^{\log_k (-|b|)} \cdot (-|a|)^{\log_k (-|c|)} = ((-|a|) \odot (-|b|)) \times \times ((-|a|) \odot (-|c|))$; On the other hand, $A^* = ((-|b|) \cdot (-|c|)) \odot (-|a|) = = ((-|b|) \cdot (-|c|))^{\log_k (-|a|)} = (-|b|)^{\log_k (-|a|)} \cdot (-|c|)^{\log_k (-|a|)} = ((-|b|) \odot (-|a|)) \times \times ((-|c|) \odot (-|a|))$. By virtue of a lemma 2.3 $A^* = ((-|a|) \odot (-|b|)) \cdot ((-|a|) \odot (-|c|))$, as was to be shown, i.e. $A = A^*$. Omitting a proof of a distributivity of operation "$\odot$" on a set R_f fractional numbers, from the theorem 2.1 and lemma 2.5 we shall conclude, that the *operation "$\odot$" is distributive on all a set* $\mathbf{R}$ *real numbers.*

Lemma 2.6. The system $\mathrm{O} = \langle o \in R_-, \bullet, \odot, k, k \neq 1 \rangle$, generated on a set R_- negative numbers by reflection $\backslash\, \omega_i \to \omega_0 \,\backslash$, is *closed.*

Proof. Let's prove a closure of a system O. Let $k > 0$. Then

$$A = (-|a|) \odot (-|b|) = (-|a|)^{\log_k (-|b|)} = (-|a|)^{\Delta \log_k |b|} = -\left((-|a|)^{\log_k |b|}\right).$$

At $\left\{\log_k^{|a|}, \log_k^{|b|}\right\} \in N_2$ the system is *closed*, as $A \in R_-$. By virtue of a commutability A it is possible to note $(-|a|) \odot (-|b|) = -\left((-|b|)^{\log_k |a|}\right)$. At $\log_k^{|a|} \in N_2$ $A \in R_-$. In case $k < 0$, $\log_k^{|a|} = \dfrac{1}{\Delta \log_{|a|} |k|} =$

$= \Delta\left(\dfrac{1}{\log_{|a|}|k|}\right)$, $\log_k^{|b|} = \Delta\left(\dfrac{1}{\log_{|b|}|k|}\right)$, i.e. $A = -\left((-|a|)^{\Delta\left(1/\log_{|b|}|k|\right)}\right) =$

$= -\left((-|b|)^{\Delta\left(1/\log_{|a|}|k|\right)}\right) = (-|a|)^{\log_{|k|}|b|} = (-|b|)^{\log_{|k|}|a|}$. At $\left\{\log_{|k|}|a|,\right.$

$\left.\log_{|k|}|b|\right\} \in N_1$ the system is *closed*. It is uneasy to prove, that the case $\left\{\log_k^{|a|} \in N_2,\right.$

$\left.\log_k^{|b|} \in N_1\right\}$ can be eliminated from reviewing. Really, $\log_k^{|a|} = n_2 \in N_2$,

$\log_k^{|b|} = n_1 \in N_1$, i.e. $|a| = k^{n_2}$ and $|b| = k^{n_1}$.

Let $k < 0$. Then $|a| -$ exists, as $k^{n_2} > 0$, and $|b| -$ does not exist. The choice k is arbitrary $(k \in \mathbf{R})$, i.e. the last case should be eliminated from reviewing.

So, the system $\mathrm{O} -$ *is closed.*

Theorem 2.2. *The system* $O = \left\langle o \in \mathbf{R}, \cdot, \odot, k, k \neq 1, \log_{|k|}|o| \in \mathbf{Z} \right\rangle$, *generated in an outcome of image* $\backslash \omega_1 \to \omega_0 \backslash$ *set* $\mathbf{R}$ *real numbers, will form a field.*

Not stopping on a proof of the theorem 2.2, we shall remark, that according to lemmas 2-6 and theorem 2.1 systems $\mathrm{O} -$ is associative, is commutative and is distributive of rather reflexive multiplication $(\odot)$, has left and right unit, left and right inverse element. At an elemination of zero equal, in our case, 1 $\left(a \odot 1 = a^{\log_k 1} = 1\right)$, we shall receive a system O, being a field.

Thus, immediately from lemmas 2-6 and theorems 2,2 follows, that *on all a set* $\mathbf{R}$ *real numbers the system* O *is a field.*

Corollary. The ordered pair $\left(\mathrm{O}^*, \rho\right)$, where $\mathrm{O}^* -$ nonempty subset obtained by image $\mathbf{R} \backslash \omega_1 \to \omega_0 \backslash \mathrm{O}^*$, and $\rho -$ ratio "$\odot$" on a field $\mathbf{R}$, is a serially *ordered set.*

Really, in case of an elemination from a set O Operations $(\bullet)$ and $(\odot)$, we shall receive a nonempty subset of values of outcomes ω- reflections to ourselves of all set $\mathbf{R}$ real numbers, and from $a \to a'$ follows $a' \to a$.

The operation "$\odot$" is *reflexive* $\left((a,a)\in\rho\right.$ for anyone $a\in\mathbf{R}$ and $\left.a\rho a\equiv(a\odot a)\in\mathbf{R}\right)$; is *antisymmetric*, as if $(a,b)\in\rho$ and $(b,a)\in\rho$, $a=b$, and if $a\rho b$ and $b\rho a$, $a=b$; is *transitive*, as $(a,b)\in\rho$ and $(b,c)\in\rho$ follows $(a,c)\in\rho$, and if $a\rho b$ and $b\rho c$, $a\rho c$. If $a\odot b=a^{\log_k b}$ and $b\odot c=b^{\log_k c}$, $a\odot c=a^{\log_k c}$, as $A=a^{\log_k b}$, $B=b^{\log_k c}$, $C=a^{\log_k c}$, $\log_k A=(\log_k a)\cdot(\log_k b)$; $\log_k B=(\log_k b)\cdot(\log_k c)$.

Whence $\dfrac{\log_k B}{\log_k A}=\dfrac{\log_k c}{\log_k a}=x;\quad x=\log_A B$

$$\log_k c=x\cdot\log_k a=\log_A B\cdot\log_k a$$

$$C=a^{\log_k c}=a^{(\log_k a)\cdot(\log_A B)}=p^{\log_A B}\in\mathbf{R}\ \left(p=a^{\log_k a}\right),$$

If $(a,b,c,k)\in\mathbf{R}$ and $k\neq 1$, as was to be shown.

Theorem 2.3. *Outcomes of reflections* $\backslash\omega_1\to\omega_0\backslash$ *fields* $\mathbf{R}_\Delta$ Δ*-numbers and* $\mathbf{R}_-$ *−negative numbers are the accordingly fields* $\mathbf{R}_-$ *negative numbers and* R_f *fractional numbers.*

Proof. By eliminating from ω_1 all mathematical objects, except for Δ-numbers and operations given it a field, we shall receive $\omega_1\equiv\{R_\Delta,+,\cdot,\dots\}$. Is realizable isomorphic reflection $\backslash\omega_1\to\omega_0\backslash$ operations:

$$(+,\cdot)\backslash\omega_1\to\omega_0\backslash(\cdot,\odot).$$

a). Let $(a,b,k)\in R_+$, $k\neq 1$, $(\log_k b,\log_k a)\in\mathbf{Z}$. Then $\Delta a\backslash\omega_1\to$ $\to\omega_0\backslash k^{\Delta a}=-(k^a)\in R_-$; $(\Delta a+\Delta b)\backslash\omega_1\to\omega_0\backslash k^{\Delta a+\Delta b}=k^{\Delta a}\cdot k^{\Delta b}=$ $=(-k^a)\cdot(-k^b)\in R_+$; $\Delta a\cdot\Delta b\backslash\omega_1\to\omega_0\backslash k^{\Delta a\cdot\Delta b}=\left(k^{\Delta a}\right)^{\Delta b}=$

$$=\left(-k^a\right)^{\Delta b}=-\left(\left(-k^a\right)^b\right)=\begin{cases}-k^{a\cdot b}\in R_-, & b\in N_2\\ k^{a\cdot b}\in R_+, & b\in N_1\end{cases},$$

i.e. at reflection $\backslash\omega_1\to\omega_0\backslash$ Δ-numbers are transformed in negative, and, the magnitude (module) them varies $\left(|a|=a,\left|k^{\Delta a}\right|=k^a,\quad a\backslash\omega_1\to\omega_0\backslash k^a\right)$.

As $\Delta a+\Delta b=(a+b)\in\mathbf{R},\quad \Delta a\cdot\Delta b=\begin{cases}\Delta(a\cdot b)\in\mathbf{R}_\Delta,\ b\in\mathbf{N}_2\\ a\cdot b\in\mathbf{R},\ b\in\mathbf{N}_1\end{cases}$; That

till analogs (in view of a raise of a rank of operations) we shall note $(-a')\cdot(-b')=a'\cdot b'>0$,

$$((a'\cdot b')\in R_+);\qquad x=(-a')\odot(-b')=(-a')^{\log_k(-b')}=(-a')^{\Delta\log_k b'}=$$

$$=-(-a')^{\log_k b'}=\begin{cases}\log_k b'\in N_2,\ x<0\ (x\in R_-)\\ \log_k b'\in N_1,\ x>0\ (x\in R_+)\end{cases}.$$

Thus, the image of operations $\{+,\cdot\}\setminus\omega_1\to\omega_0\setminus\{\cdot,\odot\}$ and Δ-numbers in negative numbers $(\Delta\setminus\omega_1\to\omega_0\setminus R_-)$ logically is justified by a rule of signs and compression of ranges of outcomes of binary operations. So the outcome addition Δ-numbers is a real number obtained by reflexive reflection of appropriate its value (geometrically - of length $d=|\Delta a+\Delta b|=|a+b|$ of a segment direct) concerning the element $(-\infty)$. The product Δ-numbers is Δ-number (if the second multiplicand - even number), or real number (if the second multiplicand - odd number). Similarly to addition Δ-numbers a product of negative numbers any more negative number, and positive. The reflexive multiplication "$\odot$" negative numbers is or negative number (if second “a reflexive multiplicand” $\log_k b'-$even number), or positive (if second “a reflexive multiplicand” $\log_k b'-$odd number).

b). Let $k\neq 1$ and $(a,b,k,\log_k b,\log_k a)\in\mathbf{Z}$. Then $(-a)\setminus\omega_1\to$ $\to\omega_0\setminus k^{-a}=(:(k^a))\in R_f$, $((-a)+(-b))\setminus\omega_1\to\omega_0\setminus k^{-a-b}=(k^{-a})\times$ $\times(k^{-b})=(:(k^a))\cdot(:(k^b))=(:(k^{a+b}))\in R_f$; $(-a)\cdot(-b)\setminus\omega_1\to$ $\to\omega_0\setminus k^{(-a)\cdot(-b)}=(:k^a)\cdot(:k^b)=\dfrac{1}{k^{a\cdot b}}\in R_f$, i.e. at image $\setminus\omega_1\to\omega_0\setminus$ the negative numbers are reflection fractional. The theorem is proved.

The note. Let's remark, that $(-a)+(-b)=-(a+b)\in R_-$, $(-a)\cdot(-b)=$ $=a\cdot b\in R_+$; $(:a)\cdot(:b)=:(a\cdot b)\in R_f$, $(:a)\odot(:b)=(:a)^{\log_k(:b)}=$ $=a^{\log_k b}\notin R_f$.

Above-stated allows to make a conclusion, that $R_{-} \in \omega_1$, and $R_f \in \omega_0$ (at appropriate eleminations).[13]

Lemma 2.7. Anyone $\omega-$reflection of numbers $\backslash \omega_i \to \omega_j \backslash$ is *isomorphic*.

Proof. From a lemma 2.1 follows, that $\omega-$reflection $\backslash \omega_i \to \omega_0 \backslash$ set $\mathbf{R}$ is *bijection*. The lemma 2.7 generalizes earlier explained outcomes of a research.

Let $a' \in \omega_i$ and $a' > 0$. Then there is in ω_0 a unique image a' $\left(a' \backslash \omega_i \to \omega_0 \backslash \left(i + \mathrm{slog}_{k_1} a'\right) * k_1,\ k_1 \neq 1,\ k_1 > 0\right)$. And, on the contrary: for everyone $a'' \in \omega_0$ there is a sole element $x \in \omega_i$, and $x \equiv \left(\mathrm{slog}_{k_1} a'' - i\right) * k_1$. It follows from the definition of function $y = \mathrm{slog}_k x$. Similarly, reverberate any number c' $(c' > 0)$ from ω_j in ω_0, we shall receive unique it an image $\left(j + \mathrm{slog}_{k_2} c'\right) * k_2,\ k_2 \neq 1,$ $k_2 > 0$. Let $\left(j + \mathrm{slog}_{k_2} c'\right) * k_2 = \left(i + \mathrm{slog}_{k_1} a'\right) * k_1$. Whence $c' = \left(\left(\mathrm{slog}_{k_2}\left(\left(i + \mathrm{slog}_{k_1} a'\right) * k_1\right)\right) - j\right) * k_2$. Have received an outcome of reflections of number a' from ω_i in ω_j: $a' \backslash \omega_i \to \omega_j \backslash c'$, where $c' -$ image of number a' in space ω_j $(c' \in \omega_j)$, and, this image sole by virtue of the definition of function $y = \mathrm{slog}_k x$.

Let $a', b' \in \omega_i$. Then their images in ω_j will be accordingly c' and d' $\left(a' \overset{\omega_i}{\underset{\omega_j}{\to}} c',\ b' \overset{\omega_i}{\underset{\omega_j}{\to}} d' \right)$, and the images of a product $p' = a' \cdot b'$ and reflexive product $r' = a' \odot b'$ can be noted so:

$$p' \ \backslash \omega_i \to \omega_j \backslash \ \left(\left(\mathrm{slog}_{k_2}\left(\left(i + \mathrm{slog}_{k_1}(a' \cdot b')\right) * k_1\right)\right) - j\right) * k_2,$$

[13] Certainly, the theorem 2.3 can be proved easier and laconic. The proof, reduced in the text, had the purposes:

a) to explain some facts about connections in plants at reflection $\backslash \omega_1 \to \omega_0 \backslash$;

b) to illustrate a possibility of study $\omega-$reflection, using the elementary plants - numbers,

c) to pay attention to a classification of numbers at $\omega-$reflections on related isomorphic groups (thus the justified passage of operations in the strict correspondence with their hierarchy) is automatically realized logically.

$$r' \ \backslash\omega_i \to \omega_j\backslash \ \left(\left(\mathrm{slog}_{k2}\left(\left(i+\mathrm{slog}_{k1}(a'\odot b')\right)*k_1\right)\right)-j\right)*k_2,$$

i.e. the image $\backslash\omega_i \to \omega_j\backslash$ is *isomorphic*.

This proof can be lead and on a set $\mathbf{R}-$negative numbers. Thus $(c', d') \in \mathbf{R}_\Delta$.

Lemma 2.8. The isomorphic image of a set $\mathbf{R}$ negative numbers $\backslash\omega_0 \to \omega_1\backslash$ concerning the element $(-\infty)$ is *bijection* at $k > 0,\ k \neq 1$.

Proof. Using lemmas 2.1, 2.7 we shall mark the following fact: if to each element $a_i \quad (a_i \in \mathbf{R}_-)$ to compare one element $b_i \quad (b_i \in \mathbf{R}_\Delta)$, on a property of uniqueness of reflection $(\mathbf{R}_- \to \mathbf{R}_\Delta)$ image $\mathrm{Im}\varphi = \varphi(R) = \{\varphi(a_i) | a_i \in \mathbf{R}\},\quad \varphi(R) = \mathbf{R}_\Delta$, as $a_i \ \backslash\omega_{-1} \to \omega_0\backslash \ \log_k(-|a_i|) \in \mathbf{R}_\Delta$, and $\mathbf{R}_\Delta \cap \mathbf{R}_- = \{-\infty\}$.

Above-stated testifies, that the indicated reflection a surjection and injection (from $\varphi(a) = \varphi(a')$ Follows $a = a'$), i.e. *bijection*.[14]

Theorem 2.4. The system $\mathrm{O} = \langle o \in \mathbf{R}, \cdot, \odot, k \neq 1, \{o_1, o_2, \log_k a_1, \log_k a_2\} \in \mathbf{Z}\rangle$ is *Abelian* group.

Proof. According to a lemma 2.2 and lemma 2.4 systems O are commutative group with left and right unit, and also with the left and right inverse by the element a^{-1}.

For a system O and operation "$\odot$" the **conservation law** is fair:

$c\odot a = c^{\log_k a}$, $c\odot b = c^{\log_k b}$. Let's take the logarithm obtained expressions and we shall equate them: $\log_k(c\odot a) = \log_k c \cdot \log_k a = \log_k c \cdot \log_k b$. Whence follows, that $a = b$. Similarly, $a\odot c = a^{\log_k c}$ and $b\odot c = b^{\log_k c}$, i.e. from $\log_k a \cdot \log_k c = \log_k b \cdot \log_k c \Rightarrow a = b$. So, from $c\odot a = c\odot b \Rightarrow a = b$, $a\odot c = b\odot c \Rightarrow a = b$.

In group O the right and left division is univalently defined.

At last, from $c\odot x = b$ or $x\odot c = b$ the unique solution of these equations $x = k^{\log_c b}$ follows.

All above-stated in view of a lemma 2.8 testifies that the system O is *Abelian* group.

Lemma 2.9. The operation (+) *is alternative, is commutative and is associative* on a set Δ-numbers.

Let's prove alternative:

$$(\Delta a + \Delta b) + \Delta b = (a + b) + \Delta b = \Delta(a + 2\cdot b),$$

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[14] It is uneasy to prove, that the *isomorphic reflection of a set* $\mathbf{R}$ *of real numbers concerning the element* $(-\infty)$ *is bijection at* $k > 0,\ k \neq 1$. However, it will not $\omega-$reflection and the proof is based on properties of operation "o".

$$\Delta a+(\Delta b+\Delta b)=\Delta a+2\cdot b=\Delta(a+2\cdot b)\text{, i.e.}$$

$(\Delta a+\Delta b)+\Delta b=\Delta a+(\Delta b+\Delta b)$, that proves *right* alternative; $(\Delta a+\Delta a)+\Delta b=\Delta(2\cdot a+b)$, $\Delta a+(\Delta a+\Delta b)=\Delta(2\cdot a+b)$, i.e. $(\Delta a+\Delta a)+\Delta b=\Delta a+(\Delta a+\Delta b)$, that proves *left* alternative. The operation (+) is commutative, as $\Delta a+\Delta b=a+b$ and $\Delta b+\Delta a=b+a$, but $a+b=b+a$.

Let's prove, that the operation (+) is associative, i.e. $\Delta a+(\Delta b+\Delta c)= =(\Delta a+\Delta b)+\Delta c$. Really, $\Delta a+(\Delta b+\Delta c)=\Delta a+(b+c)=\Delta(a+b+c)$, and $(\Delta a+\Delta b)+\Delta c=(a+b)+\Delta c=\Delta(a+b+c)$.

Lemma 2.10. The operation (+) is *distributive* concerning operation "$\circ$" on a set $\mathbf{R}_0$ numbers.

Proof. a). Let $(a,b,c)\in\mathbf{R}$

$$a+(b\circ c)\in\mathbf{R}=\begin{cases}a+b+1, & c<b\\ a+c+1, & c>b\\ a+c+2, & c=b\end{cases},\quad (b\circ c)+a=\begin{cases}a+b+1, & c<b\\ a+c+1, & c>b\\ a+c+2, & c=b\end{cases},$$

b). Let $(\Delta a,\Delta b,\Delta c)\in\Delta$

$$\Delta a+(\Delta b\circ(\Delta c))=\begin{cases}\Delta a+(\Delta b+1)=\Delta a+\Delta(b+1)=a+b+1, & \Delta b<\Delta c;\\ \Delta a+(\Delta c+1)=a+c+1, & \Delta c<\Delta b;\\ \Delta a+(\Delta c+2)=a+c+2, & \Delta c=\Delta b.\end{cases}$$

$$(\Delta b\circ\Delta c)+\Delta a=\begin{cases}a+b+1, & \Delta b<\Delta c;\\ a+c+1, & \Delta c<\Delta b;\\ a+c+2, & \Delta c=\Delta b,\end{cases}$$

As was to be shown, as at all $(a,b,c)\in\mathbf{R}_0$ the operation "+" is *distributive* concerning operation "$\circ$".

Lemma 2.11. The operation $(+)$ On a set $\mathbf{R}_\Delta$ Δ-numbers has left and right unit, and also left and right inverse element.

Proof. As $\Delta a+0=\Delta(a+0)=\Delta a$ and $0+\Delta a=\Delta(0+a)=\Delta a$, "0" - it is left and right unit. Let's remark, that $(+)-$ *"the identified multiplication"* plays a role of multiplication on a set $\mathbf{R}_\Delta$.

$\Delta(-a)-$ is the left and right inverse by the element:

$\Delta a+\Delta(-a)=a-a=0$ and $\Delta(-a)+\Delta a=-a+a=0$.

Theorem 2.5. $("\circ","\Delta"-\text{field})$. *The system of mathematical objects* $\mathrm{O}=\langle o\in\mathbf{R}_\Delta,\circ,+,-\infty,0\rangle$, *generated in an outcome of reflection on a set* Δ*-numbers* $\backslash\,\omega_1\to\omega_0\,\backslash$ *will derivate a field.*

Proof. The system O is an algebraic system, as she represents a set with defined on him by operations ("$\circ$" and "+") and ratioes (for example, >, <, =). A role of operation of multiplication, as already it was marked, in the given system plays ordinary addition $(+)$ operands.

Though the system is not closed concerning operation "+" $(\Delta a+\Delta b=$ $=(a+b)\notin\Delta)$ according to lemmas 2, 8, 9, 10, 2.8, 2.9, 2.10, 2.11 all remaining conditions for group are carried out, as: an associativity, presence of unit and inverse of the element. By virtue of a commutability of operation, identified to addition

$$a\circ b=b\circ a=\begin{cases}a+1, & a>b\\ b+1, & b>a\\ a+2, & a=b\end{cases};$$

has a place a property of an additivity, i.e. considered system - *Abelian* group. From lemmas 2.10, 2.11 is concluded, that this system is associative - commutative a ring with identity, sets of which nonempty elements will derivate group concerning operation identified to multiplication, i.e. the researched system a **field** (under condition of an elemination " of zero ", with which is the element $(-\infty)$).

Corollary 1. The groups O and $\mathbf{R}$ are *isomorphic*. a). Really, let $\Delta a,\Delta b\in\mathrm{O}$ $(\Delta a,\Delta b\in\mathbf{R}_\Delta)$, $k>0, k\neq 1$ and $\Delta a\,\backslash\,\omega_1\to\omega_0\,\backslash\,k^{\Delta a}=-k^a\in$ $\in\mathbf{R}_-\subset\mathbf{R}$, $\Delta b\,\backslash\,\omega_1\to\omega_0\,\backslash\,k^{\Delta b}=-k^b$.

Then $(\Delta a+\Delta b)\,\backslash\,\omega_1\to\omega_0\,\backslash\,k^{(\Delta a+\Delta b)}=k^{\Delta a}\cdot k^{\Delta b}=\left(-k^a\right)\cdot\left(-k^b\right)=$ $=k^{a+b}$. As $\Delta a+\Delta b=a+b$, $(\Delta a+\Delta b)\,\backslash\,\omega_1\to\omega_0\,\backslash\,k^{(\Delta a+\Delta b)}\equiv$ $\equiv(a+b)\,\backslash\,\omega_1\to\omega_0\,\backslash\,k^{a+b}$, i.e. the groups O and $\mathbf{R}$ are isomorphic on the identified multiplication (ordinary addition).

b). We shall check isomorphism O and $\mathbf{R}$ on ordinary multiplication:

$$(\Delta a\cdot\Delta b)\,\backslash\,\omega_1\to\omega_0\,\backslash\,k^{(\Delta a\cdot\Delta b)},$$

$$\Delta a\cdot\Delta b=\begin{cases} a\cdot b, & b\in N_2 \\ \Delta(a\cdot b), & b\in N_1 \end{cases};\ k^{\Delta a\cdot\Delta b}=\begin{cases} k^{a\cdot b}, & b\in N_2 \\ k^{\Delta(a\cdot b)}=-\left(k^{a\cdot b}\right), & b\in N_1 \end{cases},$$

On the other hand, $a\cdot b\setminus\omega_1\to\omega_0\setminus k^{a\cdot b}$, where $k^{a\cdot b}\in\mathbf{R}_+$ at $a>0$ both $b>0$ and $k^{a\cdot b}\in\mathbf{R}_-$ at $a>0$, $b<0$ etc., i.e. O and $\mathbf{R}$ are isomorphic on ordinary multiplication.

Corollary 2. The set $\mathbf{R}_\Delta-$numbers is a *complement* of a set $\mathbf{R}$ real numbers.

Really, $\mathbf{R}_\Delta\subset\mathbf{R}_0$, $\mathbf{R}\subset\mathbf{R_0}$. As, $\mathbf{R}_\Delta\cap\mathbf{R}=\{-\infty\}$, that, by eliminating the element $(-\infty)$ from $\mathbf{R}_\Delta$ and $\mathbf{R}$, we shall receive $\mathbf{R}_\Delta^*\cap\mathbf{R}^*=\varnothing$, where $\mathbf{R}_\Delta^*,\mathbf{R}^*-$set $\mathbf{R}_\Delta,\mathbf{R}$ with the eliminated element $(-\infty)$, i.e. $\mathbf{R}_\Delta=\mathbf{R}_0|\mathbf{R}=\{x:x\notin\mathbf{R}\}$ or $\mathbf{R}_\Delta=\mathbf{R}^d=\{x\in\mathbf{R}_0\|x|\wedge|y|=0,\ \forall y\in\mathbf{R}\}$. $\mathbf{R}_\Delta-$linear subspace $\mathbf{R}_0$ such, that: from $x\in\mathbf{R}$ and $|y|\le|x|$ follows $y\in\mathbf{R}$ etc.

Corollary 3. $\mathbf{R}_0-$*overfield* $\mathbf{R}_\Delta$ and $\mathbf{R}$.

a). As $\mathbf{R}_0=\mathbf{R}_\Delta\cup\mathbf{R}$, $\mathbf{R}_\Delta\cap\mathbf{R}=\{-\infty\}$, $\mathbf{R}_\Delta\subset\mathbf{R}_0$ and $\mathbf{R}_\Delta-$field of numbers with binary operations: *identified in the addition* ("$\circ$") *and multiplication* ("+"), given on $\mathbf{R}_0$, $\mathbf{R}_\Delta\equiv\mathbf{R}_\Delta^\sigma=\{x\in\mathbf{R}_0|\sigma(x)=x\}-$*subfield* $\mathbf{R}_0$, if $\sigma-$automorphism. (In our case, σ the reflection $\setminus\omega_1\to\omega_0\setminus$ and is put in the correspondence, if the element $\Delta x_i\setminus\omega_1\to\omega_0\setminus\Delta x'_i\left(k^{\Delta x_i}\right)$, at the inverse reflection $\Delta x'_i\setminus\omega_0\to\omega_1\setminus\Delta x$, i.e. is obtained the same element. Moreover, $\Delta x_i\setminus\omega_1\to\omega_1\setminus\Delta x_i$ and $\Delta x'_i\setminus\omega_0\to\omega_0\setminus\Delta x'_i$).

b). $\mathbf{R}_\Delta-$*complement* of a set of real numbers $\mathbf{R}$ up to $\mathbf{R}_0$: $\mathbf{R}_\Delta=\mathbf{R}_0\setminus\mathbf{R}$ at observance of a duality principle $\mathbf{R}_\Delta=\mathbf{R}_0\setminus\mathbf{R}\Rightarrow\mathbf{R}_\Delta\left(U_\xi R_\xi\right)=\cap\left(\mathbf{R}_\Delta R_\xi\right)$. By virtue of a *homeomorphism* of reflection $\setminus\omega_1\to\omega_0\setminus$ the space $\mathbf{R}_0$ is a *topological direct sum* of spaces $\mathbf{R}$ and $\mathbf{R}_\Delta$, i.e. $\mathbf{R}_\Delta-$this direct topological addition of a subspace $\mathbf{R}$, though is present some specific transformation of criterions of a complementability $\mathbf{R}$, connected with revising of the essence of the element $(-\infty)$.

So, $\mathbf{R}_0-$this *overfield* $\mathbf{R}_\Delta$ and $\mathbf{R}$.

The note. Obviously, by analogy to the extension of a field $\mathbf{R}$ up to $\mathbf{R}_0$ $(\mathbf{R}_0=\mathbf{R}\cup\mathbf{R}_\Delta)$ there are extensions of a field $\mathbf{R}_0$ up to $\mathbf{R}_1$ $(\mathbf{R}_1=\mathbf{R}_0\cup\mathbf{R}_{\Delta})$ etc. Infinite the spectrum of additions $\mathbf{R}_i/\mathbf{R}_{i-1}$ is uneasy logically to justify and to give dilatation mathematical exposition for $i=1,2,3,\ldots,m$, $\left(i=1,\ \lim\limits_{m\to\infty}m\right)$, $m=\mathrm{var}$.

Theorem 2.6. ($\circ\,\Delta-$algebra). *The system* $\mathrm{O}=\langle o\in\mathbf{R}_{\Delta},+,\cdot,-\infty,0\rangle$, *is a ring with operators (algebra) above a field* $\mathbf{R}$ *Real numbers.*

Proof. $\mathbf{R}-$the ring with identity concerning multiplication is associative-commutative. For any elements $a\in\mathbf{R}$, $\Delta b\equiv o\in\mathrm{O}$ the product is univalently defined:

$$a\cdot(\Delta b)=\Delta(a\cdot b)\in\mathrm{O}.$$

Let $(a,b)\in\mathbf{R}$, $(\Delta a',\Delta b')\in\mathrm{O}$. Then

a). $1\cdot\Delta a=\Delta(1\cdot a)=\Delta a\in\mathrm{O}$;

b). $(a+b)\cdot\Delta a'=a\cdot\Delta a'+b\cdot\Delta a'$; $a\cdot(\Delta a'+\Delta b')=a\cdot(\Delta(a'+b'))=$
$=\Delta(a\cdot(a'+b'))=\Delta(a\cdot a'+a\cdot b')=\Delta a\cdot a'+\Delta a\cdot b'=a\cdot\Delta a'+a\cdot\Delta b'$;
i.e. $a\cdot(\Delta a'+\Delta b')=a\cdot\Delta a'+a\cdot\Delta b'$.

c). $a\cdot(b\cdot\Delta a')=a\cdot\Delta(b\cdot a')=\Delta(a\cdot b\cdot a')$, $(a\cdot b)\cdot\Delta a'=\Delta(a\cdot b\cdot a')$,
i.e. $a\cdot(b\cdot\Delta a')=(a\cdot b)\cdot\Delta a'$.

d). $A=a\cdot(\Delta a'\cdot\Delta b')=a\cdot(\Delta((\Delta a')\cdot b'))=a\cdot(\Delta(a'\cdot b'))=\Delta(a\cdot a'\cdot b')$ at $b'\in N_2$ and $A=a\cdot(\Delta(\Delta(a'\cdot b')))=a\cdot a'\cdot b'$ at $b'\in N_1$.

On the other hand, $(a\cdot\Delta a')\cdot\Delta b'=\Delta(a\cdot a')\cdot\Delta b'=\Delta((\Delta(a\cdot a'))\cdot b')=$
$=\begin{cases}\Delta(a\cdot a'\cdot b'), & b'\in N_2\\ a\cdot a'\cdot b', & b'\in N_1\end{cases}$, i.e. $a\cdot(\Delta a'\cdot\Delta b')=(a\cdot\Delta a')\cdot\Delta b'$. Besides
$A=\Delta a'\cdot(a\cdot\Delta b')$, as $A=\Delta a'\cdot\Delta(a\cdot b')=\Delta((\Delta a')\cdot a\cdot b')=$

$$=\begin{cases}\Delta a'\cdot a\cdot b', & b'\in N_2\\ a'\cdot a\cdot b', & b'\in N_1\end{cases}.$$

Thus, the system O satisfies to all conditions of algebra above $\mathbf{R}$, as was to be shown.

Lemma 2.12. The images $\backslash\omega_i\to\omega_{i+1}\backslash$ are invariant concerning an index "i" $i\in\mathbf{Z}$ and are *isomorphic*.[15]

Proof. Let's designate $\varphi\equiv\backslash\omega_{-1}\to\omega_0\backslash$, $\varphi-$one-to-one image of a set Δ on a set $\mathbf{R}$ by virtue of, as already it was specified, uniqueness of function $y=\mathrm{slog}_k a$. $\mathbf{R}\backslash\omega_0\to\omega_{-1}\backslash\Delta$, and the function of image $\backslash\omega_0\to\omega_{-1}\backslash$ is equal $(-1+\mathrm{slog}_k x)*k=\log_k x$ $(k>0, k\neq 1)$, i.e. for anyone $x=a\in\mathbf{R}$ it is possible to note:

[15] Though in lemmas 2.1-2.7 was proved изоморфность $\omega-$images, in connection with it explicitly understands a lemma 2.12 again this problem with a dominating role in proofs of offered statements.

$$|a| \setminus \omega_{-1} \to \omega_0 \setminus \log_k |a|.$$

Let $a_1, a_2 \in \mathbf{R}$, $|a_1| \setminus \omega_{-1} \to \omega_0 \setminus \log_k |a_1|$, $|a_2| \setminus \omega_{-1} \to \omega_0 \setminus \log_k |a_2|$. Then $|a_1| \cdot |a_2| \setminus \omega_{-1} \to \omega_0 \setminus \log_k (|a_1| \cdot |a_2|) = \log_k |a_1| + \log_k |a_2|$. In a common case $\prod_{i=1}^{n} |a_i| \setminus \omega_{-1} \to \omega_0 \setminus \sum_{i=1}^{n} \log_k |a_i|$. Therefore, $\setminus \omega_{-1} \to \omega_0 \setminus -$ isomorphism.

Similarly, $|a_1| \odot |a_2| \setminus \omega_{-1} \to \omega_0 \setminus \log_k |a_1| \cdot \log_k |a_2| = \log_k (|a_1|)^{\log_k |a_2|} = \log_k (|a_1| \odot |a_2|)$ or $|a_1| \odot |a_2| = |a_1|^{\log_k |a_2|} \setminus \omega_{-1} \to \omega_0 \setminus \log_k (|a_1|)^{\log_k |a_2|} = \log_k (|a_1| \odot |a_2|)$. By designating ω_0 for ω_1, and ω_{-1} for ω_0, we shall receive: for all $a \in \mathbf{R}$ $\log_k |a| \setminus \omega_1 \to \omega_0 \setminus k^{\log_k |a|} = |a|$, and $|a| \setminus \omega_0 \to \omega_1 \setminus \log_k |a|$.

Sequentially changing an index "i" by magnification it on “1”, we shall receive $\setminus \omega_{-1} \to \omega_0 \setminus \equiv \setminus \omega_0 \to \omega_1 \setminus \equiv \ldots \equiv \setminus \omega_i \to \omega_{i+1} \setminus$, i.e. the reflection $\setminus \omega_i \to \omega_{i+1} \setminus$ is invariant concerning an index "i".

So, if $(a_i, a'_i) \in \omega_i$, from $\begin{cases} |a_i| \setminus \omega_i \to \omega_{i+1} \setminus \log_k |a_i| \\ |a'_i| \setminus \omega_i \to \omega_{i+1} \setminus \log_k |a'_i| \end{cases}$ follows, that

$|a_i| \cdot |a'_i| \setminus \omega_i \to \omega_{i+1} \setminus \log_k (|a_i| \cdot |a'_i|) = \log_k |a_i| + \log_k |a'_i|$ and

$|a_i| \odot |a'_i| \setminus \omega_i \to \omega_{i+1} \setminus \log_k (|a_i| \odot |a'_i|) = \log_k (|a_i|)^{\log_k |a'_i|} = \log_k |a_i| \times$ $\times \log_k |a'_i|$, that confirms *isomorphism* of image $\setminus \omega_i \to \omega_{i+1} \setminus$ on ordinary and reflexive multiplyings, and also invariance φ rather "i".

Corollary. The image $\setminus \omega_i \to \omega_{i+1} \setminus$ is *isomorphic* on any operation of multiplication (identified, ordinary, reflexive etc.). The hierarchy of operations is those: $\ldots, \circ, +, \cdot, \odot, \ldots$. Any of these operations can be accepted for multiplication at appropriate adjustment of a rank of other operations. However, it is not necessary to forget, that the operation “$\circ$” as against operations $\{+, \cdot, \odot\}$ is not of adequate operation easier than addition obtained a way $\omega-$procedure, i.e., if $+ \setminus \omega_1 \to \omega_0 \setminus \bullet \setminus \omega_1 \to \omega_0 \setminus \odot$ etc., that, reverberate "+" $\setminus \omega_0 \to \omega_1 \setminus$, in an outcome we shall not receive operation “$\circ$”. So, “$\circ$” - does not enter into a series of operations identical to multiplication, and is one of establishing (fundamental) operations.

Lemma 2.13 (about a hierarchy of numbers). The set $\mathbf{R}_0$ contains a subset $\left\{\mathbf{R}_0^i\right\}$ $\left(\mathbf{R}_0 \supset \left\{\mathbf{R}_0^i\right\}\right)$, consisting from related on a structure and properties of classes of numbers with the following hierarchy: $\left\{\mathbf{R}_0^i\right\} = \left\{\cdots \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir} \cdots\right\}$.

Proof.[16] Any set of numbers from $\left\{\cdots \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir} \cdots\right\}$ on a set $\mathbf{R}_0$ $\left(\mathbf{R}_0 = \mathbf{R}_\Delta \cup \mathbf{R}\right)$ can be considered as an outcome of a sequential circuit homogeneous $\omega-$reflections. Really,

$$\mathbf{R}_\Delta \backslash \omega_1 \to \omega_0 \backslash \mathbf{R}_-, \quad \mathbf{R}_- \backslash \omega_1 \to \omega_0 \backslash \mathbf{R}_f,$$

$$\mathbf{R}_f \backslash \omega_1 \to \omega_0 \backslash \mathbf{R}_{ir} \text{ Or } \mathbf{R}_\Delta \backslash \omega_{i+3} \to \omega_{i+2} \backslash \mathbf{R}_-,$$

$$\mathbf{R}_- \backslash \omega_{i+2} \to \omega_{i+1} \backslash \mathbf{R}_f, \ \mathbf{R}_f \backslash \omega_{i+1} \to \omega_i \backslash \mathbf{R}_{ir}.$$

Let $\left\{\mathbf{R}_0^i\right\} = \left\{\cdots \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir} \cdots\right\}$, and, $\left\{\mathbf{R}_0^i\right\} \subset \mathbf{R}_0$, as $\mathbf{R}_0^i$ is formed from $\mathbf{R}_0$ elemination of some classes of numbers.

The theorem 2.7 (about a factor set).

In a set $\mathbf{R}_0$ *the factor set* $\left\{\mathbf{R}_0^i\right\}$ *with induced classes* $\ldots \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir} \cdots$ *is inserted.*

Proof. All classes $\left(\cdots \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir} \cdots\right)$ set $\left\{\mathbf{R}_0^i\right\}$ are equivalence classes on any fixed equivalence relation, i.e. on a binary ratio possessing properties of a reflexivity, symmetry and transitivity. Moreover, the passage from one class to anyone to another is connected to appropriate image of addition and multiplication in the similar transformed operations. (For example, "*identified*" addition and multiplication are reverberate in *ordinary* addition and multiplication, which, in turn, are reverberate in the *reflexive* forms of these operations. Division (:) at reflection $\backslash \omega_1 \to \omega_0 \backslash$ is transformed in reflexive $(\triangle)$). The image of one class $\left\{\mathbf{R}_0^i\right\}$ in anyone another is carried out with the help of functions $\varphi = \left(i + \mathrm{slog}_k a\right) * k, \quad k > 0, \quad k \neq 1$. For example, mentioned above sequential images of classes in a circuit $\mathbf{R}_\Delta \xrightarrow{\varphi_1} \mathbf{R}_- \xrightarrow{\varphi_1} \mathbf{R}_f \xrightarrow{\varphi_1} \mathbf{R}_{ir}$ are realized with the help of the functions $\varphi_1 = \left(1 + \mathrm{slog}_k a\right) * k$, and $\left.\begin{matrix} \mathbf{R}_\Delta \to \mathbf{R}_f \\ \mathbf{R}_- \to \mathbf{R}_{ir} \end{matrix}\right\} \ \varphi_2 = \left(2 + \mathrm{slog}_k a\right) * k$ etc.

[16] The detailed proof of a lemma 2.13 is not reduced in connection with an evidence of outcomes $\omega-$reflection and proof of a lemma 2.12.

In view of lemmas 2.12 and 2.13 $\{\mathbf{R}_0^i\}$ is a *factor set* with induced classes $\ldots\mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir}\ldots$. According to a lemma 2.13 $\{\mathbf{R}_0^i\}\subset\mathbf{R}_0$, i.e. $\{\mathbf{R}_0^i\}$ is inserted in $\mathbf{R}_0$.

Theorem 2.8. *The image of any class in a factor set* $\{\mathbf{R}_0^i\}$ *is canonical image.*

Proof. The proof immediately follows from the theorem 2.7. Let's consider a proof on examples. Let $\Delta a\in\mathbf{R}_\Delta$, $k>0$, $k\neq 1$. Then the image $\mathbf{R}_\Delta\to\mathbf{R}_0^i$ generates a set (class) identified on $\omega-$reflection of the elements: $A_1=\left\{\Delta a;-\left(k^a\right);\ :\left(k^{k^a}\right);\ \triangle\left(k^{k^{k^a}}\right)\right\}$.

Really, $\Delta a\setminus\omega_1\to\omega_0\setminus k^{\Delta a}=-\left(k^a\right)$; $-\left(k^a\right)\setminus\omega_1\to\omega_0\setminus k^{-\left(k^a\right)}=$

$$=1\Big/k^{k^a}=:\left(k^{k^a}\right);\ \left(:\left(k^{k^a}\right)\right)\setminus\omega_1\to\omega_0\setminus k\triangle\left(k^{k^{k^a}}\right)=\triangle\left(k^{k^{k^a}}\right).$$

Let's image $\mathbf{R}_-$ in $\mathbf{R}_0^i$. An outcome will be the following class of the identified elements: $A_2=\left\{-a;\dfrac{1}{k^a};\triangle\left(k^{k^a}\right)\right\}$. Really, $-a\setminus\omega_1\to$

$$\to\omega_0\setminus k^{-a}=\frac{1}{k^a}=:k^a,\ \left(:\left(k^a\right)\right)\setminus\omega_1\to\omega_0\setminus k\triangle\left(k^{k^a}\right)=\triangle\left(k^{k^a}\right).$$

Similarly, for fractional numbers:

$$\mathbf{R}_f\to\mathbf{R}_{\bar{0}}\Rightarrow A_3=\left\{(:a);\ \triangle\left(k^a\right)\right\}.$$

The regularities of shaping of classes A_i are obvious.

All classes $A_1=\left\{\Delta a;-\left(k^a\right);:\left(k^{k^a}\right);\triangle\left(k^{k^{k^a}}\right)\right\}$, $A_2=\left\{-a;:\left(k^a\right);\triangle\left(k^{k^a}\right)\right\}$, $A_3=\left\{(:a);\ \triangle\left(k^a\right)\right\}$ are equivalence classes, as for anyone A_i all conditions necessary for equivalence classes are fulfilled. The elements - preimages (elements their forming), therefore all this *canonical images* enter into all A_i.

Theorem 2.9 (about a tower of fields).

The sets $\mathbf{R}_0, \{\mathbf{R}_0^i\}, \mathbf{R}'_0, \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir}$ *makes homogeneous on* ω*-image of a tower of fields.*

Proof. Because of above-stated line-up of fields $\mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir}$ represents a discrete spectrum of fields interconnected ω-image. The hierarchy them requires observance of sequential passage $\mathbf{R}_\Delta \to \mathbf{R}_- \to \mathbf{R}_f \to \mathbf{R}_{ir}$, where each act of passage is described by the same function of image and is equivalent on transformation of objects.

The set $\{\mathbf{R}_0^i\}$ is an overfield for any object - set from a spectrum of fields $\{\dots, \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir}, \dots\}$, i.e. $\left\{\{\mathbf{R}_0^i\} \supset \mathbf{R}_\Delta; \{\mathbf{R}_0^i\} \supset \mathbf{R}_-; \{\mathbf{R}_0^i\} \supset \mathbf{R}_f;\right.$ $\left.\{\mathbf{R}_0^i\} \supset \mathbf{R}_{ir}\right\}$. In turn, $\mathbf{R}_0$ is the extension of a field $\{\mathbf{R}_0^i\}$, i.e. *overfield* $\{\mathbf{R}_0^i\}$. From $\mathbf{R}_0 \supset \{\mathbf{R}_0^i\} \;\Rightarrow\; \mathbf{R}_0 -$ *overfield* of each of objects of a field $\{\mathbf{R}_0^i\}$.

Let's generate *towers of fields*: $\mathbf{R}_0 \supset \{\mathbf{R}_0^i\} \supset \mathbf{R}_\Delta$; $\mathbf{R}_0 \supset \{\mathbf{R}_0^i\} \supset \mathbf{R}_-$; $\mathbf{R}_0 \supset \{\mathbf{R}_0^i\} \supset \mathbf{R}_f$; $\mathbf{R}_0 \supset \{\mathbf{R}_0^i\} \supset \mathbf{R}_{ir}$. All of them differ by the last element. The set of these elements, as is known, is connected with ω-reflection. The theorem is proved.

Theorem 2.10 (about an ideal).

The set $\mathbf{I} = \{\Delta b \in \mathbf{R}_\Delta, b \in N_2\}$ *is a left ideal of a ring* $\mathbf{R}_\Delta$ $\Delta-$*numbers* $(\mathbf{I} \subset \mathbf{R}_\Delta)$ *on ordinary multiplication.*

Proof. Let $a, b \in \mathbf{Z}$. Then for anyone $\Delta a \in \mathbf{R}_\Delta$ and $\Delta b \in \mathbf{I}$ has a place a relation $\Delta a \cdot \Delta b \in \mathbf{R}_\Delta$, as $\Delta a \cdot \Delta b = \Delta((\Delta a) \cdot b) = \Delta(a \cdot b) \in \mathbf{R}_\Delta$ for $b \in N_2$. Moreover, $\Delta(a \cdot b) \in \mathbf{I}$ that $a, b \in N_2$. I.e. from $\mathbf{R}_\Delta \cdot \mathbf{I} = \{\Delta a \cdot \Delta b | \Delta a \in \mathbf{R}_\Delta, \Delta b,$ $\Delta a \cdot \Delta b \in \mathbf{I}\} \Rightarrow \mathbf{R}_\Delta \cdot \mathbf{I} = \mathbf{I}$. Therefore, $\mathbf{I}-$*left ideal*, as was to be shown.

The note. $\Delta b \cdot \Delta a = \Delta((\Delta b) \cdot a) = b \cdot a \notin \mathbf{I}$ at $a \in N_1$, i.e. $\mathbf{I}-$is not a right ideal.

Lemma 2.14. The outcome of a taking the logarithm of a set $\mathbf{R}^i$ is a set $\mathbf{R}^{i-1}$, where $\mathbf{R}^i \equiv \{\mathbf{R}_0^i\}$.

Proof.

Let's consider special cases: $\mathbf{R}^i = \{\dots, \mathbf{R}_{\triangle}, \mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \dots\}$, where $\mathbf{R}_f = \mathbf{R}^j$, $\mathbf{R}_- = \mathbf{R}^{j-1}$, $\mathbf{R}_\Delta = \mathbf{R}^{j-2}$, $\mathbf{R}_{\triangle} = \mathbf{R}^{j-3}$. Let $a \in \mathbf{R}_+$, $a \neq \{0; 1\}$. For anyone a the relations are fair: $(:a) \in \mathbf{R}_f = \mathbf{R}^j$, $\ln(:a) = (-\ln a + 0) \in \mathbf{R}_- = \mathbf{R}^{j-1}$, i.e. $\ln(\mathbf{R}^j) = \mathbf{R}^{j-1}$.

Similarly, for $a \in \mathbf{R}_-$, i.e. $a \in \mathbf{R}^{j-1}$ and $\ln(-|a|) = \Delta 0 + \ln|a| = \Delta \ln|a|$. Then $\ln\{-a_i\} = \{\Delta \ln a_i\} = \mathbf{R}_\Delta = \mathbf{R}^{j-2}$, i.e. $\ln \mathbf{R}^{j-1} = \mathbf{R}^{j-2}$;

$\ln(\Delta a) = \ln a + \underline{\triangle}0 = \underline{\triangle}\ln a \in \mathbf{R}_{\underline{\triangle}} = \mathbf{R}^{j-3}$, i.e.

$$\ln\left(\mathbf{R}^{j-2}\right) = \mathbf{R}^{j-3}.$$

In a common case, $\ln\left(\mathbf{R}^{i}\right) = \mathbf{R}^{i-1}$.

Corollary. $\mathbf{R}_+^{\mathbf{R}_-} = \mathbf{R}_f$; $\mathbf{R}_+^{\mathbf{R}_\Delta} = \mathbf{R}_-$; $\mathbf{R}_+^{\mathbf{R}_{\underline{\triangle}}} = \mathbf{R}_\Delta$.

Theorem 2.11. ***If*** $a, b \in \mathbf{R}$, $\mathbf{R}_0^i$ ***– set of numbers (classes) obtained*** ω ***– reflection of the elements of a hierarchical line-up, constructed on an indication of reflection*** $\backslash \omega_i \to \omega_0 \backslash$ ***and represented in a lemma 2.13, and*** $(a_i, b_i) \in \mathbf{R}_0^i$, $\odot_i$ ***– operation of multiplication for the given class,***

$$\boxed{a_i \odot_i b_i = a \odot_i b}.$$

Proof. Not stopping on a common proof of the theorem (it rather complicatedly also requires a special nomenclature and indexation), we shall consider some most spreaded special cases.

a). Let $a_i = \Delta a$, $b_i = \Delta b$, where $\Delta a, \Delta b \in \mathbf{R}_\Delta$. Then $\odot_i \equiv (+)$ – "*identified*" multiplication of the first order.

$\boxed{\Delta a + \Delta b = a + b}$ according to properties Δ – numbers.

b). If $(a_i, b_i) \in \mathbf{R}_-$, $\odot_i \equiv (\,\cdot\,)$ – *ordinary* multiplication and $\boxed{(-|a|)\cdot(-|b|) = |a|\cdot|b|}$.

c). Let $(a_i, b_i) \in \mathbf{R}_f$, $\odot_i \equiv (\odot)$ – *reflexive* multiplication of the *first* order $(i = 1)$:

$$(:a)\odot(:b) = (:a)^{\log_k(:b)} = a^{\log_k b} = a \odot b,$$

$$\boxed{(:a)\odot(:b) = a \odot b}.$$

d). Let $a_i = \underline{\triangle}a$, $b_i = \underline{\triangle}b$, i.e. $\odot_i \equiv \boxtimes$ – *reflexive* multiplication of the *second* order $(i = 2)$. Then

$$(\underline{\triangle}a)\boxtimes(\underline{\triangle}b) = \left(k^{\left(1/\log_k a\right)}\right)\boxtimes\left(k^{\left(1/\log_k b\right)}\right) =$$

$$= k^{k^{\left(\left(\log_k \log_k k^{\left(1/\log_k a\right)}\right)/\left(\log_k \log_k k^{\left(1/\log_k b\right)}\right)\right)}} =$$

$$= k^{k^{\left((-\log_k \log_k a)/(-\log_k \log_k b)\right)}} = k^{k^{\left((\log_k \log_k a)/(\log_k \log_k b)\right)}} = a \boxtimes b,$$

$$\boxed{(\underline{\triangle}a)\boxtimes(\underline{\triangle}b) = a \boxtimes b}.$$

e). Let $a_i = ⌑a$, $b_i = ⌑b$, i.e. $\odot_i \equiv \blacklozenge_3$ − *reflexive* multiplication of the *third* order $(i=3)$. Then by designating for $A = k^{k^{\left(1/\log_k \log_k a\right)}}$ and $B = k^{k^{\left(1/\log_k \log_k b\right)}}$, we shall receive:

$$(⌑a)\blacklozenge_3(⌑b) = k^{k^{k^{\left(\frac{\log_k \log_k \log_k A}{\log_k \log_k \log_k B}\right)}}} = k^{k^{k^{\left(\frac{-\log_k \log_k \log_k a}{-\log_k \log_k \log_k b}\right)}}} = a\blacklozenge_3 b,$$

$$\boxed{(⌑a)\blacklozenge_3(⌑b) = a\blacklozenge_3 b},$$

as was to be shown.

Lemma 2.15. At $a \in \mathbf{R}_+$, $a \neq 1$ the equality is fair

$$\left\{a^{\mathbf{R}_+^i}\right\} = \left\{a^{\mathbf{R}_+}\right\}^{i-1},$$ [16]

Where $i = 1,2,3,\ldots$ and $\mathbf{R}_+^1 \equiv :\mathbf{R}_+$, $\mathbf{R}_+^2 \equiv -\mathbf{R}_+ = \mathbf{R}_-$; $\mathbf{R}_+^3 \equiv \Delta\mathbf{R}_+$, $\mathbf{R}_+^4 \equiv ◬\mathbf{R}_+, \ldots$.

Proof. Let's consider special cases:

a). $\left\{a^{\mathbf{R}_-}\right\} = \left\{a^{-\mathbf{R}_+}\right\} = \left\{\frac{1}{a^{\mathbf{R}_+}}\right\} = \left\{:\left(a^{\mathbf{R}_+}\right)\right\} = :\left\{a^{\mathbf{R}_+}\right\}$. Whence,

$$\left\{a^{\mathbf{R}_+^2}\right\} = \left\{a^{\mathbf{R}_+}\right\}^1.$$

b). $\left\{a^{\Delta\mathbf{R}_+}\right\} = \left\{-\left(a^{\mathbf{R}_+}\right)\right\} = -\left\{a^{\mathbf{R}_+}\right\}$, i.e. $\left\{a^{\mathbf{R}_+^3}\right\} = \left\{a^{\mathbf{R}_+}\right\}^2$.

c). $\left\{a^{◬\mathbf{R}_+}\right\} = \left\{\Delta\left(a^{\mathbf{R}_+}\right)\right\} = \Delta\left\{a^{\mathbf{R}_+}\right\}$; $\left\{a^{\mathbf{R}_+^4}\right\} = \left\{a^{\mathbf{R}_+}\right\}^3$ etc. In a common case, $\left\{a^{\mathbf{R}_+^i}\right\} = \left\{a^{\mathbf{R}_+}\right\}^{i-1}$.

---

[16] In terms of similar $\left\{a^{\mathbf{R}_+^i}\right\}$ both $\left\{a^{\mathbf{R}_+}\right\}^{i-1}$ numbers i and $i-1$ mean an index of a set, instead of degree.

Lemma 2.16. If $\mathbf{R}^j$ – *global* sets[17] , where $\mathbf{R}^0 \equiv \mathbf{R}$, $\mathbf{R}^1 \equiv \mathbf{R}_\Delta$, $\mathbf{R}^2 \equiv \mathbf{R}_\triangle \ldots$, $\mathbf{R}^j + \mathbf{R}^0 = \mathbf{R}^j$.

Proof. Let's consider special cases.

a). Let $a, b \in \mathbf{R}$, $(a+b) \in \mathbf{R}$, i.e. $\mathbf{R}^0 + \mathbf{R}^0 = \mathbf{R}^0$.

b). Let $a, b \in \mathbf{R}$, $\Delta a + b = \Delta(a+b) \in \mathbf{R}_\Delta$, i.e. $\mathbf{R}^1 + \mathbf{R}^0 = \mathbf{R}^1$.

c). If $a, b \in \mathbf{R}$, $\triangle a + b = \triangle(a+b) \in \mathbf{R}_\triangle$, i.e. $\mathbf{R}^2 + \mathbf{R}^0 = \mathbf{R}^2$ etc.

Investigating sets of numbers obtained ω-reflections, i.e. systematized on ω-factor, it is possible to notice that:

$\mathbf{R}_- =]-\infty; 0[$, $\lim\limits_{a \to \infty} (-a) = (-\infty)$, i.e. the limit is the zero element of a set Δ-numbers, and $\lim\limits_{a \to 0} (-a) = 0$ – the second extreme element is a zero element of a set of numbers $\mathbf{R}_-$.

$\mathbf{R}_f =]0; 1[$, $\lim\limits_{a \to \infty} \left(\frac{1}{a}\right) = 0$ – zero element of negative numbers; $\lim\limits_{a \to 1} \left(\frac{1}{a}\right) = 1$ – it is the zero element of fractional numbers $(:a)$. $\mathbf{R}_\triangle =]1; k[$, $\lim\limits_{a \to \infty} (\triangle a) = 1$ – zero element of numbers $(:a)$, $\lim\limits_{a \to k} (\triangle a) = k^{1/\log_k k} = k$.

The zero element $\triangle$ – numbers is equal k.

$\mathbf{R}_{¤} =]^2k; k[$, $\lim\limits_{a \to \infty} (¤a) = \lim\limits_{a \to \infty} k^{k^{(1/\log_k \log_k a)}} = k$ – it is the zero element of numbers $¤\triangle$ etc. $\left(\lim\limits_{a \to {}^2k} ¤a = {}^2k\right)$.

---

[17] $\{\mathbf{R}^j\} = \{\mathbf{R}, \mathbf{R}_\Delta, \ldots\}$.

The note 1. It is uneasy to lead also similar analysis of sets of numbers $\mathbf{R}_{\Delta_i}$ with the purpose of deriving intervals, in which these numbers "are located" also of determination of the zero elements of researched sets.

The note 2. At reviewing the given problem it is necessary to take into account dominating binary operations (addition and multiplication) for a researched class of numbers. For example, for a set $\mathbf{R}-$negative numbers it there will be ordinary addition and multiplication $(-a+0=-a;\ (-a)\cdot 0=0)$, and for fractional numbers - reflexive operations, i.e. $(:a)\cdot 1=(:a);\ (:a)\odot 1=1$ etc.

§ 2.6 Classifications of operations and numbers on ω-factor. [50]

Not stopping explicitly on the carried out operations research with two operands, we state the fact of existence infinite of a spectrum of these operations.

The known part of these operations is reduced in a table 1.

`Table 1.` **Operations with two operands** $\left(n\in\mathbf{Z}\wedge i\in\{1,2,3\}\right)$[18].

n \ *i*	1	2	3
...	...	...	...
0	a∘b=c	cΔb=a	cΔa=b
1	a+b=c	c-b=a	c-a=b
2	a·b=c	c/b=a	c/a=b
3	a^b=c	$\sqrt[b]{c}=a$	$\log_a c=b$
4	$^b a$=c	$^b\hat{\surd}$ c=a	$\mathrm{slog}_a c=b$
...	...	...	...

The fact of a prolongation of a cycle of the definitions of operations in a leg of magnification n $(n>4)$ is quite obvious. And, in it infinities of operations quite pellucid. Really, we shall take operation $n=5$. At $i=1$ operation $\underline{^b}a=\left.{}^{\cdot^{\cdot^{a}}}{}^{a}a\right\}b$ (by analogy $\underbrace{a\cdot a\cdot a\cdot\ldots\cdot a}_{b}=a^b$ and $^b a=\left.a^{a^{\cdot^{\cdot^{a}}}}\right\}b$). For example, $\underline{^3}2={}^{2^2}2={}^4 2=2^{16}$. At $i=2$ $\underline{^b}\hat{\surd}c=a$, and at $i=3$ $\mathrm{sslog}_a c=b$ (from operation $\mathrm{sslog}_a c=b$ follows $\underline{^b}a=c$).

[18] $n\in\mathbf{Z}$ - it is a classical case, in a common case $n\in\mathbf{R}$.

The drop $n<0$ Too should conduct to infinities of operations, though this situation needs difficulty mathematically to be described. It is necessary also to notice, that at $n \notin \mathbf{Z}$, probably, there are operations, which exposition is based on the other principle (at $n \in \mathbf{Z}$ operations were formed on a method of analogies).

As it is visible from a table, in infinities a set of operations the transpostion of ratioes of inversion $(i=2,3)$ is realized.

Extrapolation of operations in a leg of magnification of their index $(n>4)$ is complicated, on the one hand, in connection with necessity to operate with sets possessing specific and little explore properties, on the other hand, with origin of a series of alternate and paradoxical situations, which only partially can be solvable by methods of the descriptive theory of sets and way inclusion in known axiomaticses (Peano etc.) of complementary improvements for areas there are enough of large magnitudes and tending to infinity. It is necessary to notice, that the deriving of outcomes of operations at $n=4$ does not represent large complexity.

The given classification of operations not unique. Let's consider a classification of operations and numbers on ω-factor. As follows from a lemma 2.13, any set from $\{\mathbf{R}_\Delta, \mathbf{R}_-, \mathbf{R}_f, \mathbf{R}_{ir}\}$ On a set $\mathbf{R}_0$ – is an outcome of a sequential circuit homogeneous ω-reflections, as

$$\mathbf{R}_\Delta \setminus \omega_1 \to \omega_0 \setminus \mathbf{R}_- \setminus \omega_1 \to \omega_0 \setminus \mathbf{R}_f \setminus \omega_1 \to \omega_0 \setminus \mathbf{R}_{ir}$$

This circuit can be continued to the left or to the right ad infinitum. Thus $\mathbf{R}_\Delta =]\Delta\infty, -\infty[$, $\mathbf{R}_- =]-\infty, 0[$, $\mathbf{R}_f =]0, 1[$. The set $\mathbf{R}_{ir}$, as it was specified, represents infinities a spectrum infinite of sets of a type:

$\mathbf{R}_{\text{◬}} =]1, k[$, where $k \in \mathbf{R}_0$ (in the elementary case $k \in \mathbf{R}$) and $k \neq 1$. ◬ – the numbers turn out under the formula $\text{◬}a = k\text{◬}a = k^{1/\log_k a}$; $\mathbf{R}_{\text{¤}} = \left]k, {}^2k\right[$, where $\text{¤}a = {}^2k\,\text{¤}\,a = k^{k^{(1/\log_k \log_k a)}}$; $\mathbf{R}_{\text{✧}} = \left]{}^2k, {}^3k\right[$, where $\text{✧}a = \; = {}^3k\,\text{✧}\,a = k^{k^{k^{(1/\log_k \log_k \log_k a)}}}$ etc.

The set of operations $\{\ldots; \underline{\triangle}; \Delta; -; \div; \text{◬}; \text{¤}; \text{✧}; \ldots\}$ and set of numbers $\{\ldots; \mathbf{R}_{\underline{\triangle}}; \mathbf{R}_\Delta; \mathbf{R}_-; \mathbf{R}_f; \mathbf{R}_{\text{◬}}; \mathbf{R}_{\text{¤}}; \mathbf{R}_{\text{✧}}; \ldots\}$ represents most logically justified sequence of classes of operations and numbers generated on ω-factor. Let's remark, that the above-stated circuit of operations and numbers allows us to select in a set of positive numbers $\mathbf{R}_+$ sets of numbers, which on a significance correspond to such classes of numbers as negative $\mathbf{R}_{-}$ or fractional $\mathbf{R}_f$ $\left(\mathbf{R}_f = \left\{ :a| :a = \frac{1}{a}, \quad a \in \mathbf{R} \right\}\right)$. Moreover, dominating operation for these numbers simultaneously is defined. Actually, the operations of a type $\triangle, \Delta, ◬, ⌑, ✧$ and numbers $\mathbf{R}_{\triangle}$, $\mathbf{R}_{\Delta}$, $\mathbf{R}_{◬}$, $\mathbf{R}_{⌑}$, $\mathbf{R}_{✧}$ are mathematical objects of a new nature.

Infinities of each of sets of a type $\mathbf{R}_{◬} \equiv \{k◬a\} \equiv \{◬a\}$, $\mathbf{R}_{⌑} \equiv \left\{{}^{2}k⌑a\right\} \equiv \{⌑a\}$, $\mathbf{R}_{✧} \equiv \left\{{}^{3}k✧a\right\} \equiv \{✧a\}$ is reached by two paths: by a modification of values of factor k and basic component a.

Theorem 2.12. *If* $\mathbf{R}^i$ *one of sets in infinities a spectrum of sets of numbers classified on* ω*-indication, i.e. in a series* $\ldots; \mathbf{R}_{\triangle}; \mathbf{R}_{\Delta}; \mathbf{R}_{-}; \mathbf{R}_f;\ \mathbf{R}_{◬}; \mathbf{R}_{⌑}; \mathbf{R}_{✧}; \ldots$, *where* i *— number of a set under condition of fixing any of a set as basic,*

$$\underbrace{\log_k \log_k \ldots \log_k}_{j} \mathbf{R}^i = \mathbf{R}^{i-j},$$

where $\mathbf{R}^{i-j}$ *— set in indicated to a series deleted on* j *— of units to the left from set* $\mathbf{R}^i$.

Proof. In a series of sets of numbers $\ldots; \mathbf{R}_{\triangle}; \mathbf{R}_{\Delta}; \mathbf{R}_{-}; \mathbf{R}_f;$ $\mathbf{R}_{◬}; \mathbf{R}_{⌑}; \mathbf{R}_{✧}; \ldots$ Everyone will be derivated from consequent by reflection $\backslash \omega_{i+1} \to \omega_i \backslash$. By virtue of a lemma 2.12 it is possible to replace $\backslash \omega_{i+1} \to \omega_i \backslash$ on $\backslash \omega_1 \to \omega_0 \backslash$. Then $\mathbf{R}^i \backslash \omega_1 \to \omega_0 \backslash k^{\mathbf{R}^i}$. After a taking the logarithm we shall receive $\log_k \mathbf{R}^i \backslash \omega_1 \to \omega_0 \backslash \mathbf{R}^i$. $\mathbf{R}^{i-j}$ — this set obtained from a set $\mathbf{R}^i$ way j acts of inverses ω- images, i.e. images of a type $\backslash \omega_0 \to \omega_1 \backslash$. Taking into account a lemma 2.14, we shall note $\log_k \mathbf{R}^{i-j} \backslash \omega_1 \to \omega_0 \backslash \mathbf{R}^{i-j+1}$ and $\log_k \mathbf{R}^i = \mathbf{R}^{i-1}$. Whence follows, that

$$\underbrace{\log_k \log_k \ldots \log_k}_{j} \mathbf{R}^i = \mathbf{R}^{i-j}$$

In summary we shall remark, that in the chapter "Miscellany" (application) to the basic text of the book some additional facts about numbers of a new nature are explained.

§ 2.7 Problematics of numbers of a new nature

The represented above outcomes of mathematical searching of numbers of a new nature (numbers of a type $\mathbf{R}_{\triangle}; \mathbf{R}_{\Delta}; \mathbf{R}_{◬}; \mathbf{R}_{⌑}; \mathbf{R}_{✧}$ and others) are only initial attempt of a *large* mathematical research. Certainly, in small work it is impossible even to enumerate all set of problems, originating in this direction. Moreover, some problems, which were already solved by the author are not included in the text of the book. A part of problems is enumerated only below which, under the judgement of the author, arise in this research:

1). It is necessary mathematically to justify and to analyze all set of *operations* "easier" than addition. To find common regularities in these operations, and also to find and to prove the new invariant formulas.

2). To study a spectrum of numbers located on numerical direct for $(-\infty)$. To find dilatations of a property and to give an axiomatics of numbers of a type $\mathbf{R}_{\triangle}, \mathbf{R}_{\Delta}$ etc. *To create the common theory* Δ*-algebras.* In particular, to show operations above numbers in a set Δ-algebras.

3). To specify and to expand examples of proofs in ω-symbolics. Thus to carry out a cycle of proofs from a point of view of higher algebra in a plane of *strict* set-theoretic exposition.

To investigate topology ω- Spaces.

4). To receive more precise representation about a spectrum of irrationals of a type $\mathbf{R}_{\triangle}; \mathbf{R}_{\text{¤}}; \mathbf{R}_{\text{✧}}$ and others. To find *common properties* them and *operation* above these numbers.

To create algorithms of an evaluation of these numbers with a high exactitude and definition, by what ω-class posesses that or other irrational, i.e. to decide the inverse task on a comparison that is solved in the given work.[19]

More precisely to prove an irrationality of mathematical structures of a type $\triangle a = k^{1/\log_k a}$, $\text{¤} a = k^{k^{\left(1/\log_k \log_k a\right)}}$ etc. In particular, to define, at what relations k and a the numbers of a type $\triangle a$ cease to be irrational. To find irrationals, not inherings to a set ω-classes.

5). To give a classification to all infinities to a set of numbers, inherings infinities to the extension of a field of real numbers, i.e. to systematize all information about overfields, for which the field $\mathbf{R}$ real numbers is a field.

6). To explain compression of fields at ω-passages (for example, $\mathbf{R}_{\Delta} \setminus \omega_1 \to \omega_0 \setminus \mathbf{R}_{-}$, $\mathbf{R}_{-} \setminus \omega_1 \to \omega_0 \setminus \mathbf{R}_{f}$, i.e. in a series $\mathbf{R}_{f}; \mathbf{R}_{-};$ $\mathbf{R}_{\triangle}; \mathbf{R}_{\Delta}$ etc. from left to right infinities "will increase", as $\mathbf{R}_{-}$ includes all negative fractional numbers as separate object; similarly, $\mathbf{R}_{\Delta}$ contains object consisting of a set $\Delta(\mathbf{R}_{-})$ etc.).

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[19] For example, the author did not manage to prove a membership of numbers π, e, to any from $\omega-$classes of irrationals, though such classes is indefinite much.

7). To generalize all material and to state reasons about practical it application.

8). To investigate functions on Δ-fields $(\mathbf{R}_{\triangle}; \mathbf{R}_{\Delta}; \dots)$.

9). Describing operations $n > 4$ to find numbers of a new nature.

Even the incomplete enumeration of originating problems directs on an idea on their insolubility. Moreover, can appear the desire generally to refuse the theory, offered in the given book, of the extension of a field of real numbers. However, *this* extension - *objective reality*. In particular, the origin of a set of fractional numbers by a way ω-reflection of negative numbers $\mathbf{R}_{-} \setminus \omega_1 \rightarrow \omega_0 \setminus \mathbf{R}_f$ naturally puts a problem on existence of numbers, from which similar by a mode it is possible to receive negative numbers, i.e. $\mathbf{R}_{-} \setminus \omega_0 \rightarrow \omega_1 \setminus \mathbf{R}_{\Delta}$ etc. The impression about an artificiality ω-transformations can be created, as all outcomes ω-reflections are on one numerical axes. Serious the study of the given problem testifies that such geometric interpretation ω-reflections of numerical fields only underlines structural unity of these transformations. At the same time distinction in outcomes ω-passages of numbers and accordingly operations speaks about ω-hierarchies of numbers and operations. Under the judgement of the author offered in the chapter 2 materials is a *global* area for speculation and creativity. Thus are useful as a path offered the author, and alternate a path - Attempt to desroy the theory of the extension of a field of real numbers. The path of refusaling new, nonstandart, untraditional is a quite natural expression of variance with originating *true*. As a rule, this the path only strengthens and promotes it becoming and recognizing.

CHAPTER 3. NEW INTEGRO-DIFFERENTIAL OBJECTS

§ 3.1 Basic principles of shaping ω-images of a derivative

In a basis of deriving ω-images of a derivative of function $f(x)$ lay *appropriate* ω-reflections of global object O_j $(j \in \mathbf{Z})$ which the formula of a derivative is

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

and elementary objects o_k, included in structure O_j $\left(O_j = \bigcup_{k=1}^{n} o_k\right)$. Let's designate $O_j = \Psi(o_k) = \Psi(x, f(x), f'(x), \Delta x, 0, \lim, \quad +, -, \div, =, \rightarrow)$, where $\Psi -$ defined sequence (system) of connections of elementary objects o_k, under which is understood not only number, variable, function, but also operation (operation), and also ratio. In particular, in our case, $o_1 \equiv x$, $o_2 \equiv f(x), \ldots, o_9 \equiv \div$. The objects $o_6 \equiv \lim$, $o_{10} \equiv (=)$, $o_{11} \equiv (\rightarrow)$ are invariant concerning spaces ω. It is necessary to pay attention to specifity ω-reflection of object $o_2 \equiv f(x)$, which at ω-passage can save the initial aspect at an entry of a derivative, and all modifications will be automatically produced in an image of a derivative at observance of *two principles*:

1) Appropriate ω*-transformation of objects of a set* $O'_k = \{\Delta x, 0, +, -, \div\}$*;*

2) Preservation of an initial sequence of connections between objects Ψ.

For example, $f'(x) \backslash \omega_1 \rightarrow \omega_0 \backslash \overset{'}{-}f(x)$, where $\overset{'}{-}f(x) -$ image in ω_0 derivative $f'(x)$, "were" in space ω_1. For right shaping of an image $\overset{'}{-}f(x)$ is realizable the following reflections of elementary objects: $\Delta x \backslash \omega_1 \rightarrow \omega_0 \backslash \delta x$ ($\Delta x = x - x_0$, and $\delta x = \dfrac{x}{x_0}$, where $x -$variable value of argument, and $x_0 -$constant initial it a value, and $x > x_0$, $x \neq x_0$); $0 \backslash \omega_1 \rightarrow \omega_0 \backslash k^0 = 1$, i.e. $0 \backslash \omega_1 \rightarrow \omega_0 \backslash 1$; $+ \backslash \omega_1 \rightarrow \omega_0 \backslash \bullet$; $- \backslash \omega_1 \rightarrow \omega_0 \backslash \div$; $\div \backslash \omega_1 \rightarrow \omega_0 \backslash ⟁$.

Then $$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \backslash \omega_1 \rightarrow \omega_0 \backslash \overset{'}{-}f(x) =$$

$$= \lim_{\delta x \rightarrow 1} \left(\frac{f(x \cdot \delta x)}{f(x)} \right) ⟁ \delta x \qquad (3.1)$$

The dominating procedure at designing an image $\overset{'}{-}f(x)$ derivative $f'(x)$ has become a replacement of operations $\{+; -; \div\} \backslash \omega_1 \rightarrow$ $\rightarrow \omega_0 \backslash \{\cdot, \div, ⟁\}$.

Besides at ω-reflection it is necessary to find a value, by which tends δx ($\delta x -$ it $\omega -$image Δx), and also to take into account an aspect of indeterminacy noted in an athmosphere of a limit. For example, as is known, in a usual initial derivative $\Delta x \to 0$, and aspect of indeterminacy $\frac{0}{0}$ $\left(\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\right)$; in an image $f'(x)$ increment $\delta x \to 1$, and indeterminacy 1^{∞} $\left(\lim\limits_{\delta x\to 1}\left(\frac{f(x\cdot\delta x)}{f(x)}\right)\triangle\delta x \Rightarrow\ 1\triangle 1 = 1^{1/\log_k 1} = 1^{\infty}\right)$ etc. It is necessary for a conclusion of the formula connecting ω-image and a derivative f', by disclosure of appropriate indeterminacy.

If object $o_2 = f(x)$ to reflection according to existing rules (lemma 1.1), the derivative $f'(x)$ will be similarly reflected also. So,

$$f'(x)\setminus\omega_1 \to \omega_0 \setminus k^{f'(\log_k x)} = \lim_{\delta x\to 1}\left(\frac{k^{f(\log_k(x\cdot\delta x))}}{k^{f(\log_k x)}}\right)\triangle\delta x =$$

$$= \lim_{\delta x\to 1}\left(k^{f(\log_k x+\log_k(\delta x))-f(\log_k x)}\right)^{1/\log_k(\delta x)}.$$

Let's designate $\log_k x = z$ $\left(x = k^z,\ \delta x = k^{\delta z}, \delta z = \log_k \delta x\right.$ and $\left.\lim\limits_{\delta x\to 1} = \lim\limits_{\delta z\to 0}\right)$. After a taking the logarithm of an image of a derivative $k^{f'(\log_k x)}$ we shall discover:

$$f'(z) = \lim_{\delta z\to 0}\frac{f(z+\delta z)-f(z)}{\delta z}.$$

Have received the formula of a usual derivative. It also is clear, as the essence ω-reflection consists in a modification of scales of values of functions and arguments. The aspect of function in logarithmic coordinates does not vary. For example, we shall take function $f = \sin x$. Then

$$f = \sin x \setminus \omega_1 \to \omega_0 \setminus k^{\sin\log_k x} = \mathrm{F} \quad (k \neq 1).$$

After a taking the logarithm:

$$\log_k \mathrm{F} = \sin\log_k x.$$

Let's designate $\mathrm{F}^* = \log_k \mathrm{F}$, $\log_k x = z$. Whence $\mathrm{F}^* = \sin z$. In the total have received assumed function, in logarithmic coordinates (thus not assumed function, and it ω-image) is taken the logarithm.

Let's reflection $f' = \cos x \setminus \omega_1 \to \omega_0 \setminus k^{\cos \log_k x} = \mathrm{F}'$. After a taking the logarithm:

$$\log_k \mathrm{F}' = \cos \log_\mathrm{k} x \Rightarrow \mathrm{F}'^* = \cos z,$$

as it was necessary to expect.

On occassion at designing an image of a derivative even it is expedient to realize a replacement of function $f(x)$ On it identified ω-image. For example, the image $\overset{\mathrm{o}}{f}(x)$ in ω_0 derivative noted initially in space ω_{-1} will look so:

$$\overset{\mathrm{o}}{f}(x) = \lim_{\delta x \to (-\infty)} \left(\log_k \left(k^{f\left(\log_k \left(k^x + k^{\delta x}\right)\right)} - k^{f(x)} \right) - \delta x \right) \quad (3.2)$$

In an image $\overset{\mathrm{o}}{f}(x)$ increment of argument $\delta x \to (-\infty)$, and the indeterminacy has an aspect $\infty - \infty$. Really,

$$\lim_{\delta x \to (-\infty)} \left(\log_k \left(k^{f\left(\log_k \left(k^x + k^{\delta x}\right)\right)} - k^{f(x)} \right) - \delta x \right) \Rightarrow$$

$$\Rightarrow \log_k \left(k^{f\left(\log_k \left(k^x + k^{-\infty}\right)\right)} - k^{f(x)} \right) - (-\infty) =$$

$$= \log_k \left(k^{f\left(\log_k \left(k^x\right)\right)} - k^{f(x)} \right) + \infty = (-\infty) + \infty.$$

In the given work there is no analysis of possible functions of connection between ω-spaces. The author was limited only to one exponential function k^x $(k \neq 1)$. However, *anyone continuous, monotone function* $\mathbf{F}$, *having* ***inverse*** *continuous, monotone function* $\mathbf{G}$ *in a common range of definition* $\mathbf{J}$ *can be function of connection. It is natural, that a mandatory condition* ω-*reflection* *is one-to-one the correspondence of functions* $\mathbf{F}$ *and* $\mathbf{G}$.[20] In the present work the ω-algebra generated on function k^x $(k \neq 1)$, as an example of a possibility and expediency of such approach with the purpose of ordering separate fragments, transformation of existing mathematical meanses and discovery of new mathematical objects.

Represent defined interest and images of a derivative with a saved initial scale. For example, at reflections $\backslash \omega_1 \to \omega_0 \backslash$ and $\backslash \omega_2 \to \omega_0 \backslash$ it is possible to receive pceudo-images ${}^{'}f$, ${}^{\mathrm{p}}f$. Realizing a replacement of operations $\{+ ; - ; \div\} \to \{\cdot, \div, \log\}$ $(\backslash \omega_1 \to \omega_0 \backslash)$, we shall receive

$$ {}^{'}f(x) = \lim_{\delta x \to 1} \log_{\delta x}\left(\frac{f(x \cdot \delta x)}{f(x)}\right), \tag{3.3}$$

and at a replacement of operations $\{+ ; - ; \div\} \to \{\cdot, \log, \log\}$ $(\backslash \omega_2 \to \omega_0 \backslash)$, we shall receive

$$ {}^{\mathrm{p}}f(x) = \lim_{\delta_0 x \to 1} \log_{\delta_0 x} \log_{f(x)} f\left(x^{\delta_0 x}\right) \tag{3.4}$$

It is uneasy to notice equivalence of a replacement of operations $\{+ ; - ; \div\} \to \{\cdot, \log, \log\}$ and $\{+ ; - ; \div\} \to \{\odot, \triangle, \log/\log\}$.

The note. In further it will be proved.

As already it was specified, each image of a derivative can be used for shaping the modified calculus being ω-image of a well-known means. Considering various sections of mathematics as it is uneasy to receive global objects, infinite a spectrum new transformed ω-images of these global objects, in which basis, certainly, the well-known sections (meanses) of mathematics lay. And, infinite is reached as at the expense of a qualitative modification of the order ω-reflection (from some space ω_j in space ω_i, $j \neq i$), and for the score infinite of values of factor k. Besides applying different functions of connection $\mathbf{F}$, it is possible to change in the whole structure ω-reflections, that gives inexhaustible possibilities for an operation with that either other local or global object.

However, not only ω-images, but also ω-pceudo-images (called by us as the images of a derivative which were not reduced in a scale resulting ω-space, resulting, can form the basis for creation of a specific upgraded mathematical

[20] The statement and proof of the similar theorem goes out for frameworks of the given monography.

means. Matter that all ω-pceudo-images enter as dominating components in appropriate real ω-images of a derivative, i.e. images reduced in a scale final resulting ω-space.

In summary, we shall remark, that the apparent simplicity of mathematical transformations, presence of analogies, possibility of deriving of an outcome at the expense of elementary substitutions etc. creates an impression about an *artificiality* of existence of images of a derivative, difficulty of deriving for the score ω-reflections of any practical outcomes. Certainly, to some extent, small series of substitutions partially lowers outcomes of the concept $\omega-$reflections. More the deep study of this theory in a plane of object-oriented mathematical modelling can reduce in interesting conclusions. First of all, it concerns a research infinities of spectra various Δ-numbers, functions and integro-differential objects with argument defined in area Δ-fields, and as studies of the essence of the transformed physical laws. In the present book the separate instructive primes to deriving similar outcomes and their comprehension are given only. This immense unknown space permitting to any inquisitive reader to be immersed in creativity and to achieve, probably, unique outcomes is represented, that to the author. Purely, it also is the basic purpose of the book, about what was told in “Foreword”.

§ 3.2 Images ${}^{'}f$, ${}^{\overset{'}{-}}f$, ${}^{\mathrm{p}}f$.

Research of images we begin from elementary ${}^{'}f(x)$.

The definition 3.1. *Not reduced to a scale of resulting space* ω_0 *the image* ${}^{'}f(x)$ *derivative* $f'(x)$ *continuous, monotone positive function* $f(x)$ $(f(x)>0)$, *noted in standard*[21] *aspect in space* ω_1 *names expression:*

$$
{}^{'}f(x)=\lim_{\delta x\to 1}\log_{\delta x}\left(\frac{f(x\cdot\delta x)}{f(x)}\right)
$$

In paragraph 3.1 this formula met under number (3.3).

Let's explain the formula (3.3). $x+\Delta x \setminus \omega_1 \to \omega_0 \setminus x\cdot\delta x$

[21] The standard aspect of a derivative names well-known expression of a derivative of function $f(x)$:

$$
f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.
$$

$\left(\Delta x = x - x_0 \setminus \omega_1 \to \omega_0 \setminus \delta x = \dfrac{x}{x_0}\right)$, and $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} \setminus \omega_1 \to \omega_0 \setminus$

$$\setminus \omega_1 \to \omega_0 \setminus \left(\frac{f(x \cdot \delta x)}{f(x)}\right) \triangle \delta x = \left(\frac{f(x \cdot \delta x)}{f(x)}\right)^{1 / \log_k(\delta x)}$$

$(k \in \mathbf{R}_+, k \neq 1)$.

Let's take the logarithm this expression:

$$\log_k \left(\frac{f(x \cdot \delta x)}{f(x)}\right)^{1 / \log_k(\delta x)} = \frac{\log_k \left(\dfrac{f(x \cdot \delta x)}{f(x)}\right)}{\log_k(\delta x)} = \log_{(\delta x)} \left(\frac{f(x \cdot \delta x)}{f(x)}\right).$$

The formula 3.3 whence follows.

The replacement of operations $\{+ ; - ; \div\} \to \{\cdot , \div , \log\}$ at shaping an image ${}'f(x)$ derivative $f'(x)$, connected with a rescaling of object, influences all global mathematical objects represented as the formulas, theorems and separate sections of mathematics.

Let's establish connection between ${}'f(x)$ and $f'(x)$.

Theorem 3.1. *Let function $f(x)$ has a derivative $f'(x)$, at $x \neq 0$ and $f(x) \neq 0$ the connection between an image of function ${}'f(x)$ and derivative of function is installed with the help of formulas:*

$${}'f(x) = \frac{x \cdot f'(x)}{f(x)} \tag{3.5}$$

Proof. Let's uncover indeterminacy in the formula (3.3) on a rule of the L'Hospital:

$${}'f(x) = \lim_{\delta x \to 1} \frac{\left(\ln f(x \cdot \delta x) - \ln f(x)\right)'_{\delta x}}{\left(\ln \delta x\right)'_{\delta x}}.$$

Here derivatives undertake on a variable δx, instead of on x (x, as it is uneasy to guess, in the given situation does not vary). Then

$$
{}^{'}f(x)=\lim_{\delta x\to 1}\frac{\dfrac{1}{f(x\cdot\delta x)}\cdot\left(f'(x\cdot\delta x)\right)_{\delta x}\cdot(x\cdot\delta x)'_{\delta x}-\left(\ln f(x)\right)'_{\delta x}}{\dfrac{1}{\delta x}}=
$$

$$
=\lim_{\delta x\to 1}\frac{f'_{\delta x}(x\cdot\delta x)\cdot x}{f(x\cdot\delta x)}\cdot\delta x=\frac{x\cdot f'(x)}{f(x)},
$$

As was to be shown.

From the formula (3.5) we shall note some properties ${}^{'}f(x)$:

$$
{}^{'}f^{n}(x)=n\cdot{}^{'}f(x)\quad(n=\text{const});\quad {}^{'}f(\varphi(x))={}^{'}f_{\varphi}\cdot{}^{'}\varphi_{x};\quad {}^{'}\left(\prod_{i=1}^{n}f(x_i)\right)=
$$

$$
=\sum_{i=1}^{n}\left({}^{'}f(x_i)\right);\quad {}^{'}(f(x)+g(x))={}^{'}(f+g)=\frac{f\cdot f'+g\cdot g'}{f+g};\quad {}^{'}\left(\frac{f}{g}\right)={}^{'}f-{}^{'}g;
$$

$$
{}^{'}\left(\log_{k}^{x}\right)=\frac{1}{\ln x};\ {}^{'}(\sin x)=x\cdot ctg\,x;\ {}^{'}(\cos x)=-x\cdot tg\,x;\ {}^{'}(tg\,x)=\frac{2x}{\sin 2x};
$$

$$
{}^{'}(ctg\,x)=-\frac{2x}{\sin 2x};\qquad {}^{'}(c\pm x)=\frac{x}{c\pm x}\qquad(c=const);\qquad {}^{'}\left(x^{n}\right)=n;
$$

$$
{}^{'}\left(a^{x}\right)=x\cdot\ln a;\ {}^{'}\left(e^{x}\right)=x;\ {}^{'}\left(\log_{\varphi}^{f}\right)=\frac{{}^{'}f}{\ln f}-\frac{{}^{'}\varphi}{\ln\varphi};\ {}^{'}(c+n\cdot x)=\frac{n\cdot x}{c+n\cdot x};
$$

$$
{}^{'}\left(c+k\cdot x^{n}\right)==\frac{k\cdot n\cdot x^{n}}{c+k\cdot x^{n}};\ {}^{'}\left(f^{\varphi}\right)={}^{'}\varphi\cdot\ln f^{\varphi}+{}^{'}f\cdot\varphi\ \text{etc.}\qquad(3.6)
$$

This table can be extended by the following generalization.

Theorem 3.2. *If* $f(g(x))$, $g(x)$, ${}^{'}g_{x}$, ${}^{'}f_{g}$, ${}^{'}f_{x}$ *are continuous and will not be converted in 0 in some area* $\mathbf{D}$ $(x\in\mathbf{D})$, *in this area*

$$
{}^{'}f_{x}={}^{'}f_{g}\cdot{}^{'}g_{x}\qquad(3.7)
$$

Proof. From (3.3) we shall note ${}^{'}f_{g}$:

$$ {}^{'}f_{g} = \lim_{\delta g \to 1} \log_{\delta g}\left(\frac{f(g\cdot\delta g)}{f(g)}\right) = \lim_{\delta g \to 1} \frac{\log_{\delta x}\left(\frac{f(g\cdot\delta g)}{f(g)}\right)}{\log_{\delta x}\delta g}. $$

As $\delta g = \frac{g(x\cdot\delta x)}{g(x)}$ $\lim\limits_{\delta x \to 1} \frac{g(x\cdot\delta x)}{g(x)} = 1$,

$$ {}^{'}f_{g} = \lim_{\delta x \to 1} \frac{\log_{\delta x}\left(\frac{f(g\cdot\delta g)}{f(g)}\right)}{\log_{\delta x}\delta g} = \lim_{\delta x \to 1} \frac{\log_{\delta x}\left(\frac{f(g\cdot\delta g)}{f(g)}\right)}{\log_{\delta x}\left(\frac{g(x\cdot\delta x)}{g(x)}\right)}, \text{ and} $$

$$ {}^{'}g_{x} = \lim_{\delta x \to 1} \log_{\delta x}\frac{g(x\cdot\delta x)}{g(x)}. $$

Let's discover ${}^{'}f_{g}\cdot{}^{'}g_{x}$:

$$ {}^{'}f_{g}\cdot{}^{'}g_{x} = \lim_{\delta x \to 1}\left(\frac{\log_{\delta x}\left(\frac{f(g\cdot\delta g)}{f(g)}\right)}{\log_{\delta x}\left(\frac{g(x\cdot\delta x)}{g(x)}\right)}\cdot\log_{\delta x}\left(\frac{g(x\cdot\delta x)}{g(x)}\right)\right). $$

In connection with that the expression ${}^{'}f_{g}\cdot{}^{'}g_{x}$ has not a determinancy of an aspect $\frac{\log_{1}^{1}}{\log_{1}^{1}}\cdot\log_{1}^{1}$, it is possible to note

$$'f_g \cdot 'g_x = \frac{\left(\lim_{\delta x \to 1} \log_{\delta x}\left(\frac{f(g\cdot\delta g)}{f(g)}\right)\right)\cdot\left(\lim_{\delta x\to 1}\log_{\delta x}\left(\frac{g(x\cdot\delta g)}{g(x)}\right)\right)}{\lim_{\delta x\to 1}\log_{\delta x}\left(\frac{g(x\cdot\delta x)}{g(x)}\right)} =$$

$$= \lim_{\delta x\to 1}\log_{\delta x}\left(\frac{f(g\cdot\delta g)}{f(g)}\right) = \lim_{\delta x\to 1}\log_{\delta x}\frac{f\left(g\cdot\frac{g(x\cdot\delta x)}{g(x)}\right)}{f(g)} =$$

$$= \lim_{\delta x\to 1}\log_{\delta x}\frac{f(g(x\cdot\delta x))}{f(g)} = \lim_{\delta x\to 1}\log_{\delta x}\frac{f(x\cdot\delta x)}{f(x)} = 'f_x .$$

The theorem is proved.

Because of ω-pceudo-image of a derivative $'f(x)$ it is possible to construct any more global mathematical object. For example, we shall formulate the modified Lagrange's theorem: *if the function f differentiable in each point of convex area $\mathbf{G}$ n-mensural of Euclidean space, for each pair of points $x=(x_1,\dots,x_n)\in\mathbf{G}$, $x+\Delta x=(x_1+\Delta x_1,\dots,x_n+\Delta x_n)\in\mathbf{G}$ exists such point $\xi=(\xi_1,\dots,\xi_n)$, laying on a segment with extremities x and $x+\Delta x$, that*

$$\frac{f(x+\Delta x)}{f(x)} = \prod_{i=1}^{n}\left(1+\frac{\Delta x_i}{x_i}\right)^{'f_{x_i}(\xi_i)} . \qquad (3.8)$$

Let's prove a Lagrange's theorem for function of one variable.

Theorem 3.3 (theorem of ratioes).

If the function $f(x)$ is continuous on a segment $[a,b]$ and differentiable in all interior points of this segment, inside a segment $[a,b]$ one point ε $(a<\varepsilon<b)$, that will be discovered on an extremely measure

$$\frac{f(b)}{f(a)} = \left(\frac{b}{a}\right)^{'f(\varepsilon)} \qquad (3.9)$$

Proof. Let's designate Q number $\log_{b/a}\left(\frac{f(b)}{f(a)}\right)$ and we shall consider auxiliary function

$$F(x)=\frac{f(x)\big/ f(a)}{\left(\frac{x}{a}\right)^{Q}}=\frac{f(x)}{f(a)\cdot\left(\frac{x}{a}\right)^{Q}}.$$

$'F(\varepsilon)=0$ (according to the second upgraded theorem the Lagrange[22]).

$$'F(x)='\left(\frac{f(x)}{f(a)\cdot\left(\frac{x}{a}\right)^{Q}}\right)=\left(\frac{1}{f(a)\big/ a^{Q}}\right)\cdot\left('f(x)-Q\right)=0$$

Signifies $'F(\varepsilon)=\frac{a^{Q}}{f(a)}\cdot\left('f(\varepsilon)-Q\right)=0$, i.e. $Q='f(\varepsilon)$ and

$\frac{f(b)}{f(a)}=\left(\frac{b}{a}\right)^{'f(\varepsilon)}$, as was to be shown.

In chapter 5 "a miscellany (the applications)" are reduced some more proof of images of the theorems of the analysis.

Observing a strict sequence of an appropriate replacement of operations (for example, for $\backslash\omega_1\to\omega_0\backslash$ and $\omega-$pceudo-image $'f(x)$ this replacement is those:

$\{+; - ; \div ; \sum_i \; ; f' ; \int f\cdot dx ; \ldots\}$ $\backslash\omega_1\to\omega_0\backslash$ $\{\,\cdot\, ; \div ; \log ; \prod_i \; ; 'f ;$ $\ln\int\left(\delta x\right)^{x\cdot f} ; \;\ldots\}$**), it is possible to copy in new terms all calculus.** Certainly, in this case, it is not necessary to forget, that the procedure of updating of the analysis is rather simplis, as it is enough to take the logarithm a final outcome and to lead some changes of variables as we shall receive usual expressions from the well-known analysis.

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[22] Special case here is reduced it.

Despite of it, ω-images of the analyses not only qualitatively differ from usual, as in a basis them the identified replacement of operations connected to nonlinearity of transformations lays but sometimes comprise completely new interesting outcomes.

The definition 3.2. *Not reduced to a scale* ω_0 *the image of a derivative of function* $f = f(x)$, *obtained way of image in* ω_0 *derivative from space* ω_2 *names expression:*

$$\mathrm{p}_{f(x)} = \lim_{\Delta x \to k} \log_{\log_k \Delta x} \log_k \left(f(x \odot \Delta x) \triangle f(x) \right) \quad (3.10)$$

p_f is formed by a replacement of operations $\{+, -, :\} \to \left\{ \odot, \triangle, \frac{\log}{\log} \right\}$ in terms of a derivative f'. The formula (3.10) is an entry of an image p_f in *reflexive* terms.

Lemma 3.1. The image p_f in usual terms is noted by expression:

$$\mathrm{p}_{f(x)} = \lim_{\delta_0 x \to 1} \log_{\delta_0 x} \log_{f(x)} f\left(x^{\delta_0 x}\right)$$ [23]

Proof. By designating in (3.10) $\delta_0 x \equiv \Delta x$ and $A = \log_k \left(f(x \odot \Delta x) \triangle f(x) \right)$, we shall transform A. Thus is used the definition of operations "$\odot$" and "$\triangle$". Then $A = \log_k \left(f\left(x^{\alpha}\right) \triangle f \right) = \log_k \left(\sqrt[\log f / \log k]{f\left(x^{\alpha}\right)} \right) = \dfrac{\log_k f\left(x^{\alpha}\right)}{\log_k f} = \log_f f\left(x^{\alpha}\right)$,

where $\alpha = \log_k \Delta x \quad (\Delta x \to k)$, i.e.

$$\mathrm{p}_{f(x)} = \lim_{\alpha \to 1} \log_{\alpha} \log_{f(x)} f\left(x^{\alpha}\right) \qquad (*)$$

Let's replace a variable α an $\delta_0 x$, taking into account that $\delta_0 x$ tends to 1 $(\delta_0 x \to 1)$. In an outcome (*) will coincide with (3.4), as was to be shown.

Let's prove a upgraded rule of the L'Hospital.

The lemma 3.2 Let $f_1 > 0$, $f_2 > 0$ also exists $\lim\limits_{(f_1, f_2) \to 1} \dfrac{{}'f_1}{{}'f_2}$,

[23] In an item 3.1 these formulas met under number (3.4).

$$\lim_{(f_1,f_2)\to 1} \log_{f_2} f_1 = \lim_{(f_1,f_2)\to 1} {}'f_1 / {}'f_2 . \qquad (3.11)$$

The proof can be lead by two modes:

a). $$\lim_{(f_1,f_2)\to 1} \log_{f_2} f_1 = \lim_{(f_1,f_2)\to 1} \frac{\ln f_1}{\ln f_2} = \lim_{(f_1,f_2)\to 1} \frac{(\ln f_1)'}{(\ln f_2)'} =$$

$$= \lim_{(f_1,f_2)\to 1} \frac{x\cdot(\ln f_1)'}{x\cdot(\ln f_2)'} = \lim_{(f_1,f_2)\to 1} \frac{'f_1}{'f_2}$$, as was to be shown.

b). $'f(x) = \log_{\delta x} \delta f(x)$ is an entry of an image $'f$ through differentials. She will be quite coordinated with an image of a differential of function:

$$\delta f(x) = \delta x^{'f(x)}$$

Then from $$\lim_{(f_1,f_2)\to 1} \log_{f_2} f_1 \Rightarrow \lim_{(\delta f_1,\delta f_2)\to 1} \log_{\delta f_2} \delta f_1 =$$

$$= \lim_{(f_1,f_2)\to 1} \log_{(\delta x)^{'f_2}} (\delta x)^{'f_1} = \lim_{(f_1,f_2)\to 1} \frac{'f_1}{'f_2} \cdot \log_{(\delta x)} (\delta x) = \lim_{(f_1,f_2)\to 1} \frac{'f_1}{'f_2} .$$

Theorem 3.4. *The not reduced images of a derivative* $'f$ *and* ^{p}f *are connected by a relation* $^{p}f = {}'f \cdot \log_f^x$ (3.12).

Proof. Let's output the formula (3.12) two modes: using expressions ^{p}f in *reflexive* (3.10) and *usual* (3.4) terms. a). In reflexive terms for disclosure of indeterminacy of a type "$\log_1^1$" in (3.10) is aplicable upgraded (replacement f' on $'f$) rule of the L'Hospital:

$$^{p}f(x)=\lim_{\Delta x\to k}\frac{'\left(\log_k\left(f(x\odot\Delta x)\triangle f(x)\right)\right)}{'\left(\log_k\Delta x\right)}=\lim_{\Delta x\to k}\frac{'\left(f(x\odot\Delta x)\triangle f(x)\right)\cdot\ln\Delta x}{\ln\left(f(x\odot\Delta x)\triangle f(x)\right)}.$$

Let's remark, that

$$'\left(f(x\odot\Delta x)\triangle f(x)\right)={}'\left(\sqrt[(\log f(x)/\log k)]{f(x\odot\Delta x)}\right)=^{24}$$

$$=\left(\log_k f(x)\right)^{-1}\cdot{}'f(x\odot\Delta x),\text{ i.e.}$$

$$^{p}f(x)=\lim_{\Delta x\to k}\frac{\ln\Delta x\cdot{}'f(x\odot\Delta x)}{\log_k f(x)\cdot\ln\left(f(x\odot\Delta x)\triangle f(x)\right)}=$$

$$=\lim_{\Delta x\to k}\frac{\log\Delta x}{\log\left(f(x\odot\Delta x)\triangle f(x)\right)}\cdot\lim_{\Delta x\to k}\frac{'f(x\odot\Delta x)}{\log_k f(x)}.$$

From an entry of an image $'f$ a derivative through differentials follows:

$$'f(x\odot\Delta x)=\lim_{\Delta x\to k}\log_{\delta(x\odot\Delta x)}^{\delta f(x\odot\Delta x)}\cdot\lim_{\Delta x\to k}\log_{\delta(\Delta x)}^{\delta(x\odot\Delta x)}.$$

Then $^{p}f=\dfrac{1}{\log_k f(x)}\cdot\lim\limits_{\Delta x\to k}\log_{\delta(x\odot\Delta x)}^{\delta f(x\odot\Delta x)}\cdot\lim\limits_{\Delta x\to k}\log_{\delta(\Delta x)}^{\delta(x\odot\Delta x)}$

As $\log_{\delta x}^{\delta f}={}'f$,

$$^{p}f=\frac{1}{\log_k f(x)}\cdot\lim_{\Delta x\to k}{}'f(x\odot\Delta x)\cdot{}'(x\odot\Delta x)_{\Delta x}.$$

---

24 $'\left(x^a\right)=a\ (a=const)$ and $'\left(\left(f(x\odot\Delta x)\right)^{\log k/\log f}\right)=\left(\log_k f\right)^{-1}\times$
$\times\,'f(x\odot\Delta x)$.

In connection with that $'(x\odot\Delta x)_{\Delta x} = '\left(\Delta x^{\log_k(x)}\right)_{\Delta x} = \log_k x$,

$$^{\mathrm{p}}f = 'f(x)\cdot\log_f x,$$

As was to be shown.

b). Let's output the formula (3.12), using an image $^{\mathrm{p}}f$, noted in usual terms (3.4). For disclosure of indeterminacy "$\log_1^1$" is aplicable a rule of the L'Hospital for a derivative f':

$$^{\mathrm{p}}f(x) = \lim_{\Delta x\to 1}\frac{\left(\ln\log_f f\left(x^{\Delta x}\right)\right)'_{\Delta x}}{\left(\ln(\Delta x)\right)'_{\Delta x}} = \lim_{\Delta x\to 1}\frac{\Delta x\cdot\left(\log_f f\left(x^{\Delta x}\right)\right)'_{\Delta x}}{\log_f f\left(x^{\Delta x}\right)} =$$

$$= \lim_{\Delta x\to 1}\frac{\Delta x\cdot\left(\ln f\left(x^{\Delta x}\right)\right)'_{\Delta x}}{\ln f\left(x^{\Delta x}\right)}.$$

Under the definition of an image $'f$ $\left('f = {x\cdot f'}/{f} = x\cdot(\ln f)'\right)$:

$$^{\mathrm{p}}f(x) = \lim_{\Delta x\to 1}\frac{'\left(f\left(x^{\Delta x}\right)\right)'_{\Delta x}}{\ln f\left(x^{\Delta x}\right)} = \lim_{\Delta x\to 1}\frac{'\left(f\left(x^{\Delta x}\right)\right)_{x^{\Delta x}}\cdot{}'\left(x^{\Delta x}\right)_{\Delta x}}{\ln f\left(x^{\Delta x}\right)}.$$

Let's remark, that $\lim_{\Delta x\to 1}{}'\left(x^{\Delta x}\right)_{\Delta x} = \ln x$ $\left({}'\left(a^x\right) = x\cdot\ln a\right.$, where $\left.a = const\right)$. Then $^{\mathrm{p}}f = \frac{\ln x}{\ln f(x)}\cdot\lim_{\Delta x\to 1}\log^{\delta f\left(x^{\Delta x}\right)}_{\delta\left(x^{\Delta x}\right)} = 'f(x)\cdot\log_f x$, as was to be shown.

The definition 3.3 *Reduced to a scale* ω_0 *image of a derivative from function* $f = f(x)$, *obtained way of reflection in* ω_0 *derivative from space* ω_1 *is named expression:*

$$\overset{\prime}{-}f(x)=\lim_{\Delta x\to 1}\frac{f(x\cdot\Delta x)}{f(x)}\triangle(\Delta x) \qquad (3.13)$$

$\overset{\prime}{-}f$ is formed by a replacement of operations $\{+,-,\div\}\to\{\cdot,\div,\triangle\}$ in terms of a derivative f'.

Theorem 3.5 *Reduced* $\overset{\prime}{-}f$ *is connected to a not reduced image* $'f$ *relation:*

$$\overset{\prime}{-}f=k^{'f}. \qquad (3.14)$$

Proof. Let's take the logarithm (3.13):

$$\log_k \overset{\prime}{-}f(x)=\lim_{\Delta x\to 1}\log_k\left(\frac{f(x\cdot\Delta x)}{f(x)}\triangle(\Delta x)\right).$$

We shall transform this expression, by designating $\log_k \overset{\prime}{-}f(x)=C$. From the definition of an operation $\triangle$ follows, that

$$C=\lim_{\Delta x\to 1}\log_k\left(\frac{f(x\cdot\Delta x)}{f(x)}\right)^{\frac{1}{\log_k(\Delta x)}}=\lim_{\Delta x\to 1}\left(\log_k^{\Delta x}\right)^{-1}\cdot\log_k\left(\frac{f(x\cdot\Delta x)}{f(x)}\right)=$$

$$=\lim_{\Delta x\to 1}\log_{\Delta x}\left(\frac{f(x\cdot\Delta x)}{f(x)}\right),$$ i.e. $C='f(x)$ (under the definition $'f$).

Whence $\overset{\prime}{-}f=k^{'f}$, as was to be shown.

The note. It is easy to note a table. For example, $\overset{\prime}{-}f$: $\overset{\prime}{-}\left(x^n\right)=k^n$,

$$\overset{\prime}{-}(u\cdot v)=\overset{\prime}{-}u\cdot\overset{\prime}{-}v, \qquad \overset{\prime}{-}\left(\frac{u}{v}\right)=\frac{\overset{\prime}{-}u}{\overset{\prime}{-}v}, \qquad \overset{\prime}{-}\left(\log_a x\right)=k^{1/\ln x}=\exp\left(\frac{1}{\log_k x}\right),$$

$\overset{\prime}{-}\left(a^x\right)=a^{x/\log_k e}=\left(a^x\right)^{\ln k}$, $\overset{\prime}{-}\left(e^x\right)=k^x$, $\overset{\prime}{-}(\sin x)=k^{x\cdot ctg\,x}$ And т.д..

Theorem 3.6 *Let* $f_1>0,\ f_2>0$ *also exists* $\lim\limits_{(f_1,f_2)\to 1} \underline{'f_1} ⟁ \underline{'f_2}$,

$$\lim_{(f_1,f_2)\to 1} f_1 ⟁ f_2 = \lim_{(f_1,f_2)\to 1} \underline{'f_1} ⟁ \underline{'f_2} \quad (3.15)$$

Proof. Through differentials the image $\underline{'f}$ Is noted so:

$$\underline{'f} = \delta f(x) ⟁ \delta x \qquad (3.17)$$

Then $\lim\limits_{(f_1,f_2)\to 1} f_1 ⟁ f_2 = \lim\limits_{(f_1,f_2)\to 1} \delta f_1 ⟁ \delta f_2 =$

$$= \lim_{(f_1,f_2)\to 1} \left(\underline{'f_1} \odot \delta x\right) ⟁ \left(\underline{'f_2} \odot \delta x\right) = \lim_{(f_1,f_2)\to 1} \underline{'f_1} ⟁ \underline{'f_2},$$

as was to be shown.

We shall discover an image $\underline{'f}$ derivative from a composite function.

Theorem 3.7 *Let* $f_1>0,\ f_2>0$ *also exist* $\underline{'f_2},\ 'f_1(f_2(x))$,

$$\underline{'f_1}(f_2(x)) = \left(\underline{'f_2}(x)\right)^{'f_1(f_2(x))} \qquad (3.17)$$

Proof. Let's discover an image of a derivative $\underline{'f_1}(f_2(x))$ in *reflexive* terms: $\underline{'f_1}(f_2(x)) = \delta(f_1(f_2(x))) ⟁ \delta x =$

$$= \left(\delta f_1(f_2(x)) ⟁ \delta f_2(x)\right) \odot \left(\delta f_2(x) ⟁ \delta x\right) = f_1(f_2(x)) \odot \underline{'f_2}(x),$$

where f – derivative from a primitive $f_1(x)$, and

$$\underline{'f_2}(x) = \delta f_2 ⟁ \delta x$$

Then $\underline{'f_1}(f_2(x)) = k^{'f_1(f_2(x)) \cdot 'f_2(x)} = \left(k^{'f_2(x)}\right)^{'f_1(f_2(x))} =$

$$=\left(\underline{'f_2}(x)\right)^{'f_1(f_2(x))}$$, as was to be shown.

Note. The proof (3.17) can be realized, using a *usual* nomenclature:

$$\underline{'f_1}(f_2(x))=k^{'f_1(f_2(x))}=k^{'f_1(f_2(x))_{f_2}\cdot 'f_2(x)}=\left(\underline{'f_2}(x)\right)^{'f_1(f_2(x))}$$

Theorem 3.8 *A reduced image in* ω_0 *derivative of logarithmic function from space* ω_1 *in reflexive terms is noted so:* $\overset{'}{\text{-}y}=\Delta x\odot \operatorname{ilog}_a\left(k^e\right)$, (3.18)

where $y=\log_a x$ $\left(\log_a x\setminus\omega_1\to\omega_0\setminus\operatorname{ilog}_a x\right)$.

Proof. An image in ω_0 derivative of logarithmic function $y=\log_a x$, noted in ω_1 : $\underline{y}\cdot\underline{\Delta y}=\operatorname{ilog}_a(x\cdot\Delta x)$, where $\underline{y}=\operatorname{ilog}_a x$, since $\underline{y}$ – image y in ω_0, and

$$\underline{\Delta y}=\frac{\operatorname{ilog}_a(x\cdot\Delta x)}{\operatorname{ilog}_a x}=\operatorname{ilog}_a(x\cdot\Delta x)\blacktriangle x=\operatorname{ilog}_a((x\blacktriangle x)\cdot(\Delta x\blacktriangle x))=$$

$=\operatorname{ilog}_a(k\cdot(\Delta x\blacktriangle x))$. By designating $\Delta x\blacktriangle x=\alpha$ $(\Delta x=\alpha\odot x)$, we shall note:

$$\overset{'}{\text{-}y}=\lim_{\Delta x\to 1}\left((\alpha\blacktriangle(\Delta x))\odot\operatorname{ilog}_a(k\cdot\alpha)^{\blacktriangle\alpha}\right)=\lim_{\Delta x\to 1}\left(\Delta x\odot\operatorname{ilog}_a(k\cdot\alpha)^{\blacktriangle\alpha}\right)=$$

$$=\Delta x\odot\lim_{\alpha\to 1}\operatorname{ilog}_a(k\cdot\alpha)^{\blacktriangle\alpha}=\Delta x\odot\operatorname{ilog}_a\left(\lim_{\alpha\to 1}(k\cdot\alpha)^{\blacktriangle\alpha}\right)=\Delta x\odot\operatorname{ilog}_a k^e,$$

as was to be shown.

The note. Let's discover ω-image of a remarkable limit $\lim\limits_{\alpha\to 0}(1+\alpha)^{1/\alpha}=e$. It is obvious, that $e\setminus\omega_1\to\omega_0\setminus k^e$, and

$$\lim_{\alpha\to 0}(1+\alpha)^{1/\alpha} \setminus \omega_1 \to \omega_0 \setminus \mathrm{L} = \lim_{\alpha_1\to 1}(k\cdot\alpha_1)^{\to(k\triangle\alpha_1)} \quad\Rightarrow\quad \log_k \mathrm{L} =$$

$$= \lim_{\alpha_1\to 1}(1+\log_k\alpha_1)^{\log_k p}, \quad \lim_{\alpha_1\to 1} p = \lim_{\alpha_1\to 1} \triangle\alpha_1 = \lim_{\alpha_1\to 1} k^{1/\log_k\alpha_1} = \infty,$$

i.e. $\log_k \mathrm{L} = e$ and $\mathrm{L} = \lim\limits_{\alpha_1\to 1}(k\cdot\alpha_1)^{\to(k\triangle\alpha_1)} = k^e$.

(we shall explain this equality, translating *reflexive* symbolics in *usual*. As $a^{\to b} = a^{(\log_k a)^{\log_k b}}$, $(k\cdot\alpha_1)^{\to(k\triangle\alpha_1)} = (k\cdot\alpha_1)^{(\log_k(k\cdot\alpha_1))^{\log_k(k\triangle\alpha_1)}}$. Noticing, that $\log_k(k\cdot\alpha_1) = 1+\log_k\alpha_1$, and $k\triangle\alpha_1 = k^{1/\log_k\alpha_1}$ and by designating $\log_k\alpha_1 = \alpha_2$, we shall receive:

$$\mathrm{L} = \lim_{\alpha_1\to 1}(k\cdot\alpha_1)^{(1+\alpha_2)^{1/\alpha_2}} = \lim_{\alpha_1\to 1}(k\cdot\alpha_1)^e = k^e,$$

as $\lim\limits_{\alpha_1\to 1}\alpha_2 = \lim\limits_{\alpha_1\to 1}\log_k\alpha_1 = 0$, and $\lim\limits_{\alpha_1\to 1}(1+\alpha_2)^{1/\alpha_2} = e$).

The equality $\lim\limits_{\alpha_1\to 1}(k\cdot\alpha_1)^{\to(k\triangle\alpha_1)} = k^e$ is proved.

§ 3.3 Image $\overset{0}{f}$.

Let in ω_{-1} there is a derivative of function $f(x)$:

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

For shaping it of an image $\overset{0}{f}(x)$ in space ω_0 is necessary for mirror $\setminus\omega_{-1}\to\omega_0\setminus$ all components (objects o_k) formula of a derivative. It is known, that the reflection $\setminus\omega_1\to\omega_0\setminus$ function $f(x)$ will be noted as $k^{f(\log_k x)}$, i.e. $f(x)\setminus\omega_1\to\omega_0\setminus k^{f(\log_k x)}$. Then the inverse reflection of function $k^{f(\log_k x)}$ will be function $f(x)$: $k^{f(\log_k x)}\setminus\omega_0\to\omega_1\setminus f(x)$.

As it is visible, *for the inverse reflection it is necessary to replace* $\log_k x$ *on* x *and instead of* $k^{f(x)}$ *to note* $f(x)$. By virtue of equivalence (look "Lemma 2.12") reflections $\backslash \omega_0 \to \omega_1 \backslash$ and $\backslash \omega_{-1} \to \omega_0 \backslash$ (ω-reflection, as it was specified above, is invariant concerning an index ω-spaces) it is possible to note:

$$k^{f\left(\log_k x\right)} \backslash \omega_{-1} \to \omega_0 \backslash f(x).$$

Let's image $\backslash \omega_{-1} \to \omega_0 \backslash$ object

$$\Phi(x) = \log_k\left(k^{f\left(\log_k\left(k^x + k^{\delta x}\right)\right)} - k^{f(x)}\right).$$

For this purpose according to above-stated we shall change the elements of object $\Phi(x)$: $k^x \backslash \omega_{-1} \to \omega_0 \backslash x$, $k^{\delta x} \backslash \omega_{-1} \to \omega_0 \backslash \Delta x$ (δx turns in Δx, as the limit varies, by which the increment of argument) tends,

$$k^{f\left(\log_k\left(k^x + k^{\delta x}\right)\right)} \backslash \omega_{-1} \to \omega_0 \backslash f(x + \Delta x),$$

$$k^{f(x)} \backslash \omega_{-1} \to \omega_0 \backslash f(x).$$

The note. The apparent inconsistency $k^{f\left(\log_k x\right)} \backslash \omega_{-1} \to \omega_0 \backslash$ $\backslash \omega_{-1} \to \omega_0 \backslash f(x)$ and $k^{f(x)} \backslash \omega_{-1} \to \omega_0 \backslash f(x)$ is explained simply.

In the first case, the function $k^{f\left(\log_k x\right)}$ is transformed to two stages (phase):

a) the reduction of a value of argument, with which is $\log_k x$ to a scale of space ω_0 $\left(\log_k x \backslash \omega_{-1} \to \omega_0 \backslash x\right)$;

b) reduction of values of function to a scale ω_0 $k^{f(x)} \backslash \omega_{-1} \to \omega_0 \backslash f(x)$.

In the second case, ω – reflection is carried out only in one stage; as the first phase is already realized. So,

$$\Phi(x) \backslash \omega_{-1} \to \omega_0 \backslash f(x + \Delta x) - f(x).$$

In this case, it would be possible to note $\Phi(x) = \log_k Z$, where $Z = k^{f\left(\log_k\left(k^x + k^{\delta x}\right)\right)} - k^{f(x)}$. Whence $\log_k Z \backslash \omega_{-1} \to \omega_0 \backslash \underline{Z}$, where $\underline{Z}$ – image of function $\log_k Z$. By analogy to the above-stated mirror of object $\Phi(x)$ is carried out in two stages: at first is reflection Z that Z – composite function, instead of simple argument; then, considering an image Z $(\underline{Z})$ as an explanatory variable is reflection $\log_k \underline{Z}$, omitting $\log_k$. However, for completion of shaping of an image $\overset{0}{f}(x)$ derivative $f'(x)$, it is required to install two facts: a) to what value the increment of argument δx tends in a limit; b) to what the operation of division will turn at reflection $\backslash \omega_0 \to \omega_{-1} \backslash$.

Both of the fact are installed from the formula of a derivative. Really,

$\Delta x \backslash \omega_{-1} \to \omega_0 \backslash \log_k \Delta x \quad \Rightarrow \quad \lim\limits_{\Delta x \to 0} \log_k \Delta x = (-\infty)$ but

$\Delta x \backslash \omega_{-1} \to \omega_0 \backslash \delta x$, i.e. $\delta x \to (-\infty)$. At last, $- \backslash \omega_0 \to \omega_{-1} \backslash \div$, i.e.

$\div \backslash \omega_{-1} \to \omega_0 \backslash -$. Then $f'(x) \backslash \omega_{-1} \to \omega_0 \backslash \overset{0}{f}(x) =$

$$= \lim_{\delta x \to (-\infty)} \left(\log_k \left(k^{f\left(\log_k\left(k^x + k^{\delta x}\right)\right)} - k^{f(x)} \right) - \delta x \right).$$

All this allows to give the following definition.

The definition 3.4. *The image in* ω_0 *derivative* $f'(x)$, *noted in space* ω_{-1} *names expression*

$$\overset{o}{f}(x) = \lim_{\delta x \to (-\infty)} \left(\log_k \left(k^{f\left(\log_k\left(k^x + k^{\delta x}\right)\right)} - k^{f(x)} \right) - \delta x \right)$$

In terms ω_{-1}-space $\overset{o}{f}(x)$ looks so:

$$\overset{o}{f}(x) = \lim_{\delta x \to 0} \left(f(x \oplus \delta x) \ominus f(x) \right) \varnothing \, \delta x,$$

i.e. in this case at reflection $\backslash \omega_{-1} \to \omega_0 \backslash$ the following replacement of operations is realized:

$$\{+, -, \div\} \to \{\oplus, \ominus, \varnothing\},$$

where $\oplus, \ominus, \varnothing$ – accordingly images of operations $+, -, \div$ at reflection $\backslash \omega_{-1} \to \omega_0 \backslash$.

Theorem 3.9. *The image* $\overset{o}{f}(x)$ *function* $f(x)$, *continuous, monotone on some gap of area it of the definition* I $(J \in I)$, *is connected to an initial derivative* $f'(x)$ *following formula, fair on gap* J:

$$f'(x)=k^{\overset{\circ}{f}(x)-f(x)+x}, \qquad (3.19)$$

where k *– conversion coefficient between spaces* ω_{-1} *and* ω_0 $(k \neq 1)$.

Proof. All conditions for function $f(x)$ and $k \in \mathbf{R}_+$, $k \neq 1$ let are carried out. Let's designate for A a limit

$$\lim_{\delta x \to (-\infty)}\left(\log_k\left(k^{f\left(\log_k\left(k^x+k^{\delta x}\right)\right)}-k^{f(x)}\right)-\delta x\right).$$

The available indeterminacy has an aspect $\infty-\infty$. Let $\varphi=$ $=\log_k\left(k^{f\left(\log_k\left(k^x+k^{\delta x}\right)\right)}-k^{f(x)}\right)$, $\psi=\delta x$. Then

$$A=\lim_{\delta x \to (-\infty)}(\varphi-\psi) \;\Rightarrow\; B=k^A=k^{\left(\lim\limits_{\delta x \to (-\infty)}(\varphi-\psi)\right)}=$$

$=\lim\limits_{\delta x \to (-\infty)}\dfrac{k^{\varphi}}{k^{\psi}}$. Have received indeterminacy $\dfrac{0}{0}$, which is uncovered on a rule of the L'Hospital:

$$B=\lim_{\delta x \to (-\infty)}\left(\frac{k^{\varphi}\cdot\varphi'}{k^{\psi}\cdot\psi'}\right). \quad \text{As} \quad k^{\varphi}=k^{f\left(\log_k\left(k^x+k^{\delta x}\right)\right)}-k^{f(x)},$$

$$\varphi'=\frac{k^{f\left(\log_k\left(k^x+k^{\delta x}\right)\right)}\cdot f'\left(\log_k\left(k^x+k^{\delta x}\right)\right)}{k^{f\left(\log_k\left(k^x+k^{\delta x}\right)\right)}-k^{f(x)}},\; \psi'=1,\; k^{\psi}=k^{\delta x}.$$

$$B=\lim_{\delta x \to (-\infty)}\frac{k^{f\left(\log_k\left(k^x+k^{\delta x}\right)\right)}\cdot f'(p)\cdot p'}{k^{\delta x}},$$

where $p=\log_k\left(k^x+k^{\delta x}\right)$, i.e.

$$B=\frac{k^{f(x)}\cdot f'(x)}{\lim\limits_{\delta x \to (-\infty)}\left(k^x+k^{\delta x}\right)}=\frac{k^{f(x)}\cdot f'(x)}{k^x}.$$

Whence $A=\log_k B = f(x) - x + \log_k f'(x) \quad \Rightarrow$

$$\Rightarrow \quad \overset{0}{f}(x) - f(x) + x = \log_k f'(x), \text{ i.e.}$$

$$f'(x) = k^{\overset{0}{f}(x) - f(x) + x},$$

As was to be shown.

The note. From a comparison of expressions $\boxed{f'(x) = k^{\overset{0}{f}(x) - f(x) + x}}$ **and** $\boxed{\overset{'}{-}f = k^{(f' \cdot x)/f}}$ **the identity** f' **and** $\overset{'}{-}f$ **for** $\omega-$**spaces of a different index follows in view of a modification order (rank) of operations. Really, by replacing accordingly in the formula (3.19) operations** $\{ - ; + \}$ **on** $\{ \div ; \cdot \}$**, we shall receive instead of an assotiation** f' **both** $\overset{0}{f}$ **assotiation** $\overset{'}{-}f$ **and** f'**.**

Uneasy, to make a table $\overset{0}{f}(x)$. For example, for $f(x) = c = const$ $\overset{0}{f} = -\infty$; for $f = x$ $\overset{0}{f} = 0$; for $f = x^n$ $\overset{0}{f} = x^n - x + (n-1) \cdot \log_k x + \log_k n$; for $f = a^x$ $\overset{0}{f} = a^x - x + {}$ ${} + x \cdot \log_k a + \log_k \ln a$; $f = e^x$ $\overset{0}{f} = e^x - 0{,}57 \cdot x$; $\overset{0}{\overbrace{(f+\varphi)}} = (f+\varphi) - {}$ ${} - x + \log_k (f+\varphi)'$; $\overset{0}{\overbrace{(f \cdot \varphi)}} = f \cdot \varphi - x + \log_k (f \cdot \varphi)'$ etc.

The shaping of an image $\overset{0}{f}$ allows to supplement item 3.1 present chapters statement of the following theorem, which will help, to some extent, deeper to understand the common concept ω-reflections of complicated functional constructions.

Theorem 3.10. *Let on a set of arguments* $X = \{x_1, x_2, \dots, x_n\}$ *with ranges of definition* $I = \{i_1, i_2, \dots, i_n\}$ *the composite function*

$Y = \Psi\left(f\left(\varphi\left(x_1, x_2, \dots, x_n\right)\right)\right)$ *is given, and there is an area* J, *in which all functions* (ψ, f, φ) *are defined. Then at reflection* $\backslash \omega_j \to \omega_i \backslash$ *with function of connection* $F(x)$ *between spaces* ω_j *and* ω_i *the image* Y^* *function* Y *as expression turns out:*

$$Y^* = F\left(\psi\left(F^{-1}\left(f\left(F\left(\varphi\left(F^{-1}(x_1), F^{-1}(x_2), \dots, F^{-1}(x_n)\right)\right)\right)\right)\right)\right). \quad (3.20)$$

Not stopping on a proof of this theorem, which goes out for frameworks of the given monography, we shall mark some important moment:

a). The shaping anyone ω-image of a composite function begins with image of a set of arguments X:

$$X = \{x_1, x_2, \dots, x_n\} \backslash \omega_j \to \omega_i \backslash X^* = \left\{F^{-1}(x_1), F^{-1}(x_2), \dots, F^{-1}(x_n)\right\},$$

i.e. each argument x_k it is necessary to note as function $F^{-1}(x_k)$, inverse function of connection $F(x)$.

b). Then the ω-image of function (first internal function) as φ is $F\left(\varphi\left(F^{-1}(x_1), F^{-1}(x_2), \dots, F^{-1}(x_n)\right)\right)$, i.e. the function of connection F is noted, and argument it is the function φ with the transformed arguments $\left(F^{-1}(x_1), F^{-1}(x_2), \dots, F^{-1}(x_n)\right)$.

c). At last, is noted sequentially all internal functions (in our case, it only f – second internal function) and external function ψ. Thus there is a sequential alternation of assotiations F and F^{-1}. In the total, in an entry appear direct function F and inverse to her F^{-1}.

d). The proof of the theorem is based on marked in a lemma 1.1 fact:

$$f(x) \backslash \omega_j \to \omega_i \backslash F\left(f\left(F^{-1}(x)\right)\right),$$

where $F(x)$ – function of connection.

The gradually complicating in the correspondence about (3.20) structure of function $f(x)$, is possible to receive a required outcome. For example, elementary ω – reflection of function $f(x_1, x_2, \dots, x_n)$ will be those:

$$f(x_1, x_2, \dots, x_n) \backslash \omega_1 \to \omega_0 \backslash$$

$$\backslash \omega_1 \to \omega_0 \backslash F\left(f\left(F^{-1}(x_1), F^{-1}(x_2), \dots, F^{-1}(x_n)\right)\right).$$

The note. To construct function the sum exterior is necessary so that and all internal functions was odd.

§ 3.4 Adjacent ω-space and image f_{k_2}'.

In connection with an arbitrary of a choice of a constant k (unique conditions, which she should satisfy with is $k \neq 1$ and, in the elementary case, $k \in \mathbf{R}$) on this indication (k) it is possible to receive infinite a spectrum adjacent ω-spaces. Changing a value k, we always obtain new ω-space.

Let known mathematical objects are located in spaces ω_i and ω_i', which are connected to space ω_1 function k^x $\left(k_1^x - \text{for } \omega_i \text{ and } k_2^x - \text{ for } \omega_i'\right)$. Let's consider the procedure of reflection of objects from space ω_i' in space ω_i (or on the contrary). If object is the number a (in ω_1), it is obvious $a \setminus \omega_1 \rightarrow \omega_i \setminus k_1^a$ and $a \setminus \omega_1 \rightarrow \omega_i' \setminus k_2^a$. Then $k_2^a \setminus \omega_i' \rightarrow \omega_i \setminus k_1^a$. By designating $a_i' = k_2^a$, $a_i = k_1^a$, we shall receive $a = \log_{k_2} a_i' = \log_{k_1} a_i$, i.e. $a_i' = k_2^{\log_{k_1} a_i}$, and $a_i = k_1^{\log_{k_2} a_i'}$. So,

$$a_i' \setminus \omega_i' \rightarrow \omega_i \setminus k_1^{\log_{k_2} a_i'} = \left(a_i'\right)^{1/p} \quad (3.21)$$

$$a_i \setminus \omega_i \rightarrow \omega_i' \setminus k_2^{\log_{k_1} a_i} = \left(a_i\right)^{p_1},$$

where $p_1 = \log_{k_1} k_2$.

Let in ω_i the function $y = f\left(x_1, x_2, \ldots, x_n\right)$ is given. Let's discover the formula ω-conversion of this function in some space ω_i'.

Theorem 3.11. *If in space ω_i the function $f\left(x_1, x_2, \ldots, x_n\right)$ is noted, at ω-reflection it $\setminus \omega_i \rightarrow \omega_i' \setminus$ from one space in adjacent[26] space ω_i' the*

[26] So, ω – spaces $\left(\omega_i\right)$ with a various value k in function of connection k^x we shall name adjacent. Passage from one adjacent ω – space to another we shall name horizontal ω – reflection.

function f *is transformed in following* $\sqrt[p]{f\left(x_1{}^p, x_2{}^p, \ldots, x_n{}^p\right)}$, *where* $p = \log_{k_2} k_1$, *and* k_1 *and* k_2 *- accordingly conversion factors between spaces* ω_i, ω_i' *and space* ω_1, *i.e.*

$$\boxed{f(x_1, x_2, \ldots, x_n) \setminus \omega_i \to \omega_i' \setminus \sqrt[p]{f\left(x_1{}^p, x_2{}^p, \ldots, x_n{}^p\right)}} \quad (3.22)$$

Proof. Let factors (parameters) of passage from ω_1 in spaces ω_i both ω_i' are accordingly equal k_1 and k_2. Let's image function f from ω_i in ω_1:

$$f(x_1, x_2, \ldots, x_n) \setminus \omega_i \to \omega_1 \setminus \log_{k_1} f\left(k_1{}^{x_1}, k_1{}^{x_2}, \ldots, k_1{}^{x_n}\right).$$

Let's image an obtained image of function f in space ω_i':

$$\log_{k_1} f\left(k_1{}^{x_1}, k_1{}^{x_2}, \ldots, k_1{}^{x_n}\right) \setminus \omega_1 \to \omega_i' \setminus$$

$$\setminus \omega_1 \to \omega_i' \setminus \ k_2{}^{\log_{k_1} f\left(k_1{}^{x_1^*}, k_1{}^{x_2^*}, \ldots, k_1{}^{x_n^*}\right)}$$

The arguments $(x_1, x_2, \ldots, x_n)$ vary in $\left(x_1{}^*, x_2{}^*, \ldots, x_n{}^*\right)$, where $x_1{}^* = \log_{k_2} x_1$, $x_2{}^* = \log_{k_2} x_2$, ... , $x_n{}^* = \log_{k_2} x_n$, i.e.

$$f(x_1, x_2, \ldots, x_n) \setminus \omega_i \to \omega_i' \setminus$$

$$\setminus \omega_i \to \omega_i' \setminus k_2{}^{\log_{k_1} f\left(k_1{}^{\log_{k_2} x_1}, k_1{}^{\log_{k_2} x_2}, \ldots, k_1{}^{\log_{k_2} x_n}\right)} =$$

$$= k_2{}^{\log_{k_1} f\left((k_1)^{\frac{\log_{k_1} x_1}{\log_{k_1} k_2}}, (k_1)^{\frac{\log_{k_1} x_2}{\log_{k_1} k_2}}, \ldots, (k_1)^{\frac{\log_{k_1} x_n}{\log_{k_1} k_2}} \right)} =$$

$$= k_2{}^{\log_{k_1} f\left(x_1{}^p, x_2{}^p, \ldots, x_n{}^p\right)}, \text{ where } p = \log_{k_2} k_1.$$

At last, we shall transform $k_2^{\log_{k_1} f\left(x_1^{\,p}, x_2^{\,p}, \ldots, x_n^{\,p}\right)} =$
$= k_2^{\left(\left(\log_{k_2} f\left(x_1^{\,p}, x_2^{\,p}, \ldots, x_n^{\,p}\right)\right)/\log_{k_2} k_1\right)} = \sqrt[p]{f\left(x_1^{p}, x_2^{p}, \ldots, x_n^{p}\right)}$,
as was to be shown.

Theorem 3.12. *If function* $f(x)$*, it a derivative* $f'(x)$ *and pseudoimage by a derivative* ${}^{'}f(x)$ *are continuous also differentiable in some area* D *and in this area* $x \neq 0$ *and* $f(x) \neq 0$*, the image in* ω_0' *derivative* $f'(x)$*, noted in space* ω_0 *is defined under the formula*

$$f'_{k_2}(x) = \frac{f(x)}{x} \cdot \left(\frac{x \cdot f'_{k_1}(x)}{f(x)} \right)^p, \qquad (3.23)$$

where $f'_{k_1} \equiv f'(x)$, $p = \log_{k_2} k_1$, k_1 *and* k_2 *−factors of connection of adjacent spaces* ω_0 *and* ω_0' *with space* ω_1.

Proof. Reflection $f'(x)$ from space ω_1 in adjacent ω_0 and ω_0', we shall receive the following images:

$$f'(x) \setminus \omega_1 \to \omega_0 \setminus k_1^{\,{}^{'}f_{k_1}(x)} = {}^{'}_{-}f_{k_1}$$

$$f'(x) \setminus \omega_1 \to \omega_0' \setminus k_2^{\,{}^{'}f_{k_2}(x)} = {}^{'}_{-}f_{k_2}.$$

From the formula (3.22) follows:

$$k^{\varphi} \setminus \omega_0' \to \omega_0 \setminus \sqrt[p_1]{k^{\varphi^{p_1}}} = k^{\left(\varphi^{p_1}\right)/p_1},$$

where $p_1 = \dfrac{1}{p} = \log_{k_1} k_2$, $k = const$, $\varphi = \varphi(x)$ - some continuous function.

Image in ω_0 derivative $f'(x)$, noted in ω_1:

$$\overset{'}{\underset{-}{}}f_{k_1} = k_1^{\,'f_{k_1}(x)} \Rightarrow k_1^{f'\cdot x/f} = \overset{'}{\underset{-}{}}f_{k_1} \Rightarrow f' = \frac{f}{x}\cdot \log_{k_1} \overset{'}{\underset{-}{}}f_{k_1}.$$

Let's discover:

$$k_2^{\,'f_{k_2}(x)} \setminus \omega_0' \to \omega_0 \setminus k_2^{\left(\left({}'f_{k_2}(x)\right)^{p_1}\right)/p_1}.$$

On the other hand,

$$\overset{'}{\underset{-}{}}f_{k_2} \setminus \omega_0' \to \omega_0 \setminus \overset{'}{\underset{-}{}}f_{k_1}, \text{ i.e.}$$

$$\overset{'}{\underset{-}{}}f_{k_1} = k_2^{\left(\left({}'f_{k_2}(x)\right)^{p_1}\right)/p_1} \Rightarrow k_1^{f'\cdot x/f} = k_2^{\left(\left({}'f_{k_2}(x)\right)^{p_1}\right)/p_1} \Rightarrow$$

$$\Rightarrow f'\cdot x/f = \log_{k_1} k_2^{\left(\left({}'f_{k_2}(x)\right)^{p_1}\right)/p_1} \Rightarrow$$

$$\Rightarrow f'(x) = \frac{f(x)}{x}\cdot \log_{k_1} k_2^{\left(\left({}'f_{k_2}(x)\right)^{p_1}\right)/p_1} =$$

$$= \frac{f(x)}{x}\cdot \frac{\left({}'f_{k_2}(x)\right)^{p_1}}{p_1}\cdot \log_{k_1} k_2 = \frac{f(x)}{x}\cdot \sqrt[p]{{}'f_{k_2}(x)} \Rightarrow$$

$$\Rightarrow \left(\frac{f'(x)\cdot x}{f(x)}\right)^p = {}'f_{k_2}(x), \text{ i.e. } \left({}'f_{k_1}(x)\right)^{\log_{k_2} k_1} = {}'f_{k_2}(x).$$

As $\quad {}'f_{k_2}(x)=\dfrac{x\cdot f'_{k_2}(x)}{f(x)}$, $\quad f'(x)\equiv f'_{k_1}(x)$, and

$p=\dfrac{1}{p_1}=\log_{k_2}k_1$, $f'_{k_2}(x)=\dfrac{f(x)}{x}\cdot\left(\dfrac{x\cdot f'_{k_1}(x)}{f(x)}\right)^p$, as was to be shown.

The note. Because of above-stated it is possible to note an general image in ω_0 derivative obtained at image from *any* space ω_i:

$$f' \setminus \omega_1 \to \omega_0 \setminus {}^{[i]}f = \lim_{\delta x\to\alpha} {}^{i+1}_{3}\Re_{\delta x}\left({}^{i}_{3}\Re_f\left(f\left({}^{i}_{1}\Re^{\delta x}_{x}\right)\right)\right), \quad (3.24)$$

where α – neutral (zero) element, $\Re$ – numeral entered for an entry of the formula invariant rather of ω-spaces (look "Labels").

If there is a reflection in ω_0 from adjacent space ω_i', and the entry is carried on in terms of an image ${}^{[i]}f$, it is necessary expression (3.24) to substitute in (3.23) instead of f'_{k_1}. Then ${}^{[i]}f_{k_2}(x) \equiv f'_{k_2}(x)$ will be a unknown quantity by an outcome, i.e. the procedure of the registration of a contiguity of space in this case is rather simples.

The integration in adjacent spaces is of interest, as the specifity of the obtained formulas at ω-reflections of integrated objects, allows sometimes to solve rather challenges. In the present chapter the small fragment of this unfinished research is reduced only, but, under the judgement of the author, it is enough of the underwritten text to receive initial skills and impulse for creative development of the given direction of an integration.

Let's begin from well-known reasonings. Let in space ω_0 the definite integral $I=\int\limits_{\alpha}^{\beta} f(x)\,dx$ is given. Geometrically it can be interpreted as square S_* curvilinear trapezoid $ABCD(a)$.

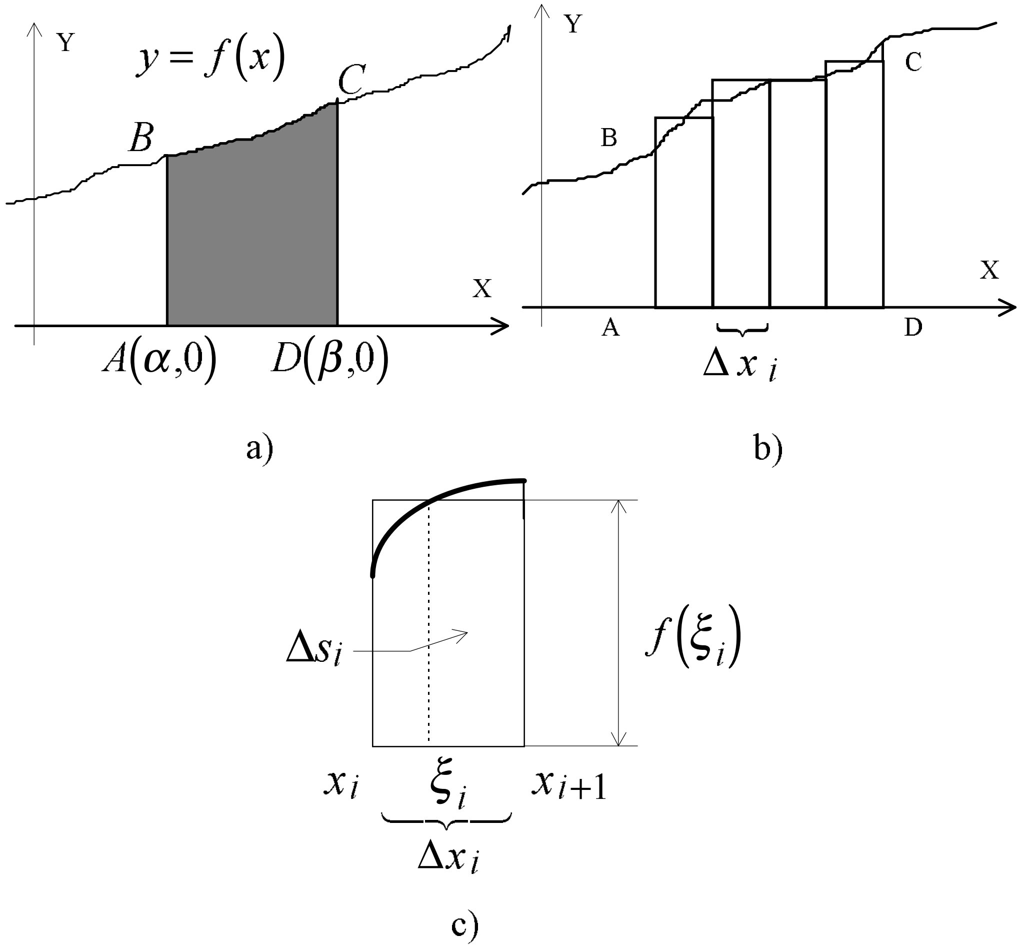

Fig. 2. A curvilinear trapezoid ABCD (a), it approximation by a graduated figure (b) and image of a low level cell Δs_i in space ω_0.

For deriving such outcome in a calculus S_* approximate by square of a graduated figure (δ) and then take it a limit provided that length of a maximum elementary segment $(\max \Delta x_i)$ tends to zero, i.e.

$$I = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i) \cdot \Delta x_i = \int_{\alpha}^{\beta} f(x)\,dx = \Phi(x)\Big|_{\alpha}^{\beta} = \Phi(\beta) - \Phi(\alpha),$$

where $\xi_i \in \Delta x_i$; $\alpha, \beta -$ limits of an integration; $\Phi(x) -$ primitive function $(\Phi'(x) = f(x))$.

The square S_* Graduated figure, approximating square of a curvilinear trapezoid, is a composite function $S_* = \varphi\left(f(\xi_i) \cdot \Delta x_i\right)$.

The reflection of a sum S_* is possible after a proof of the following theorem.

Theorem 3.13. *If in space* ω_0 *the continuous positive composite function* $y=\varphi(f(x))$ *is given, it* $\omega-$*image in adjacent space is equal* $\sqrt[p]{\varphi(f(x^p))}$, *i.e.*

$$\varphi(f(x))\setminus\omega_0\to\omega_0'\setminus\sqrt[p]{\varphi(f(x^p))}.$$ [27]

Proof. Considering three spaces ω_0, ω_0', ω_1 and reflecting objects in these spaces it is possible to note:

$$\varphi(f(x))\setminus\omega_0\to\omega_0'\setminus k_2^{\log_{k_1}\varphi\left(k_1^{\log_{k_2}f_*}\right)},$$

where $f_*=k_2^{\log_{k_1}f\left(k_1^{\log_{k_2}x}\right)}$ - ω-image of function $f(x)$ $\left(f(x)\setminus\omega_0\to\omega_0'\setminus k_2^{\log_{k_1}f\left(k_1^{\log_{k_2}x}\right)}\right)$. Whence,

$$k_1^{\log_{k_2}f_*}=k_1^{\log_{k_2}k_2^{\log_{k_1}f\left(k_1^{\log_{k_2}x}\right)}}=f\left(k_1^{\log_{k_2}x}\right),$$ i.e.

$$k_2^{\log_{k_1}\varphi\left(k_1^{\log_{k_2}f_*}\right)}=k_2^{\log_{k_1}\varphi\left(f\left(k_1^{\log_{k_2}x}\right)\right)};\qquad k_1^{\log_{k_2}x}=$$

$$=x_*\ \Rightarrow\ \log_{k_1}x_*=\log_{k_2}x\ \Rightarrow\ \frac{\ln x_*}{\ln k_1}=\frac{\ln x}{\ln k_2}\ \Rightarrow\ \ln x_*=$$

---

[27] At a replacement $p=\log_{k_2}k_1$ on $p_1=\frac{1}{p}=\log_{k_1}k_2$ it is necessary to interchange the names of spaces, i.e. instead of $\setminus\omega_0\to\omega_0'\setminus$ it is necessary to write $\setminus\omega_0'\to\omega_0\setminus$.

$= \log_{k_2} k_1 \cdot \ln x \;\Rightarrow\; x_* = x^p$.

In an outcome, $k_2^{\log_{k_1} \varphi\left(f\left(x^p\right)\right)} = \varphi_* \;\Rightarrow\; \log_{k_2} \varphi_* =$

$= \log_{k_1} \varphi\left(f\left(x^p\right)\right) \;\Rightarrow\; \dfrac{\ln \varphi_*}{\ln k_2} = \dfrac{\ln \varphi\left(f\left(x^p\right)\right)}{\ln k_1} \;\Rightarrow\; \varphi_* = \sqrt[p]{\varphi\left(f\left(x^p\right)\right)}$, as was to be shown.

The note. Let's remark, that the product of variables at image $\backslash \omega_0 \to \omega_0' \backslash$ turns too to a product, that it is uneasy to prove.

From the theorem 3.13 follows, that $S_* \backslash \omega_0 \to \omega_0' \backslash S_*^0 =$

$= \sqrt[p]{\sum\limits_{i=1}^{n} f\left(\xi_i^p\right) \cdot \Delta x^p}$.

Let's discover $\lim\limits_{\Delta x \to 0} S_*^0$: $\lim\limits_{\Delta x \to 0} S_*^0 = \lim\limits_{\Delta x \to 0} \sqrt[p]{\sum\limits_{i=1}^{n} f\left(\xi_i^p\right) \cdot \Delta x^p} =$

$$= \sqrt[p]{\lim_{\Delta x \to 0} \sum_{i=1}^{n} f\left(\xi_i^p\right) \cdot \Delta x^p} = \sqrt[p]{\int_{\alpha}^{\beta} f\left(x^p\right) d\, x^p},$$

i.e. ω-the image of a definite integral in adjacent space ω_0' will be:

$$I = \int_{\alpha}^{\beta} f(x) dx \backslash \omega_0 \to \omega_0' \backslash I_* = \sqrt[p]{\int_{\alpha}^{\beta} f\left(x^p\right) d\, x^p}, \qquad (3.25)$$

where $I_* - \omega$-image of an integral obtained by a way $\backslash \omega_0 \to \omega_0' \backslash$.

Let's image a primitive function $\Phi(x)$:

$$\Phi(x) \backslash \omega_0 \to \omega_0' \backslash \sqrt[p]{\Phi\left(x^p\right)}$$

This outcome coincides with the formula (3.25). Really, we shall replace $x^p = z$. Then

$I=\int f(x)\,dx=\Phi(x)\setminus\omega_0\to\omega_0{}'\setminus I_*=\sqrt[p]{\int f(z)\,dz}=\sqrt[p]{\Phi(z)}$, as $\int f(z)\,dz=\Phi(z)$ by virtue of invariancy of an aspect of the first differential of function, so, invariancy of an aspect of an integral.

Despite of an apparent simplicity the problem of an integration in adjacent spaces is rather problematic. At it to development the reader can find a wide field for creativity …

Applying the formula (3.23) it is simple to note a table of images of derivatives in adjacent space $\omega_0{}'$. For example, $\left(x^n\right)'_{k_2}=n^p\cdot x^{n-1}$. The index k_2 means, that the derivative is taken in space $\omega_0{}'$. Really, as $f'_{k_2}=\left(\frac{f^{1/p-1}}{x^{1/p-1}}\cdot f'_{k_1}\right)^p$, $\left(x^n\right)'_{k_2}=\left(\frac{x^{n\cdot(1/p-1)}}{x^{1/p-1}}\cdot n\cdot x^{n-1}\right)^p=n^p\cdot x^{n-1}$.

$$(\sin x)'_{k_2}=\left(\frac{(\sin x)^{(1/p)-1}}{x^{(1/p)-1}}\cdot\cos x\right)^p=\left(\frac{\sin x}{x}\right)^{1-p}\cdot\cos^p x,$$

$$(arctg\,x)'_{k_2}=\left(\frac{arctg\,x}{x}\right)^{1-p}\cdot\frac{1}{\left(1+x^2\right)^p}\quad\text{etc.}$$

Knowing a table of derivatives in adjacent spaces, it is uneasy to make a similar table for ω-images of integrals; designating them $\int_{k_2}$:

$$\int_{k_2}x^{n-1}\,\delta x=\frac{x^n}{n^p},$$ [28]

$$\int_{k_2}\left(\frac{\sin x}{x}\right)^{1-p}\cdot\cos^p x\,\delta x=\sin x,$$

[28] Arbitrary constant we not note for simplification of understanding of the essence $\omega-$images of an integral in adjacent spaces.

$$\int_{k2}\left(\frac{arctg\,x}{x}\right)^{1-p}\cdot\frac{1}{\left(1+x^2\right)^p}\,\delta x = arctg\,x \text{ etc.}$$

So,

$$\int_{k2}\left(\frac{f(x)}{x}\right)^{1-p}\cdot\left(f'(x)\right)^p\,\delta x = f(x). \qquad (3.26)$$

In this case, $\int_{k1} f'(x)\,dx = f(x)$, $\int_{k1}\equiv\int$ and

$$\int_{k2}\left(\frac{f(x)}{x}\right)^{1-p}\cdot\left(f'(x)\right)^p\,\delta x = \int_{k1} f'(x)\,dx.$$

From here it is visible, that for an entry of an integral $\int_{k2}$ it is necessary to find a primitive function $f(x)$, i.e. to solve an initial integral $\int_{k1}$.

For example, $I_{k1}=\int\cos x\,dx \setminus \omega_0 \to \omega_0' \setminus I_{k2} =$

$$=\int\left(\frac{\sin x}{x}\right)^{1-p}\cdot\cos^p x\,\delta x.$$

Then passing to a standard notation of a primitive, we shall receive

$$\int_{k2}\left(\frac{\sqrt[p]{\Phi\left(x^p\right)}}{x}\right)^{1-p}\cdot\left(\frac{1}{p}\cdot\left(\Phi\left(x^p\right)\right)^{1/p-1}\cdot\Phi'_{x^p}\left(x^p\right)\cdot p\cdot x^{p-1}\right)^p\delta x =$$

$$=\int_{k2}\left(\frac{\sqrt[p]{\Phi\left(x^p\right)}}{x}\right)^{1-p^2}\cdot\left(\Phi'_{x^p}\left(x^p\right)\right)^p\delta x = \sqrt[p]{\Phi\left(x^p\right)},$$

where $\Phi(x)-$primitive function in a usual integral $\left(I=\int f(x)dx=\Phi(x)\right)$.

If there is such function $\psi(x)$, that it the primitives in spaces ω_0 and ω_0' coincide, the following equality should be fulfilled:

$$\int_{k2}\left(\frac{\Psi(x)}{x}\right)^{1-p}\cdot\left(\Psi'(x)\right)^{p}\delta x=\Psi(x),$$

$$\text{or } \int_{k2}\left(\frac{\Psi(x)}{x}\right)^{1-p}\cdot\psi^{p}(x)\delta x=\Psi(x),$$

$$\text{where } \int_{k1}\psi(x)dx=\int\psi(x)dx=\Psi(x).$$

It is possible to try also to image $I=\int\limits_{\alpha}^{\beta}f(x)dx$ from ω_0 in ω_0' immediately. It signifies to try to note $\int_{k2}$ as a usual integral $\left(\int\equiv\int_{k1}\right)$ in space ω_0':

$$I=\int\limits_{\alpha}^{\beta}f(x)dx\setminus\omega_0\to\omega_0'\setminus\int\limits_{\sqrt[p]{\alpha}}^{\sqrt[p]{\beta}}\sqrt[p]{f\left(x^{p}\right)}\delta x$$

(The constants α and β are reflections so: $\alpha\setminus\omega_0\to$ $\to\omega_1\setminus\log_{k1}\alpha\setminus\omega_1\to\omega_0'\setminus k2^{\log_{k1}\alpha}=\alpha_*$, i.e. $\log_{k2}\alpha_*=$ $=\log_{k1}\alpha\ \Rightarrow\ \dfrac{\ln\alpha_*}{\ln k_2}=\dfrac{\ln\alpha}{\ln k_1}\ \Rightarrow\ \alpha_*=\sqrt[p]{\alpha}$).

For approximate accounts it is necessary to search for the formula containing a differential δx approximately that of the order, as dx. For example, such formula is necessary:

$$I=\int\limits_{\alpha}^{\beta}f(x)dx\setminus\omega_0\to\omega_0'\setminus I'\approx p\cdot\int\limits_{\sqrt[p]{\alpha}}^{\sqrt[p]{\beta}}\sqrt[p]{f\left(x^{p}\right)}dx \quad (3.27)$$

Example. By a method Th. Simpson we shall calculate integrals I' using their expression under the formula (3.27):

a)

$$I' = \left\{ 2 \cdot \int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{2}}} \sqrt{\sin x^2}\, \delta x \approx 0{,}6923;\ 3 \cdot \int_{\sqrt[3]{\frac{\pi}{4}}}^{\sqrt[3]{\frac{\pi}{2}}} \sqrt[3]{\sin x^3}\, \delta x \approx 0{,}6908; \right.$$

$$4 \cdot \int_{\sqrt[4]{\frac{\pi}{4}}}^{\sqrt[4]{\frac{\pi}{2}}} \sqrt[4]{\sin x^4}\, \delta x \approx 0{,}6908; \qquad 7 \cdot \int_{\sqrt[7]{\frac{\pi}{4}}}^{\sqrt[7]{\frac{\pi}{2}}} \sqrt[7]{\sin x^7}\, \delta x \approx 0{,}6913;$$

$$\left. 101 \cdot \int_{\sqrt[101]{\frac{\pi}{4}}}^{\sqrt[101]{\frac{\pi}{2}}} \sqrt[101]{\sin x^{101}}\, \delta x \approx 0{,}6929,\ \ldots \right\}.$$

b)

$$I' = 2 \cdot \int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{3}}} \sqrt{\sin x^2 + \cos x^2}\, \delta x \approx 0{,}7087$$

$$I' = 3 \cdot \int_{\sqrt[3]{\frac{\pi}{6}}}^{\sqrt[3]{\frac{\pi}{3}}} \sqrt[3]{\sin x^3 + \cos x^3}\, \delta x \approx 0{,}7027$$

$$I' = 4 \cdot \int_{\sqrt[4]{\frac{\pi}{6}}}^{\sqrt[4]{\frac{\pi}{3}}} \sqrt[4]{\sin x^4 + \cos x^4}\, \delta x \approx 0{,}7000$$

$$I' = 100 \cdot \int_{\sqrt[100]{\frac{\pi}{6}}}^{\sqrt[100]{\frac{\pi}{3}}} \sqrt[100]{\sin x^{100} + \cos x^{100}}\, \delta x \approx 0{,}6934 .$$

c) $$I' = 2 \cdot \int_{\sqrt{2}}^{\sqrt{5}} \frac{\delta x}{\sqrt{x^2 + 4}} \approx 0{,}6079$$

$$I' = 3 \cdot \int_{\sqrt[3]{2}}^{\sqrt[3]{5}} \frac{\delta x}{\sqrt[3]{x^3 + 4}} \approx 0{,}6966$$

$$I' = 4 \cdot \int_{\sqrt[4]{2}}^{\sqrt[4]{5}} \frac{\delta x}{\sqrt[4]{x^4 + 4}} \approx 0{,}7458 .$$

It is possible to prove, that $\delta x \approx p \cdot dx$ under defined conditions.
However formula (3.27) sometimes gives an erroneous outcome.
For example, if for integrals

d) $$I' = 2 \cdot \int_{1}^{\sqrt{5}} \frac{\delta x}{\sqrt{3 \cdot x^2 - 2}} \approx 1{,}1627$$

$$I' = 3 \cdot \int_{1}^{\sqrt[3]{5}} \frac{\delta x}{\sqrt[3]{3 \cdot x^3 - 2}} \approx 1{,}2933$$

$$I' = 5 \cdot \int_{1}^{\sqrt[5]{5}} \frac{\delta x}{\sqrt[5]{3 \cdot x^5 - 2}} \approx 1{,}41036 .$$

e) $$I' = 2 \cdot \int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{4}}} x \cdot \sqrt{\cos\frac{x^2}{3}}\,\delta x \approx 0{,}26$$

$$I' = 3 \cdot \int_{\sqrt[3]{\frac{\pi}{6}}}^{\sqrt[3]{\frac{\pi}{4}}} x \cdot \sqrt[3]{\cos\frac{x^3}{3}}\,\delta x \approx 0{,}30\,.$$

The error of the prognosis at an evaluation is rather insignificant, but there are functions, outcome of which integration it is impossible to predict. For example,

f) $$I' = 2 \cdot \int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{2}}} \sqrt{3 + \cos x^2}\,\delta x \approx 1{,}3518;$$

$$I' = 3 \cdot \int_{\sqrt[3]{\frac{\pi}{4}}}^{\sqrt[3]{\frac{\pi}{2}}} \sqrt[3]{3 + \cos x^3}\,\delta x \approx 1{,}0814;$$

$$I' = 4 \cdot \int_{\sqrt[4]{\frac{\pi}{4}}}^{\sqrt[4]{\frac{\pi}{2}}} \sqrt[4]{3 + \cos x^4}\,\delta x \approx 0{,}9673.$$

g) $$I' = 2 \cdot \int_{\sqrt{2}}^{\sqrt{4}} \sqrt{e^{x^2}}\,\delta x \approx 5{,}3208;$$

$$I' = 3 \cdot \int\limits_{\sqrt[3]{2}}^{\sqrt[3]{4}} \sqrt[3]{e^{x^3}}\, \delta x \approx 2{,}6524\,;$$

$$I' = 4 \cdot \int\limits_{\sqrt[4]{2}}^{\sqrt[4]{4}} \sqrt[4]{e^{x^4}}\, \delta x \approx 1{,}8845;$$

$$I' = 11 \cdot \int\limits_{\sqrt[11]{2}}^{\sqrt[11]{4}} \sqrt[11]{e^{x^{11}}}\, \delta x \approx 0{,}9928 \text{ etc.}$$

The theorem 3.14. *If* $\left\{\omega_0{}'\right\}$ *– set adjacent with* ω_0 *spaces, and* $\left\{p_1 = \log_{k_1} k_2\right\}$, *where* k_1 *and* k_2 *– appropriate factors of connection of spaces* ω_0 *and* $\left\{\omega_0{}'\right\}$ *with space of a higher rank* ω_1, *that*

$$\lim_{p_1 \to 0} \left\{\omega_0{}'\right\} = \omega_1, \tag{3.28}$$

at $k_1 = const$, $k_2 = \text{var}$.

The strict proof of the theorem falls outside the limits the given to the book. Is offered for a quotient of a proof of the theorem 3.14 digressions in the numerical methods of a solution of the differential equations. Using the known formula of the Euler, we shall note

$$y_{i+1} = y_i + (x_{i+1} - x_i) \cdot f(x_i, y_i), \tag{3.29}$$

where y_i, y_{i+1} – i and $i+1$ of a value of function; $f(x_i, y_i) = y'$ in a point (x_i, y_i).

Let formula (3.29) as object belongs to space $\omega_0{}'$. Let's image this object in adjacent space ω_0, applying the formulas (3.23) and (3.24):

$$y_{k_2}' = \lim_{\Delta x \to 0} \frac{\sqrt[p_1]{f^{p_1} \cdot \left(\left(\sqrt[p_1]{x^{p_1} + (\Delta x)^{p_1}}\right)'\right) - f^{p_1}(x)}}{\Delta x},$$

$$y_{k_2}' = \frac{f}{x} \cdot \sqrt[p_1]{\frac{f' \cdot x}{f}} \qquad \Rightarrow \qquad \left(\frac{x \cdot f_{k_2}'}{f}\right)^{p_1} = \frac{f' \cdot x}{f} \qquad \Rightarrow$$

$$\Rightarrow \quad f' = \frac{f}{x} \cdot \left(\frac{x \cdot f_{k_2}'}{f}\right)^{p_1} = \left(\frac{x}{f}\right)^{p_1 - 1} \cdot \left(f_{k_2}'\right)^{p_1}.$$

$$f(x_i, y_i) = y' = \left(\frac{x_i}{y_i}\right)^{p_1 - 1} \cdot \frac{y_{i+1}^{p_1} - y_i^{p_1}}{x_{i+1}^{p_1} - x_i^{p_1}}.$$

(Beforehand formula (3.29) have copied as $f(x_i, y_i) = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$).

So, at reflection $\backslash \omega_0' \to \omega_0 \backslash$ object (3.29) have received ω-image of the formula (3.29):

$$f(x_i, y_i) = \left(\frac{x_i}{y_i}\right)^{p_1 - 1} \cdot \frac{y_{i+1}^{p_1} - y_i^{p_1}}{x_{i+1}^{p_1} - x_i^{p_1}} \qquad (3.30)$$

Let's discover a limit (3.30) at $p_1 \to 0$:

$$f\left(x_i,y_i\right)=\lim_{p_1\to 0}\left(\frac{x_i}{y_i}\right)^{p_1-1}\cdot\frac{y_{i+1}^{p_1}-y_i^{p_1}}{x_{i+1}^{p_1}-x_i^{p_1}}=$$

$$=\frac{y_i}{x_i}\cdot\lim_{p_1\to 0}\frac{y_{i+1}^{p_1}-y_i^{p_1}}{x_{i+1}^{p_1}-x_i^{p_1}}\quad\Rightarrow\quad\frac{x_i\cdot f\left(x_i,y_i\right)}{y_i}=\lim_{p_1\to 0}\frac{y_{i+1}^{p_1}-y_i^{p_1}}{x_{i+1}^{p_1}-x_i^{p_1}}=$$ [28]

$$=\lim_{p_1\to 0}\frac{y_{i+1}^{p_1}\cdot\ln y_{i+1}-y_i^{p_1}\cdot\ln y_i}{x_{i+1}^{p_1}\cdot\ln x_{i+1}-x_i^{p_1}\cdot\ln x_i}=\frac{\ln y_{i+1}-\ln y_i}{\ln x_{i+1}-\ln x_i}\quad\Rightarrow$$

$$\Rightarrow\quad\ln y_{i+1}=\ln y_i+\frac{x_i}{y_i}\cdot f\left(x_i,y_i\right)\cdot\left(\ln x_{i+1}-\ln x_i\right).$$

Whence $$y_{i+1}=y_i\cdot\left(\frac{x_{i+1}}{x_i}\right)^{\frac{x_i\cdot f\left(x_i,y_i\right)}{y_i}} \qquad (3.31)$$

Where $\frac{x_i\cdot f\left(x_i,y_i\right)}{y_i}={}'y$ in a point $\left(x_i,y_i\right)'$.

As it is visible, (3.31) - it is an image in ω_0 formula (3.29), noted in ω_1:

$$y_i+\left(x_{i+1}-x_i\right)\cdot f\left(x_i,y_i\right)\backslash\omega_1\to\omega_0\backslash\, y_i\cdot\left(\frac{x_{i+1}}{x_i}\right)^{'f}=$$

---

[28] Uncovering indeterminacy on a rule of the L'Hospital: as $y_{i+1}=const$, and $p_1=\text{var}$, $\left(y_{i+1}^{p_1}\right)'=y_{i+1}^{p_1}\cdot\ln y_{i+1}$ etc. (the derivative from an exponential function settle).

$$= y_i \cdot \left(\frac{x_{i+1}}{x_i}\right)^{\frac{x_i \cdot f(x_i, y_i)}{y_i}}$$

, i.e. the space ω_1 is limiting adjacent space $\{\omega_0'\}$

At $p_1 \to 0$: $\lim\limits_{p_1 \to 0} \{\omega_0'\} = \omega_1$, as was to be shown.[29]

From (3.23) follows, that $\lim\limits_{p \to \infty} \frac{f}{x} \cdot \sqrt[p]{\frac{x \cdot f_{k_1}'}{f}} = k_2^{\frac{x \cdot f_{k_1}'}{f}}$,

$$\lim_{p \to \infty} \left(f\left(x_1^{1/p}, x_2^{1/p}, \ldots, x_n^{1/p}\right)\right)^{1/p} =$$

$$= k_2^{f\left(\log_{k_2} x_1, \log_{k_2} x_2, \ldots, \log_{k_2} x_n\right)},$$

as at $p_1 \to 0$ ω_0' is transformed in ω_1.

The reduced theorem proves a *continuity* of passage from adjacent ω-spaces to space of other step of a hierarchy ω-spaces. Infinite the set $\{\omega_i'\}$ adjacent spaces is connected to space ω_1 same function k^x (*analogy* of adjacent spaces). At the same time everyone ω_i' – the space differs from any other adjacent value of factor of connection k (*variability* of adjacent spaces). The compromise of analogy and variability ω-spaces is achieved by exhaustive search of all possible values k. In the total the passage to *qualitative* to new space ω_1 is obtained, the image of operations which from at passage $\backslash \omega_1 \to \omega_i \backslash$ is connected to emerging of new operations.

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[29] The above-stated fact can be used for a solution of some limits and other tasks. By noting ω – image of function in ω_0' and by taking it a limit, we shall receive ω – image of function in ω_1. For example, $\lim\limits_{p \to \infty} \sqrt[p]{\sin x^p} = k_2^{\sin \log_{k_2} x}$, $k_2 \neq 1$, $k_2 > 0$, $p = \log_{k_2} k_1$, $\lim\limits_{p \to \infty} k_2 = 1$.

At study ω-images the compromise between a *discretization* of global objects and their *continuity* is precisely observed also. Really, between two arbitrary taken ω-spaces ω_i and ω_j has a place a *homeomorphism*, i.e. one-to-one images in both legs $\left(\backslash\omega_i \leftrightarrow \omega_j\backslash\right)$ any objects. By virtue of a lack of restrictions on values of indexes i and j the existence of a *continuity* in global sets of spaces $\{\omega_i\}$, $\{\omega_j\}$ etc. is obvious. At the same time qualitative distinctions ω-images of the same objects located in different spaces ω, testifies to presence of a *discretization* in "texture" ω-spaces.

In summary we shall consider two examples.

(Though in chapter 5 the examples of a solution of the differential equations by a way ω-reflection will be illustrated.

a). Let in ω_0' the equation $f'' = x \cdot \cos x$ is noted. One of it of quotients of solutions will be function $f = 2 \cdot \sin x - x \cdot \cos x$. Let's image the equation and it a solution in ω_0, i.e.

$$\{f'' = x \cdot \cos x\} \backslash \omega_0' \to \omega_0 \backslash \left\{ f_{k_2}'' = \sqrt[k]{x^k \cdot \cos x^k} \right\}.$$

$$\{f = 2 \cdot \sin x - x \cdot \cos x\} \backslash \omega_0' \to \omega_0 \backslash \left\{ \varphi = \left(2 \cdot \sin x^k - x^k \cdot \cos x^k\right)^{1/k} \right\}$$

$$f_{k_2}'' = \left(\frac{1}{k^2} \cdot \frac{\left(f^k\right)'' \cdot x^{k-1} - (k-1) \cdot x^{k-2} \cdot \left(f^k\right)'}{x^{3 \cdot (k-1)}} \right)^{1/k}, \quad (3.32)$$

where $k = \log_{k_1} k_2$., i.e. $k \equiv p_1$. The equation (3.32) is uneasy for receiving, by taking a flexon from f_{k_2}' (3.23). The equation $f_{k_2}'' = \sqrt[k]{x^k \cdot \cos x^k}$ Rather complicated, but it a solution is already found as φ.

$$f_{k_2}'' = \sqrt[k]{\frac{Z'}{k^2 \cdot x^{2 \cdot (k-1)}} - \frac{(k-1) \cdot Z}{k^2 \cdot x^{2 \cdot k - 1}}} \quad (3.33)$$

Whether we shall check is the image $f\left(f\setminus\omega_0{}' \to \omega_0\setminus\varphi\right)$ $\varphi = $
$= \left(2\cdot\sin x^k - x^k\cdot\cos x^k\right)^{1/k}$ a solution of this equation:

$$Z = \left(2\cdot\sin x^k - x^k\cdot\cos x^k\right)' = 2\cdot k\cdot x^{k-1}\cdot\cos x^k - k\cdot x^{k-1}\times$$

$$\times\cos x^k + x^k\cdot\sin x^k\cdot k\cdot x^{k-1} = k\cdot x^{k-1}\cdot\left(\cos x^k + x^k\cdot\sin x^k\right),$$

$$Z' = k\cdot\left((k-1)\cdot x^{k-2}\cdot\left(\cos x^k + x^k\cdot\sin x^k\right) + x^{k-1}\cdot\left(-\sin x^k\times\right.\right.$$

$$\left.\left.\times k\cdot x^{k-1} + k\cdot x^{k-1}\cdot\sin x^k + x^k\cdot k\cdot x^{k-1}\cdot\cos x^k\right)\right).$$

Let $B = \frac{k-1}{k^2}\cdot\frac{Z}{x^{2\cdot k-1}} = \frac{k-1}{k}\cdot\frac{\cos x^k + x^k\cdot\sin x^k}{x^k}$;

$$A = \frac{Z'}{k^2\cdot x^{2\cdot(k-1)}} = \frac{k-1}{k}\cdot\frac{\cos x^k + x^k\cdot\sin x^k}{x^k} + x^k\cdot\cos x^k.$$

Then $A - B = x^k\cdot\cos x^k$, and from (3.33) $f_{k2}'' =$
$= \sqrt[k]{A-B} = \sqrt[k]{x^k\cdot\cos x^k}$, as was to be shown.

So, reflection the differential equation together with it by a solution from one ω-spaces in another, we shall receive the new differential equation with a ready solution. In a considered case the equation $f'' = x\cdot\cos x$ was transformed at image $\setminus\omega_0{}' \to \omega_0\setminus$ to the equation $f_{k2}'' = \sqrt[k]{\frac{Z'}{k^2\cdot x^{2\cdot(k-1)}} - \frac{(k-1)\cdot Z}{k^2\cdot x^{2\cdot k-1}}}$, and solution $f = 2\cdot\sin x - x\cdot\cos x$ in the following $\varphi = = \left(2\cdot\sin x^k - x^k\cdot\cos x^k\right)^{1/k}$. By check were convinced, that the given solution satisfies to the transformed differential equation.

b). Let in ω_0' the equation $f'' = 3\cdot x^2 - \cos x$ is given. It a solution in this space $f = \frac{x^4}{4} + \cos x$. Then in ω_0 the solution will be $f = \sqrt[k]{\frac{x^{4k}}{4} + \cos x^k}$. Let's image the equation under the formulas (3.22; 3.33):

$$\left\{f'' = 3\cdot x^2 - \cos x\right\} \quad \backslash \omega_0' \to \omega_0 \backslash \quad \left\{f_{k2}'' = \right.$$

$$\left. = \sqrt[k]{\frac{1}{k^2}\cdot\frac{\left(f^k\right)''\cdot x^{k-1} - (k-1)\cdot x^{k-2}\cdot\left(f^k\right)'}{x^{3\cdot(k-1)}}} = \sqrt[k]{3\cdot x^{2\cdot k} - \cos x^k}\right\}.$$

By check it is easy to be convinced, that $f = \sqrt[k]{\frac{x^{4k}}{4} + \cos x^k}$ is a solution of the equation $f_{k2}'' = \sqrt[k]{3\cdot x^{2\cdot k} - \cos x^k}$.

§ 3.5. ω-images of integrals.

The display elements of integrated object in adjacent spaces were above explained. In the present paragraph the examples ω-images of integrals noted in spaces of higher ranks are reduced. At first we shall define an integral of a new nature I_* because of pseudo-image $'f$ derivative:

$$I_* = \lim_{\max \delta x_i \to 1} \prod_i \left(\delta x_i\right)^{'f(\xi_i)}, \tag{3.34}$$

where $\delta x_i = \frac{x_i + \Delta x}{x_i} = 1 + \frac{\Delta x}{x_i}$ (at $\Delta x_i \to 0$ $\lim\limits_{\Delta x_i \to 0} \delta x_i = 1$), $\xi_i \in (x_i, x_i + \Delta x) \equiv (x_i, x_i \cdot \delta x_i)$.

The formula (3.34) is similar to the formula of a usual integral I in view of a modification of a rank of operations and number, by which the increment of argument tends. It is possible to note:

$$\int_{\alpha}^{\beta} (\delta x)^{f(x)} = \lim_{\max \delta x_i \to 1} \Pi_i (\delta x_i)^{f(\xi_i)},$$

where $\int -$ will designate an integral of a new nature ("a superintegral of the first sort").

Let's prove some properties I_*:

1. If the intergrand function contains a constant factor k,

$$\int_{\alpha}^{\beta} (\delta x)^{k \cdot f(x)} = \left(\int_{\alpha}^{\beta} (\delta x)^{f(x)} \right)^k \tag{3.35}$$

Proof. $$\int_{\alpha}^{\beta} (\delta x)^{k \cdot f(x)} = \lim_{\delta x_i \to 1} \Pi_i (\delta x_i)^{k \cdot f(\xi_i)} =$$

$$= \lim_{\delta x_i \to 1} \Pi_i \left((\delta x_i)^{f(\xi_i)} \right)^k = \left(\lim_{\delta x_i \to 1} \Pi_i (\delta x_i)^{f(\xi_i)} \right)^k =$$

$$= \left(\int_{\alpha}^{\beta} (\delta x)^{f(x)} \right)^k .$$

Corollary. $$\int_{\alpha}^{\beta} (\delta x)^{f(x)} = \frac{1}{\int_{\beta}^{\alpha} (\delta x)^{f(x)}} .$$

2. $$\int_{\alpha}^{\beta} (\delta x)^{\sum_{i=1}^{n} f_i(x)} = \prod_{i=1}^{n} \left(\int_{\alpha}^{\beta} (\delta x_i)^{f_i(x)} \right) \tag{3.36}$$

$$\int_{\alpha}^{\beta}(\delta x)^{\sum_{i=1}^{n} f_i(x)} = \lim_{\delta x_i \to 1} \prod_{i=1}^{n}(\delta x_i)^{\sum_{i=1}^{n} f_i(x)} = \lim_{\delta x_i \to 1} \prod_{i=1}^{n}(\delta x_i)^{f_1} \times$$

$$\times(\delta x_i)^{f_2} \cdot \ldots \cdot (\delta x_i)^{f_n} = \prod_{i=1}^{n}\left(\int_{\alpha}^{\beta}(\delta x_i)^{f_i(x)}\right),$$

As $\lim\limits_{\delta x_i \to 1} \prod\limits_{i=1}^{n} = \lim\limits_{\delta x_i \to 1} \prod\limits_{i=1}^{n}(\delta x_1)^{f_1} \cdot \lim\limits_{\delta x_2 \to 1} \prod\limits_{i=1}^{n}(\delta x_2)^{f_2} \ldots$

It is uneasy to place(install) connection between a superintegral $\int(\delta x)^{f(x)}$ And usual integral $\int f(x)dx$: [31]

3. $$\int f(x)dx = \ln\int(\delta x)^{x \cdot f(x)} \qquad (3.37)$$

$$\exp\left(\int \frac{f(x)}{x}dx\right) = \int(\delta x)^{f(x)}$$

4. If on a segment $[\alpha, \beta]$, where $\alpha < \beta$, the function $f(x)$ and $\varphi(x)$ satisfies to a condition $f(x) \le \varphi(x)$,

$$\int_{\alpha}^{\beta}(\delta x)^{f(x)} \le \int_{\alpha}^{\beta}(\delta x)^{\varphi(x)}$$

Proof. Let's consider a ratio

$$\frac{\int_{\alpha}^{\beta}(\delta x)^{\varphi(x)}}{\int_{\alpha}^{\beta}(\delta x)^{f(x)}} = \exp\left(\int_{\alpha}^{\beta}\frac{\varphi(x)}{x}dx - \int_{\alpha}^{\beta}\frac{f(x)}{x}dx\right) =$$

$$= \exp\left(\lim_{\delta x_i \to 0} \sum_{i=1}^{n} \frac{\varphi(\xi_i) - f(\xi_i)}{\xi_i}\Delta x_i\right) \ge 1, \text{ as } \sum_{i=1}^{n} \frac{\varphi(\xi_i) - f(\xi_i)}{\xi_i}\Delta x_i \ge 0.$$

[31] The proof of the formulas (3.37) is submitted to the reader.

Whence $\int\limits_{\alpha}^{\beta}(\delta x)^{\varphi(x)} \geq \int\limits_{\alpha}^{\beta}(\delta x)^{f(x)}$.

5. If m and $M-$least and greatest values of function $f(x)$ on $[\alpha,\beta]$ and $\alpha \leq \beta$,

$$\left(\frac{\beta}{\alpha}\right)^{m} \leq \int\limits_{\alpha}^{\beta}(\delta x)^{f(x)} \leq \left(\frac{\beta}{\alpha}\right)^{M} \qquad (3.38)$$

Proof. Let $m \leq f(x) \leq M$. Because of properties 4 we have

$$\int\limits_{\alpha}^{\beta}(\delta x)^{m} \leq \int\limits_{\alpha}^{\beta}(\delta x)^{f(x)} \leq \int\limits_{\alpha}^{\beta}(\delta x)^{M}.$$

But $\int\limits_{\alpha}^{\beta}(\delta x)^{m} = \left(\int\limits_{\alpha}^{\beta}\delta x\right)^{m} = \left(\frac{\beta}{\alpha}\right)^{m}$; $\int\limits_{\alpha}^{\beta}(\delta x)^{M} = \left(\frac{\beta}{\alpha}\right)^{M}$.

Whence follows (3.38).

6. If the function $f(x)$ Is continuous on a segment $[\alpha,\beta]$, on this segment such point ε will be discovered, that the equality (theorem of the mean) is fair:

$$\int\limits_{\alpha}^{\beta}(\delta x)^{f(x)} = \left(\frac{\beta}{\alpha}\right)^{f(\varepsilon)} \qquad (3.39)$$

Proof. Let for a determinancy $\alpha < \beta$. If m and M essence appropriate the least and greatest values $f(x)$ on a segment $[\alpha,\beta]$, by virtue of an inequality (3.38) we have:

$$\left(\frac{\beta}{\alpha}\right)^{m} \leq \int\limits_{\alpha}^{\beta}(\delta x)^{f(x)} \leq \left(\frac{\beta}{\alpha}\right)^{M}.$$

Whence $$m \cdot \ln\frac{\beta}{\alpha} \leq \ln\int\limits_{\alpha}^{\beta}(\delta x)^{f(x)} \leq M \cdot \ln\frac{\beta}{\alpha},$$

$$m \le \frac{1}{\ln\frac{\beta}{\alpha}} \cdot \ln \int_{\alpha}^{\beta} (\delta x)^{f(x)} \le M \text{, i.e. } m \le \mu \le M,$$

where $$\mu = \frac{1}{\ln\frac{\beta}{\alpha}} \cdot \ln \int_{\alpha}^{\beta} (\delta x)^{f(x)}.$$

As $f(x)$ is continuous on a segment $[\alpha, \beta]$, she accepts all intermediate values between m and M. Therefore, at some value ε $(\alpha \le \varepsilon \le \beta)$ will be $\mu = f(\varepsilon)$, i.e.

$$\frac{1}{\ln\frac{\beta}{\alpha}} \cdot \ln \int_{\alpha}^{\beta} (\delta x)^{f(x)} = f(\varepsilon) \quad \Rightarrow \quad \int_{\alpha}^{\beta} (\delta x)^{f(x)} = \left(\frac{\beta}{\alpha}\right)^{f(\varepsilon)},$$ as was to be shown.

Corollary. From (3.39) follows, that $f(\varepsilon) \cdot \ln\frac{\beta}{\alpha} = \int_{\alpha}^{\beta} \frac{f(x)}{x} dx$. On the other hand, $\int_{\alpha}^{\beta} \frac{f(x)}{x} dx = (\beta - \alpha) \cdot \frac{f(\kappa)}{\kappa}$. Whence $f(\varepsilon) \cdot \ln\frac{\beta}{\alpha} =$

$$= (\beta - \alpha) \cdot \frac{f(\kappa)}{\kappa} \Rightarrow \left(\frac{1}{\beta - \alpha} \cdot \ln\frac{\beta}{\alpha}\right) \cdot \kappa = \frac{f(\kappa)}{f(\varepsilon)} \Rightarrow$$

$$\Rightarrow \quad f(\varepsilon) = \frac{f(\kappa)}{\kappa} \cdot \frac{\beta - \alpha}{\ln\frac{\beta}{\alpha}} \tag{3.40}$$

The formula (3.40) determines connection between average values for an integrand noted in a superintegral and a usual integral.

7. Let $\Phi(x)-$primitive function for undersuperintegrated of function $f(x)$, i.e. $'\Phi(x)=f(x)$ and $c\cdot\Phi(x)=\int_{\alpha}^{\beta}(\delta x)^{f(x)}$. Let's remark, that $'(c\cdot\Phi(x))='(c)+'\Phi(x)='\Phi(x)$), where $c-$arbitrary constant.

Then $$\int_{\alpha}^{\beta}(\delta x)^{f(x)}=\frac{\Phi(\beta)}{\Phi(\alpha)} \qquad (3.41)$$

(It is the modified theorem of the Newton-Leibnitz).

Proof. $\Phi(x\cdot\delta x)=\int_{\alpha}^{x\cdot\delta x}(\delta t)^{f(t)}=\int_{\alpha}^{x}(\delta t)^{f(t)}\times\int_{x}^{x\cdot\delta x}(\delta t)^{f(t)}$. An increment of a function $\delta\Phi=\dfrac{\Phi(x\cdot\delta x)}{\Phi(x)}=\dfrac{\int_{\alpha}^{x}(\delta t)^{f(t)}\cdot\int_{x}^{x\cdot\delta x}(\delta t)^{f(t)}}{\int_{\alpha}^{x}(\delta t)^{f(t)}.}=$

$=\int_{x}^{x\cdot\delta x}(\delta t)^{f(t)}$. Under the formula (3.39) $\delta\Phi=\left(\dfrac{x\cdot\delta x}{x}\right)^{f(\varepsilon)}$, where $x<\varepsilon<x\cdot\delta x$. Let's discover $\log_{\delta x}\delta\Phi$:

$$'\Phi(x)=\lim_{\delta x\to 1}\log_{\delta x}\delta\Phi=\lim_{\delta x\to 1}f(\varepsilon)\qquad\left(\delta\Phi=\frac{\Phi(x\cdot\delta x)}{\Phi(x)}\right).$$ At $\varepsilon\to x$, $\delta x\to 1$, and $\lim_{\delta x\to 1}f(\varepsilon)=\lim_{\varepsilon\to x}f(\varepsilon)$.

In an aspect of a continuity of function $f(x)$ we have $\lim\limits_{\varepsilon \to x} f(\varepsilon) = f(x)$

and $'\Phi(x) = f(x) \Rightarrow \int\limits_{\alpha}^{x} (\delta x)^{f(x)} = c \cdot \Phi(x)$.

Then $\int\limits_{\alpha}^{\alpha} (\delta x)^{f(x)} = \exp\left(\int\limits_{\alpha}^{\alpha} f(t)dt \right) = 1 \quad \Rightarrow \quad c \cdot \Phi(\alpha) = 1 \quad \Rightarrow$

$\Rightarrow \quad c = \dfrac{1^{\alpha}}{\Phi(\alpha)}$. $\int\limits_{\alpha}^{x} (\delta t)^{f(t)} = \dfrac{\Phi(x)}{\Phi(\alpha)}$. At $x = \beta$ $\int\limits_{\alpha}^{\beta} (\delta t)^{f(t)} = \dfrac{\Phi(\beta)}{\Phi(\alpha)}$, as was to be shown, as by virtue of invariancy of an integral concerning an explanatory variable it is possible to note:

$$\int\limits_{\alpha}^{\beta} (\delta x)^{f(x)} = \frac{\Phi(\beta)}{\Phi(\alpha)}.$$

8. Integration[32] piecemeal. Let $u = u(x)$, $v = v(x)$, $u^{v} = \exp\left(\ln u^{v}\right) = e^{v \cdot \ln u} = e^{\int (v \cdot \ln u)' dx} = e^{\int \left(v' \cdot \ln u + (\ln u)' \cdot v \right) \cdot dx} =$

$= \int (\delta x)^{x \cdot v' \cdot \ln u + x \cdot v \cdot (\ln u)'} = \int (\delta x)^{'v \cdot v \cdot \ln u} \cdot \int (\delta x)^{'u \cdot v}$. Whence

$$\int (\delta u)^{v} = \frac{u^{v}}{\int (\delta v)^{v \cdot \ln u}} \tag{3.42}$$

The note 1. Derivation piecemeal $'\left(u^{v}\right) = {}'u \cdot v + {}'v \cdot v \cdot \ln u =$
$= v \cdot \left({}'u + {}'v \cdot \ln u \right)$.

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[32] Using terms "integration", "derivation" in a superintegration and superderivation, we pursued the purpose not only reductions of an entry, but also underlining of similarity of these operations.

Let $f_2 = \left(f_1 \cdot \varphi\right)'$. Then $\int f_2 dx = f_1 \cdot \varphi \Rightarrow e^{\int f_2 dx} = e^{f_1 \cdot \varphi} \Rightarrow$

$\int (\delta x)^{f_2 \cdot x} = e^{f_1 \cdot \varphi} \Rightarrow \left(\int (\delta x)^{\varphi' \cdot x}\right)^{f_1} = = \left(\int (\delta x)^{' \varphi \cdot \varphi}\right)^{f_1}$, that follows from the formula of an integration by parts. The useful formulas are also such $\int (\delta x)^{' \varphi \cdot \varphi} = \int (\delta x)^{\varphi' \cdot x} = e^{\varphi}$, $\int (\delta x)^{(\varphi \cdot f)' \cdot x} = e^{\varphi \cdot f}$,

$\int (\delta x)^{' \varphi \cdot \varphi \cdot \ln \varphi} = = \int (\delta x)^{' \left(e^{\varphi}\right) \cdot \ln \varphi} = \dfrac{\varphi^{\varphi}}{e^{\varphi}}$ and others, which easily can be received from explained above.

9. Approximate evaluation of superintegrals.

a). Rectangular formula.

$$\int_{\alpha}^{\beta} (\delta x)^{f(x)} \approx \exp\left(\int_{\alpha}^{\beta} \frac{f(x)}{x} dx\right) \approx \exp\left(\frac{\beta - \alpha}{n} \cdot \left(\frac{y_0}{x_0} + \frac{y_1}{x_1} + \ldots + \right.\right.$$

$$\left.\left. + \frac{y_{n-1}}{x_{n-1}}\right)\right) = \left(\prod_{i=1}^{n-1} \exp\left(\frac{y_i}{x_i}\right)\right)^{\frac{\beta-\alpha}{n}}.$$

The note 1. $\ln \int_{\alpha}^{\beta} (\delta x)^{f(x)} = \int_{\alpha}^{\beta} \frac{f(x)}{x} dx \approx \frac{\beta - \alpha}{n} \cdot \sum_{i=0}^{n-1} \frac{y_i}{x_i}$.

b). Formula of parabolas (Th. Simpson).

$$\int_{\alpha}^{\beta} (\delta x)^{f(x)} \approx \left(e^{y_0/x_0} \cdot e^{y_{2 \cdot m}/x_{2 \cdot m}} \cdot \left(e^{y_2/x_2} \cdot e^{y_n/x_n} \cdot \ldots \times \right.\right.$$

$$\left.\left. \times e^{y_{2 \cdot m-1}/x_{2 \cdot m-1}}\right)^2 \cdot \left(e^{y_1/x_1} \cdot e^{y_3/x_3} \cdot \ldots \cdot e^{y_{2 \cdot m-1}/x_{2 \cdot m-1}}\right)^4\right)^{\frac{\beta-\alpha}{6 \cdot m}}.$$

c). Evaluation of a superintegral under the formula of the Chebyshev:

$$\int_{\alpha}^{\beta}(\delta x)^{f(x)}=\left(\prod_{i=1}^{n}e^{f(x_i)}\right)^{\frac{\beta-\alpha}{n}},$$

where $x_i=\dfrac{\beta+\alpha}{2}+\dfrac{\beta-\alpha}{2}\cdot x_i\ (i=1,2,\ldots,n)$.

It is uneasy to make a table of superintegrals:

$$\int(\delta x)^{x^n}=c\cdot\exp\left(\frac{x^n}{n}\right),\quad \int(\delta x)^{1/\ln(1/x)}=\frac{c}{\ln\frac{1}{x}},\quad \int(\delta x)^{\ln x}=$$

$=c\cdot x^{\ln x/2}$, $\quad\int(\delta x)^{k}=c\cdot x^{k}$, $\quad\int(\delta x)^{x\cdot\sin x}=c\cdot e^{-\cos x}$,

$\int(\delta x)^{x/\sin x}=c\cdot tg\dfrac{x}{2}$, $\int(\delta x)^{k+x}=c\cdot x^{k}\cdot e^{x}$ $(k=const)$ etc.

As I see, the deriving of the “upgraded” analysis because of pseudo-image $'f$ Derivative and superintegral $\int(\delta x)^{f(x)}$ does not represent any difficulty. Moreover, the term “upgraded” is taken by the author in quotation markses, as, actually, it is the same calculus with a changed rank of operations (the appropriate replacement of operations $\{+;-;\div;\sum_i;$ $f';\int f\,dx;\ldots\}\rightarrow\left\{\cdot;\div;\log;\prod_i;\ 'f;\ln\int(\delta x)^{x\cdot f(x)};\ldots\right\}$) is realized.

In connection with a simplicity of connection of images of a derivative $'f$ and $\underline{'f}$ $\left(\underline{'f}=k^{'f}\right)$, obviously, construction of the analysis because of it is enough of an image $\underline{'f}$ too simply. For example, the formula Taylor's, displayed from ω_1 In space ω_0, has an aspect:

$$\boxed{f(x)=\prod_{n=0}^{\infty}\left({}^{(n)}f(a)\right)^{\left(\log_k\left(\frac{x}{a}\right)\right)^n/n!}}\qquad(3.43)$$

The upgraded analysis because of image of a derivative $\underline{'f}$ is formed by an appropriate replacement of operations $\{+;-;\div;\sum_n;$

$n!\,;\ldots\} \to \{\,\cdot\,;\div\,;\,\triangle\,;\prod_n;\ln n!\,;\ldots\}$ at preservation to constants of number of addends in $\sum_n$ and number of terms in $\prod_n$. An image in ω_0 integral $\int f(x)dx$, noted in ω_1 we shall discover so:

$$\int f(x)\cdot dx \setminus \omega_1 \to \omega_0 \setminus \int f(x) \odot \delta x = \int (\delta x)^{\log_k f(x)}.$$

It is uneasy to show, that the defined *superintegral of the first sort* [33] $\underline{I_*}$ is connected to an ordinary definite integral by a relation:

$$\underline{I_*} = \int_\alpha^\beta f(x) \odot \delta x = \exp\left(\int_\alpha^\beta \frac{\log_k f(x)}{x} dx\right), \qquad (3.44)$$

Really, from well-known expression

$$I = \int_\alpha^\beta f(x)dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^n f(\xi_i)\cdot \Delta x_i,\ x_i < \xi_i < x_i + \Delta x_i \text{ follows}$$

$$I \setminus \omega_1 \to \omega_0 \setminus I_* = \lim_{\max \delta x_i \to 1} \prod_{i=1}^n (\delta x_i) \odot f(\xi_i) =$$

$$= \lim_{\max \delta x_i \to 1} \prod_{i=1}^n (\delta x_i)^{\log_k f(\xi_i)} = \int_\alpha^\beta (\delta x)^{\log_k f(x)} = \exp\left(\int_\alpha^\beta \frac{\log_k f(x)}{x} dx\right)$$

according to a property (3.37), as was to be shown.

Let's remark, that $\int_\alpha^\beta \log_k f(x) dx = \ln \int_\alpha^\beta (\delta x)^{x\cdot \log_k f(x)}$ according to that to a property (3.37).

Similarly, $\int f(x) \odot \delta x = \exp\left(\int \frac{\log_k f(x)}{x} dx\right)$,

$$\int f(x)dx = \ln \int k^{x\cdot f(x)} \odot \delta x = \ln \int (\delta x)^{\log_k f(x)}. \qquad (3.45)$$

[33] A defined superintegral of the first sort we shall understand an image of a definite integral obtained by reflection in ω_0 from space ω_1.

As an example of application of the formulas (3.44 and 3.45) we shall reduce a rectangular formula for an approximate evaluation of a definite integral, using ω-image of an integral obtained by reflection $\backslash \omega_1 \to \omega_0 \backslash$.

From (3.44 and 3.45), and also from a proof (3.44) follows:

$$I = \int_a^b f(x)dx \approx \ln \prod_{i=1}^{n} e^{x_i f(x_i)} \odot H,$$

where H – new pitch (ω-image of a pitch h), equal $\frac{x_{i+1}}{x_i}$, as the residual is substituted with operation of division.

$$I \approx \ln \prod_{i=1}^{n} e^{x_i f(x_i)} \odot \left(\frac{x_{i+1}}{x_i} \right) = \ln \prod_{i=1}^{n} \left(e^{x_i f(x_i)} \right)^{\ln \frac{x_{i+1}}{x_i}} =$$

$$= \ln \prod_{i=1}^{n} \left(e^{x_i f(x_i) \cdot \ln \frac{x_{i+1}}{x_i}} \right) = \ln e^{\sum_{i=1}^{n} x_i f(x_i) \cdot \ln \frac{x_{i+1}}{x_i}} = \sum_{i=1}^{n} x_i f(x_i) \cdot \ln \left(\frac{x_{i+1}}{x_i} \right),$$

where $n = \frac{|b-a|}{h}$ at $x_{i+1} - x_i = h$.

So, for an approximate evaluation of a definite integral it is possible to use the formula:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} x_i \cdot f(x_i) \cdot \ln \left(\frac{x_{i+1}}{x_i} \right) \tag{3.46}$$

At a diminution of a pitch $\ln\left(\frac{x_{i+1}}{x_i}\right) = \ln\left(1 + \frac{h}{x_i}\right) \approx \frac{h}{x_i}$, i.e. (3.46) becomes the formula of a usual method of rectangles at $h >> x_i$.

From expression for *not reduced* to a scale ω_0 image of a derivative ${}^{\mathrm{p}}f$, obtained by reflection $\backslash \omega_2 \to \omega_0 \backslash$

$${}^{\mathrm{p}}f = f'(x) \cdot \log_{2_f}{}^2 x$$

It is possible to receive an entry ω-image of an integral:

$$\int(\delta x)^{f(x)}=\exp\left(\int(\delta x)^{\frac{f(x)}{\ln x}}\right)=\exp\left(\exp\left(\int\frac{f(x)dx}{\ln\left({}^{2}x\right)}\right)\right).$$

In case of an image of an integral obtained by reflection $\backslash\omega_{-1}\to\omega_0\backslash$ it is necessary to use such formulas:

$$\int f(x)+\nu x=\log_k\left(\ln k\int k^{f(x)+x}dx\right),$$

and at $k=e$ $\int f(x)+\nu x=\ln\left(\int\exp(f(x)+x)dx\right)$, where $\nu x-\omega$-image of a differential obtained by reflection $dx\ \backslash\omega_{-1}\to\omega_0\backslash\ \nu x$.

Not stopping on various aspects of "superintegration", we shall remark, that to these problems we shall return in item 3.7 of the present chapter, where the ω-images of multiple integrals, in chapter 4 are considered, where the technique of reflection of curvilinear and surface integrals understands.

§ 3.6. ω-images of derivatives higher is ordinal

For exposition ω-images of derivatives and integrals it is necessary to receive the formulas permitting to discover outcomes of reflections $\backslash\omega_i\to\omega_j\backslash$ at $i\neq j$ of integro-differential objects of a higher rank. A fragment of this research for cases of reflections $\backslash\omega_1\to\omega_0\backslash$ and $\backslash\omega_0\to\omega_0'\backslash$ is reduced below.

The theorem 3.15. *If the positive function $f(x)$ is continuous, is monotone also differentiable $n+1$ time in some range of values of argument G, the evaluation ω-image of a derivative n of the order $\underline{(n)}f(x)$ in area G is carried out on a recurrence formula*

$$\underline{(n)}f(x)=k^{x}\left(\ln\underline{(n-1)}f(x)\right), \qquad (3.47)$$

where $\underline{(n)}f(x)-$image of a derivative $f^{(n)}(x)$, obtained by reflection it $\backslash\omega_1\to\omega_0\backslash$.

Proof. Let's prove the theorem by a method of a mathematical induction. Let's consider special cases:

1). At $n=1$ $'f(x)=k^{'f(x)}=k^{(x\cdot f')/f}=k^{x\cdot(\ln f)'}$.

2). At $n=2$ $\overset{''}{-}f(x)=\overset{'}{-}\left(\overset{'}{-}f(x)\right)$. Let's designate $\overset{'}{-}f(x)=\varphi(x)$. Then

$$\overset{''}{-}f(x)=\overset{'}{-}\varphi(x)=k^{{}'\varphi(x)}=k^{\frac{x\cdot\varphi'(x)}{\varphi(x)}}=k^{x\cdot(\ln\varphi)'}=k^{x\cdot\left(\ln \overset{'}{-}f(x)\right)'}.$$

3). At $n=3$ $\overset{'''}{-}f(x)=\overset{'}{-}\left(\overset{''}{-}f(x)\right)$. Let's designate $\overset{''}{-}f(x)=\psi(x)$.

Then

$$\overset{'''}{-}f(x)=\overset{'}{-}\psi(x)=k^{{}'\psi(x)}=k^{\frac{x\cdot\psi'(x)}{\psi(x)}}=k^{x\cdot(\ln\psi)'}=k^{x\cdot\left(\ln \overset{''}{-}f(x)\right)'}$$

Is admissible, that the formula (3.47) is correct. Let's prove, that the formula in this case is correct:

$$\underline{(n+1)}f(x)=k^{x\cdot\left(\ln \underline{(n)}f(x)\right)'}.$$

Really, by designating $\underline{(n)}f(x)=\chi(x)$, we shall receive

$$\underline{(n+1)}f(x)=\overset{'}{-}(\chi(x))=k^{{}'\chi(x)}=k^{x\cdot(\ln\chi(x))'}=k^{x\cdot\left(\ln \underline{(n)}f(x)\right)'}.$$

The theorem is proved.

Corollary. For an evaluation ω-images of derivatives higher is ordinal sometimes conveniently to use the formulas expressed through argument x and function $y=\ln f(x)$:

$\overset{'}{-}f(x)=k^{x\cdot y'}$; $\qquad \overset{''}{-}f(x)=k^{\ln k\cdot x\cdot(y'+x\cdot y'')}$; $\qquad \overset{'''}{-}f(x)=$
$=k^{(\ln k)^2\cdot x\cdot\left(y'+3\cdot x\cdot y''+x^2\cdot y'''\right)}$; $\qquad \underline{(4)}f(x)=k^{(\ln k)^3\cdot x\cdot\left(y'+7\cdot x\cdot y''+\right.}$
$^{\left.+6\cdot x^2\cdot y'''+x^3\cdot y^{(4)}\right)}$; $\qquad \underline{(5)}f(x)=k^{(\ln k)^4\cdot x\cdot\left(y'+15\cdot x\cdot y''+25\cdot x^2\cdot y'''+\right.}$
$^{\left.+10\cdot x^3\cdot y^{(4)}+x^4\cdot y^{(5)}\right)}$; $\qquad \underline{(6)}f(x)=k^{(\ln k)^5\cdot x\cdot\left(y'+31\cdot x\cdot y''+90\cdot x^2\cdot y'''+\right.}$
$^{\left.+65\cdot x^3\cdot y^{(4)}+15\cdot x^4\cdot y^{(5)}+x^5\cdot y^{(6)}\right)}$ etc.

The theorem 3.16. *If the positive function* $f=f(x)$ *is continuous, is monotone also differentiable twice in some area* G *values of argument, the* *connection between a derivative of the second order* f'' $\left(f'' \equiv f''_{k_1}\right)$, *belonging to space* ω_0, *and it by an image* f''_{k_2}, *obtained reflection* $\backslash \omega_0 \to \omega_0' \backslash$ $\left(f'' \backslash \omega_0 \to \to \omega_0' \backslash f''_{k_2}\right)$, *is installed by the formula:*

$$f''_{k_2} = \left(p^2 \cdot \frac{\left(f^{1/p}\right)'' \cdot x^{1/p-1} - \left(\frac{1}{p} - 1\right) \cdot x^{1/p-2} \cdot \left(f^{1/p}\right)'}{x^{3 \cdot (1/p-1)}} \right)^p, \quad (3.48)$$

where $\left(f^{1/p}\right)'' \equiv \left(f^{1/p}\right)''_{k_1}$; $\left(f^{1/p}\right)' \equiv \left(f^{1/p}\right)'_{k_1}$; $p = \log_{k_2} k_1$;

k_1, k_2 *– factors of connection of accordingly spaces* ω_0 *and* ω_0' *with space* ω_1.

Proof. As $f'_{k_2} = \frac{f}{x} \cdot \left(\frac{x \cdot f'}{f}\right)^p$,

$$f''_{k_2} = \frac{f'_{k_2}}{x} \cdot \left(\frac{x \cdot \left(f'_{k_2}\right)'_{k_1}}{f'_{k_2}} \right)^p = \left(\frac{\left(f'_{k_2}\right)^{1/p-1} \cdot \left(f'_{k_2}\right)'_{k_1}}{x^{1/p-1}} \right)^p =$$

$$= \left(\frac{p \cdot \left(\left(f'_{k_2}\right)^{1/p}\right)'_{k_1}}{x^{1/p-1}} \right)^p = \left(\frac{p^2}{x^{1/p-1}} \cdot \left(\frac{\left(f^{1/p}\right)'_{k_1}}{x^{1/p-1}} \right)'_{k_1} \right)^p.$$

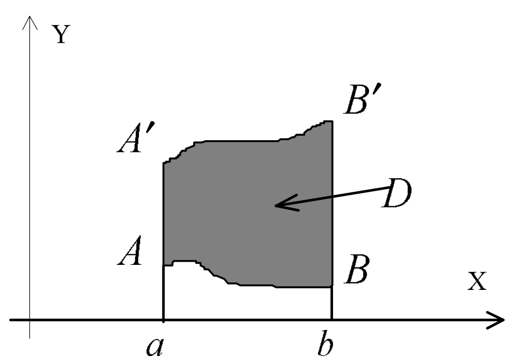
Whence

$$\left(\frac{p\cdot\left(f^{1/p}\right)'_{k_1}}{x^{1/p-1}}\right)'_{k_1} = \frac{p\cdot\left(\left(f^{\frac{1}{p}}\right)''\cdot x^{\frac{1}{p}-1} - \left(\frac{1}{p}-1\right)\cdot x^{\frac{1}{p}-2}\cdot\left(f^{\frac{1}{p}}\right)'\right)}{x^{2\cdot\left(\frac{1}{p}-1\right)}}$$ and

$$f''_{k_2} = \left(p^2\cdot\frac{\left(f^{\frac{1}{p}}\right)''\cdot x^{\frac{1}{p}-1} - \left(\frac{1}{p}-1\right)\cdot x^{\frac{1}{p}-2}\cdot\left(f^{\frac{1}{p}}\right)'}{x^{3\cdot\left(\frac{1}{p}-1\right)}}\right)^p$$, as was to be shown.

§ 3.7 ω-images of multiple integrals

Let in space ω_1 the continuous function $f(x,y)$ on right area D is given. Thus we suppose, that the area D – right in direction of an axes OY and is limited to lines: $y=\varphi_1(x)$, $y=\varphi_2(x)$, $x=a$, $y=b$.



Pic.3. Area D: $\breve{A}B-\varphi_1(x)$ $\breve{A}'B'-\varphi_2(x)$.

Let's designate a double integral in space for I:

$$I=\iint_D f(x,y)\,dx\,dy=\int_a^b\left(\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)\,dy\right)dx=\int_a^b dx\cdot\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)\,dy.$$

Let's note ω-image of a double integral through repeated:

$$I=\iint_D f(x,y)\,dx\,dy=\int_a^b dx\cdot\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)\,dy\setminus\omega_1\to\omega_0\setminus\underline{I_*}=$$

$$=\exp\left(\int_a^b\log_k\left(\exp\left(\int_{\varphi_1(x)}^{\varphi_2(x)}\frac{\log_k f(x,y)}{y}dy\right)\right)\cdot\frac{dx}{x}\right) \qquad (3.49)$$

Whence $\quad \underline{I_{**}}=\int_a^b\frac{dx}{x}\cdot\int_{\varphi_1(x)}^{\varphi_2(x)}\frac{\ln f(x,y)}{y}dy,$

where $\underline{I_{**}}=\ln^2 k\cdot\ln\underline{I_*}$.

Let's consider an example.

Let in ω_1 there is a object P, limited by surfaces is given: $z=x^3+3\cdot\ln y$, $y=x$, $y=x^2$, $x=1$, $x=2$. Volume of this object is equal: $V_p=I=\iint_D\left(x^2+3\cdot\ln y\right)dx\,dy=\int_1^2 dx\cdot\int_x^{x^2}\left(x^2+3\cdot\ln y\right)dy\approx 4{,}4648$ cube unity.

Let's discover ω-image of this double integral $\left(I\setminus\omega_1\to\omega_0\setminus\underline{I_*}\right)$:

$$\underline{I_{**}}=\ln^2 k\cdot\ln\underline{I_*}=\int_1^2\frac{dx}{x}\cdot\int_x^{x^2}\frac{x^2+3\cdot\ln y}{y}dy.$$

We realizable a replacement $x^2+3\cdot\ln y=t$:

$$\underline{I}_{**} = \int_1^2 \frac{dx}{x} \cdot \frac{1}{3} \cdot \int_{x^2+3\cdot\ln x}^{x^2+6\cdot\ln x} \ln t\,dt = \int_1^2 \frac{dx}{x} \cdot \left(t \cdot \ln t - t\right)\Big|_{x^2+3\cdot\ln x}^{x^2+6\cdot\ln x} =$$

$$= \int_1^2 \Big(\left(x^2+6\cdot\ln x\right)\cdot\ln\left(x^2+6\cdot\ln x\right) - \left(x^2+3\cdot\ln x\right)\cdot\ln\left(x^2+3\cdot\ln x\right) -$$

$$-3\cdot\ln x\Big)\cdot\frac{dx}{x} \approx 1{,}071018.$$

Thus $\underline{I}_{*} \approx 9{,}2923$ cube unity at $k = 2$.

At ω-reflection it is necessary to reflect immediately *method* (mode) representing defined selection of sequential operations. In this case, in space ω_0 it is possible to receive the formula (regularity), in which it is expedient to apply well-known numbers and functions (from ω_0). Changing $a, b, \varphi_1, \varphi_2$ in the formula (3.49), we compute *new mathematical object* – ω-image of a double integral, which has an independent value as well as double integral.

The note. If to image all elementary objects which are included in the formula of a double integral (3.49), including, and operation, we shall receive *trivial* ω-image of a double integral.

For this purpose we shall note (3.49) as

$$I_* = \int_{k^a}^{k^b} (\delta x) \odot \int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} \delta y \odot k^{f(\log_k x, \log_k y)},$$

where, $\{\Delta x, \Delta y, a, b, \varphi_1, \varphi_2, \bullet, \int\} \quad \backslash \omega_1 \to \omega_0 \backslash$

$$\backslash \omega_1 \to \omega_0 \backslash \quad \left\{\delta x, \delta y, k^a, k^b, k^{\varphi_1(\log_k x)}, k^{\varphi_2(\log_k x)}, \odot, f\right\}.$$

Whence $$I'_{1*} = \int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} \delta y \odot k^{f(\log_k x, \log_k y)} =$$

$$= \int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} (\delta y)^{\log_k k^{f(\log_k x, \log_k y)}} =$$

$$= \int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} (\delta y)^{f(\log_k x, \log_k y)} =$$

$$= \exp\left(\int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} \frac{f(\log_k x, \log_k y)}{y} dy \right).$$

Let's designate $\log_k y = Z$, and $\frac{dy}{\ln k \cdot y} = dz$, i.e.

$$I_{1*} = \exp\left(\int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} \ln k \cdot f(\log_k x, Z) dz \right) =$$

$$= \int_{k^{\varphi_1(\log_k x)}}^{k^{\varphi_2(\log_k x)}} f(\log_k x, Z) dz .$$

Let $\int_{\varphi_1(\log_k x)}^{\varphi_2(\log_k x)} f(\log_k x, Z) dz = \Phi_1(\log_k x, Z) \Big|_{Z_1}^{Z_2}$. Then

$$I_{1*} = k^{\Phi_1(\log_k x, Z)} \Bigg|_{Z_1 = \varphi_1(\log_k x)}^{Z_2 = \varphi_2(\log_k x)} =$$

$$= k^{\left[\Phi_1(\log_k x, \varphi_2(\log_k x)) - \Phi_1(\log_k x, \varphi_1(\log_k x))\right]} = k^{\Psi(\log_k x)}.$$

Therefore, $I_* = \int\limits_{k^a}^{k^b} (\delta x) \odot k^{\Psi(\log_k x)} = \int\limits_{k^a}^{k^b} (\delta x)^{\Psi(\log_k x)} =$

$$= \exp\left(\int\limits_{k^a}^{k^b} \frac{\Psi(\log_k x)}{x} dx \right).$$

By designating $\log_k x = t$, i.e. $\dfrac{dx}{x \cdot \ln k} = dt$,

and $I_* = \exp\left(\int\limits_a^b \Psi(t) \cdot \ln k \, dt \right) = k^{\left(\int\limits_a^b \Psi(t) dt \right)}$. (3.50)

Solving an integral I (3.49), we shall receive

$$I_1 = \int\limits_{\varphi_1(x)}^{\varphi_2(x)} f(x,y) dy = \Phi_1(x, \varphi_2(x)) - \Phi_1(x, \varphi_1(x)) = \Psi(x), \text{ and}$$

$I = \int\limits_a^b \Psi(x) dx$, i.e. with the accounting (3.50) $I_* = k^I$.

So, *trivial* ω-image of a double integral turns out in an outcome of the elementary substitution and does not give qualitatively new outcomes both theoretical, and practical, though a configuration and volume of a object vary (accordingly, the equations of surfaces, bounding a object vary also).

By analogy with ω-image of a double integral (3.49) we shall note ω-image of a triple integral.

$$I = \iiint\limits_V f(x,y,z) dx\, dy\, dz = \int\limits_a^b dx \int\limits_{\varphi_1(x)}^{\varphi_2(x)} dy \int\limits_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z) dz \setminus \omega_1 \to \omega_0 \setminus \underline{I_*} =$$

$$= \exp\left(\int\limits_a^b \log_k \left(\exp\left(\int\limits_{\varphi_1(x)}^{\varphi_2(x)} \log_k \left(\exp\left(\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)} \frac{\log_k f(x,y,z)}{z} dz \right) \right) \cdot \frac{dy}{y} \right) \right) \cdot \frac{dx}{x} \right).$$

(3.51)

It is obvious, that ω-image of a triple integral is a completely *new mathematical object*.

§ 3.8 Problematics of integro-differential objects new nature

It is necessary to systematize and to classify images of integro-differential objects obtained by various ω-reflection. Thus there is a series of problems:

1. Unifications of mathematical exposition ω-images by creation of special symbolics and nomenclature. Deriving common of the formulas for ω-images of derivatives of any order and integrals of any multiplicity. The rationalization of the process of deriving ω-images irrespective of number of the elementary acts ω-reflections, i.e. transition of object on a horizontal (passage in adjacent spaces \ $\omega_i \to \omega_i'$ \) and vertical (passage in ω-space of higher or lowest ranks \ $\omega_i \to \omega_j$ \) should be carried out for one universal procedure without linear search of all acts of reflections. In a fig. 4 the field ω-spaces in a system of indications k and m, where k – factor of an exponential function of connection $\left(k^x\right)$, and m – index of space (for example, $\omega_m = \left(\omega_0, \omega_1, \dots, \omega_i, \dots, \omega_j, \dots\right.$ etc.)) is represented. The well-known mathematical space ω_0 is represented as a point appropriate to an origin of coordinates. Everyone ω-the space is represented by a point on a plane.

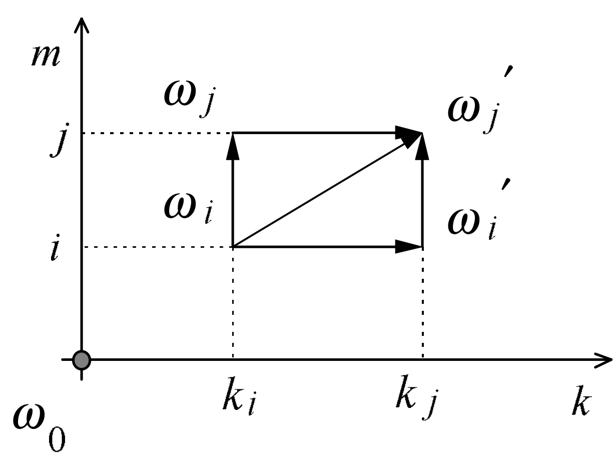


Fig. 4 ω-spaces in a system of indications k and m.

2. Considering various sections of mathematics as global objects to find them ω-images.

Shaping of the uniform scheme ω-reflections of various global and local objects (mathematical methods, theorems, formulas etc.) way ω-passage from any given space ω_i in another ω_j' $\left(\backslash \omega_i \to \omega_j' \backslash\right)$. The deriving is qualitative of a new mathematical means by a selection of an appropriate cycle ω-reflections on a horizontal and vertical.

3. Study of a problem of invariancy of objects at ω-reflection.

4. The determination of the common formulas of correlation of derivatives and integrals from them ω-images (in the present work is reduced only a few formulas of connection, but already in them it is uneasy to notice regularities in a structure and rank of operations; for example, $f' = k^{\overset{0}{f} - f + x}$ and $'f = k^{\frac{f' \cdot x}{f}}$).

5. Proof of the theorem 3.15 for a common case $\backslash \omega_i \to \omega_j \backslash$ at $i \neq j$.

6. Proof of the theorem 3.16 for a derivative of any order.

7. Creation of the concept polymeric ω-reflections (in the book the ω-transformations graphically represented as vectors in a fig. 4 are represented only two-dimensional).

8. Research of a nature adjacent ω-spaces.

9. The realization ω-reflections of integro-differential objects because of other functions of connection (in the given work is used only exponential function k^x).

10. Research of a problem of an integration in arbitrary adjacent spaces. To find the exacter and universal formulas of an entry ω-image of an integral as an initial integral, i.e. formula of *adequacy* of a type (3.27). To lead numerical check of datas of the formulas. (In the present work this problem, practically, was not studied).

11. Check of applicability ω-reflections in adjacent spaces in various mathematical method (for example, in the monography only slightly affect a problem on a determination of adjacent limits, using the theorem 3.15, and the more deep research in this direction is required).

Naturally, there are also other problems. Linking this problematics with problematics represented in chapters 4 and 5, where the new hypothesises are advanced, the various practical tasks are solved and the questions at issue are stated, it is possible to generate a circle of the tasks, which solution undoubtedly will reduce in *unique* theoretical and *practical* outcomes.

In summary, it would be necessary to mark the following fact, which the author repeatedly mentions in the monography: apparent the *triviality* ω-reflections and is *possible derivings* all outcomes by a change of variables are refuted at deep study ω-reflections of operations (since at ω-reflection the replacement of operations is produced).

CHAPTER 4. THE ELEMENTS QUASIVECTORIAL OF THE ANALYSIS

§ 4.1. The general provisions

The mathematical objects are necessary components in exposition of the physical facts. The population last makes a basis of understanding of a reality of the world, enclosing us. By virtue of susceptibility of mathematical objects ω-reflections, obviously, and to the physical laws (facts) the means ω-reflection is applicable. As one of connecting between ω-*mathematics* and ω-*physics* (understanding under it the possibility of designing infinite of spectra of any mathematical and physical objects) in the given book is quasivectorial the analysis, obtained in an outcome ω-reflections of vectorial magnitudes identified with usual vectors and vectorial functions. And, the structures *quasivectorial and vectorial analyses are similar*.

In a basis quasivectorial of the analysis the concept *quasivector* lays. Last is *ranked* on an index i motherly space ω_i, from which there is ω-reflection in derived ω_0 $\left(\backslash \omega_i \to \omega_0 \backslash\right)$ and on a parameter k. In the present chapter in quality ω_i the space ω_1, i.e. $\omega_i \equiv \omega_1$ undertakes only.

Quasivectorial the analysis allows more more scalely to perceive any physical law (or fact), and also *physical skew fields* of the material world, understanding under a physical skew field all gamma micro and macrostructure's of material objects.

In the present work the author reduces only readily available[34] primary reasonings which, will help the reader to generate the own concept of a structure of physical space as populations of physical objects.

[34] At the deep analysis of correlation of mathematical and physical objects, them $\omega-$ reflections it are taken into account the factors influencing to distinction abstract and real, and also on transformation of objects at $\omega-$transformations.

Let's consider ω-reflection of a vector $\bar{a}$.

Let in ω_1 the vector $\bar{a} = a_x \cdot \bar{i} + a_y \cdot \bar{j} + a_z \cdot \bar{k}$ is given. Then $\bar{a} \backslash \omega_1 \to \omega_0 \backslash A_* = \left(a_x \odot \bar{i}\right) \cdot \left(a_y \odot \bar{j}\right) \cdot \left(a_z \odot \bar{k}\right) = a_x^{\log_k \bar{i}} \cdot a_y^{\log_k \bar{j}} \times$ $\times a_z^{\log_k \bar{k}}$, $k \neq 1$, $k \in \mathbf{R}$.

The note. It is not necessary to confuse a vector $\bar{k}$ and scalar $\underline{\underline{k}}$, being a foundation of a log.

Let's take the logarithm A_*:

$$\log_k A_* = \log_k a_x \cdot \log_k \bar{i} + \log_k a_y \cdot \log_k \bar{j} + \log_k a_z \cdot \log_k \bar{k},$$

$k \neq 1$, $k \in \mathbf{R}$.

By analogy to expansion of a vector $\bar{a}$ on base (in a cartesian rectangular frame) we shall note:

$$\log_k A_* = \left(\log_k a_x\right) \cdot \overline{i_*} + \left(\log_k a_y\right) \cdot \overline{j_*} + \left(\log_k a_z\right) \cdot \overline{k_*}, \qquad \text{where}$$

$A_* = k^{\left(\log_k a_x\right) \cdot \overline{i_*} + \left(\log_k a_y\right) \cdot \overline{j_*} + \left(\log_k a_z\right) \cdot \overline{k_*}}$, $\log_k \bar{i} = \overline{i_*}$, $\log_k \bar{j} = \overline{j_*}$, $\log_k \bar{k} = \overline{k_*}$;

$\overline{i_*}$, $\overline{j_*}$, $\overline{k_*}$ – objects of a *new* nature, which we shall name *basis quasivectors*, though they and do not correspond to common representation about quisivectors. *Quisivector* A_* vector $\bar{a}$ the ω-image of a vector is named $\bar{a}$ at reflection it from ω_1 in ω_0 $\left(\bar{a} \backslash \omega_1 \to \omega_0 \backslash A_*\right)$.

From the definition basis quasivector follows:

$\bar{i} = k^{\overline{i_*}}$, $\bar{j} = k^{\overline{j_*}}$, $\bar{k} = k^{\overline{k_*}}$, i.e. if some factor k $\left(k \neq 1, k \in \mathbf{R}\right)$ announce in a degree, with which is basis quasivector, we shall receive an appropriate single vector (basis vector).

As basis vectors - single vectors, $\left|k^{\overline{i_*}}\right| = \left|k^{\overline{j_*}}\right| = \left|k^{\overline{k_*}}\right| = 1$. At $k > 0$ we shall receive $\left|k^{\overline{i_*}}\right| = k^{\left|\overline{i_*}\right|} = 1$. Whence $\left|\overline{i_*}\right| = 0$.

For exposition quasivector we shall discover ω-images of some performances of usual vectors and connections between them.

The theorem 4.1. *The image of the module of a vector* $\overline{a}$ $\left(\overline{a} = \sum_{i=1}^{n} a_{x_i} \cdot \overline{e_i}\right)$ *reflection* $\backslash \omega_1 \to \omega_0 \backslash$ *is defined under the formula:*

$$\left|A_*\right| = k^{\sqrt{\sum_{i=1}^{n} \log_k{}^2 a_{x_i}}}, \tag{4.1}$$

where $\left|A_*\right|$ – module quasivector of a vector $\overline{a}$.

Proof. Let a vector $\overline{a} = a_x \cdot \overline{i} + a_y \cdot \overline{j} + z_z \cdot \overline{k}$ in three-dimensional space. As is known, the module of a vector is equal:

$\left|\overline{a}\right| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$. Let's reflection it from space ω_1 in ω_0:

$$\sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} \,\backslash \omega_1 \to \omega_0 \backslash \left|A_*\right| = \left((a_x \odot a_x)(a_y \odot a_y) \times\right.$$

$$\left.\times (a_z \odot a_z)\right)^{\to (\underline{1} \triangle \underline{2})}, \; a_x \odot a_x = k^{\log_k^2 a_x} \; (k \neq 1, k \in \mathbf{R}).$$

Let's designate $p = k^{\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z}$.

As $$A^{\to b} = \underbrace{A \odot A \odot A \odot \ldots \odot A}_{\log_k b} = k^{(\log_k A)^{\log_k b}}, \quad \left|A_*\right| =$$

$$= p^{\to (\underline{1} \triangle \underline{2})} = k^{(\log_k p)^{\log_k (\underline{1} \triangle \underline{2})}}, \text{ where } (\underline{1} \triangle \underline{2}) = \left(k \triangle k^2\right) = k^{1/\log_k k^2} = k^{\frac{1}{2}},$$

$$\log_k (\underline{1} \triangle \underline{2}) = \frac{1}{2}, \text{ i.e. } p^{\to (\underline{1} \triangle \underline{2})} = k^{\left(\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z\right)^{\frac{1}{2}}}.$$

Therefore, $\left|A_*\right| = k^{\sqrt{\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z}}$, as was to be shown.

The note. The formula (4.1) can be obtained rather simple if to take into account, that $\sqrt{a_x^2 + a_y^2 + a_z^2}$ is function:

$f(a_x, a_y, a_z) = \sqrt{a_x^2 + a_y^2 + a_z^2}$, where a_x, a_y, a_z − arguments.

$$\sqrt{a_x^2 + a_y^2 + a_z^2} \setminus \omega_1 \to \omega_0 \setminus k^{\sqrt{\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z}}.$$

The theorem 4.2. *The image C_* scalar product C two vectors $\bar{a}$ and $\bar{b}$, obtained ω-reflection $\setminus \omega_1 \to \omega_0 \setminus$ is determined under the formula:*

$$C_* = k^{\sum_{i=1}^{n} \log_k a_{x_i} \cdot \log_k b_{x_i}} = \prod_{i=1}^{n} a_{x_i} \odot b_{x_i}. \tag{4.2}$$

Proof. Let in three-dimensional space ω_1 the vectors $\bar{a} = a_x \cdot \bar{i} + a_y \cdot \bar{j} + a_z \cdot \bar{k}$ and $\bar{b} = b_x \cdot \bar{i} + b_y \cdot \bar{j} + b_z \cdot \bar{k}$ are given. Let's discover ω-image of a scalar product. $C = \bar{a} \cdot \bar{b} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z \setminus \omega_1 \to \omega_0 \setminus \quad C_* = (a_x \odot b_x)(a_y \odot b_y)(a_z \odot b_z) = a_x^{\log_k b_x} \cdot a_y^{\log_k b_y} \cdot a_z^{\log_k b_z}$, $k \neq 1$, $k \in \mathbf{R}$.

Whence, $\log_k C_* = \log_k a_x \cdot \log_k b_x + \log_k a_y \cdot \log_k b_y + \log_k a_z \cdot \log_k b_z$,

$$C_* = k^{\log_k a_x \cdot \log_k b_x + \log_k a_y \cdot \log_k b_y + \log_k a_z \cdot \log_k b_z} =$$

$$= (a_x \odot b_x) \cdot (a_y \odot b_y) \cdot (a_z \odot b_z),$$

As was to be shown.

The note. $\bar{a} \cdot \bar{b} \setminus \omega_1 \to \omega_0 \setminus C_* = A_* \odot B_*$. Really,

$$A_* = \left((a_x)^{\bar{i}_*} \cdot (a_y)^{\bar{j}_*} \cdot (a_z)^{\bar{k}_*} \right), \qquad B_* = \left((b_x)^{\bar{i}_*} \cdot (b_y)^{\bar{j}_*} \cdot (b_z)^{\bar{k}_*} \right),$$

$$A_* \odot B_* = \left((a_x)^{\overline{i_*}} \cdot (a_y)^{\overline{j_*}} \cdot (a_z)^{\overline{k_*}} \right)^{\log_k \left((b_x)^{\overline{i_*}} \cdot (b_y)^{\overline{j_*}} \cdot (b_z)^{\overline{k_*}} \right)}, \quad \text{i.e.}$$

$$\log_k C_* = \left((\log_k a_x) \cdot \overline{i_*} + (\log_k a_y) \cdot \overline{j_*} + (\log_k a_z) \cdot \overline{k_*} \right) \cdot \left((\log_k b_x) \times \right.$$

$$\left. \times \overline{i_*} + (\log_k b_y) \cdot \overline{j_*} + (\log_k b_z) \cdot \overline{k_*} \right).$$

From (4.2) follows, that $\log_k C_* = \log_k a_x \cdot \log_k b_x + \log_k a_y \times \times \log_k b_y + \log_k a_z \cdot \log_k b_z$, i.e. the log ω-*image of a scalar product of two vectors* $\overline{a}$ *and* $\overline{b}$ *Is equal to a reflexive product appropriate quasivector, which, in turn, is equal to a sum of conjugate products of logs of appropriate projections of vectors* $\overline{a}$ *and* $\overline{b}$.

The image of a scalar product $\overline{a}$ and $\overline{b}$ has a commutability and distributivity. Let's remind, that thus it is necessary to take into account a rank of operations. Multiplication and addition at reflection $\backslash \omega_1 \rightarrow \omega_0 \backslash$ will accordingly be transformed in operation on an order above (reflexive multiplication and ordinary multiplication).

The theorem 4.3. *Quasivector* A_*, B_*, D_* *have properties of a* ***commutability*** *of rather reflexive multiplication and* ***distributivity*** *of rather ordinary multiplication.*

Proof. a). Let's prove a commutability A_* and B_*. Let's designate $A_* \odot B_* = C_*$, where C_* – ω-image of a scalar product. Then

$$\log_k C_* = \log_k \left(A_* \odot B_* \right) = \log_k a_x \cdot \log_k b_x + \log_k a_y \times$$

$$\times \log_k b_y + \log_k a_z \cdot \log_k b_z = \log_k \left(B_* \odot A_* \right),$$ i.e. $C_* = B_* \odot A_*$, as was to be shown.

The note. The commutability A_* and B_* follows, certainly, immediately from a commutability of operation $\odot$. The proof is reduced only because A_* and B_* - mathematical objects of a new nature.

b). We shall prove a distributivity A_*, B_*, D_*. Let's designate $A_* \odot \left(B_* \cdot D_* \right) = P_1$. Then $\log_k p_1 = \log_k \left(A_* \odot \left(B_* \cdot D_* \right) \right)$.

Let's designate $p_2 = \left(A_* \odot B_* \right) \cdot \left(A_* \odot D_* \right)$.

Then $\log_k p_2 = \log_k \left(A_* \odot B_* \right) + \log_k \left(A_* \odot D_* \right)$,

$\log_k \left(A_* \odot B_* \right) = \log_k a_x \cdot \log_k b_x + \log_k a_y \cdot \log_k b_y + $
$+ \log_k a_z \cdot \log_k b_z$, $\quad \log_k \left(A_* \odot D_* \right) = \log_k a_x \cdot \log_k d_x +$
$+ \log_k a_y \cdot \log_k d_y + \log_k a_z \cdot \log_k d_z$.

$\log_k p_2 = \log_k a_x \cdot \log_k (b_x \cdot d_x) + \log_k a_y \cdot \log_k (b_y \cdot d_y) +$
$+ \log_k a_z \cdot \log_k (b_z \cdot d_z)$.

On the other hand, $\log_k p_1 = \log_k \left(A_* \odot \left(B_* \cdot D_* \right) \right)$. As
$\log_k \left(B_* \cdot D_* \right) = \log_k B_* + \log_k D_* = (\log_k b_x) \cdot \overline{i_*} + (\log_k b_y) \cdot \overline{j_*} +$
$+ (\log_k b_z) \cdot \overline{k_*} + (\log_k d_x) \cdot \overline{i_*} + (\log_k d_y) \cdot \overline{j_*} + (\log_k d_z) \cdot \overline{k_*} =$
$= (\log_k b_x \cdot d_x) \cdot \overline{i_*} + (\log_k b_y \cdot d_y) \cdot \overline{j_*} + (\log_k b_z \cdot d_z) \cdot \overline{k_*}$,

$\log_k \left(A_* \odot \left(B_* \cdot D_* \right) \right) = \log_k a_x \cdot \log_k (b_x d_x) + \log_k a_y \cdot \log_k (b_y d_y) +$
$+ \log_k a_z \cdot \log_k (b_z d_z)$.

Then $\log_k p_2 = \log_k p_1 \quad \Rightarrow \quad p_2 \equiv p_1$, and $A_* \odot \left(B_* \cdot D_* \right) =$
$= \left(A_* \odot B_* \right) \left(A_* \odot D_* \right)$, i.e. the distributivity quasivector A_*, B_*, D_* is proved.

It is uneasy to find $\omega -$images of other performances of vectors. For example, we shall reflection $\cos\varphi$, where $\varphi -$an angle between two vectors $\overline{a}$ and $\overline{b}$, located in space ω_1. Then

$$\cos\varphi = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} \qquad \backslash \omega_1 \to \omega_0 \backslash \qquad k^{\cos\log_k \varphi_*} = k^{\left(\log_k a_x \cdot \log_k b_x + \right.}$$

$$\left. + \log_k a_y \cdot \log_k b_y + \log_k a_z \cdot \log_k b_z \right) \Big/ \left(\sqrt{\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z} \times \right.$$

$$\left. \times \sqrt{\log_k^2 b_x + \log_k^2 b_y + \log_k^2 b_z} \right), \qquad \text{i.e.} \qquad \cos\log_k \varphi_* =$$

$$= \frac{\log_k a_x \cdot \log_k b_x + \log_k a_y \cdot \log_k b_y + \log_k a_z \cdot \log_k b_z}{\sqrt{\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z} \cdot \sqrt{\log_k^2 b_x + \log_k^2 b_y + \log_k^2 b_z}} \quad (4.3)$$

Really, from (4.1) and (4.2) follows:

$$k^{\cos\log_k \varphi_*} = \left(k^{\log_k a_x \cdot \log_k b_x + \log_k a_y \cdot \log_k b_y + \log_k a_z \cdot \log_k b_z} \right) \triangle$$

$$\triangle \left(\left(k^{\sqrt{\log_k^2 a_x + \log_k^2 a_y + \log_k^2 a_z}} \right) \odot \left(k^{\sqrt{\log_k^2 b_x + \log_k^2 b_y + \log_k^2 b_z}} \right) \right).$$

After simple transformations we shall receive the formula (4.3), in which φ_* – "angle" between quasivectors A_* and B_*. So, quasivectors are objects of a new nature, which are *characterized* by some magnitude called as the *module* ($|A_*|$ and $|B_*|$), and also *orientation* in space. In particular, two quasivectors are oriented among themselves with performance of orientation as a parameter φ_*, which is identified to an angle between vectors $\bar{a}$ and $\bar{b}$.

At last, we shall discover ω-image W_* vector product $\bar{W} = \bar{a} \times \bar{b}$:

$$\bar{W} = (a_y b_z - a_z b_y)\cdot \bar{i} + (a_z b_x - a_x b_z)\cdot \bar{j} + (a_x b_y - a_y b_x)\cdot \bar{k}$$

$$\bar{W} \backslash \omega_1 \to \omega_0 \backslash W_* = \left(\frac{a_y \odot b_z}{a_z \odot b_y} \odot \bar{i} \right) \cdot \left(\frac{a_z \odot b_x}{a_x \odot b_z} \odot \bar{j} \right) \cdot \left(\frac{a_x \odot b_y}{a_y \odot b_x} \odot \bar{k} \right)$$

$$\log_k W_* = \log_k\left(\frac{a_y \odot b_z}{a_z \odot b_y}\right)\cdot \overline{i_*} + \log_k\left(\frac{a_z \odot b_x}{a_x \odot b_z}\right)\cdot \overline{j_*} +$$

$$+\log_k\left(\frac{a_x \odot b_y}{a_y \odot b_x}\right)\cdot \overline{k_*},$$

$$W_* = k^{\log_k\left(\frac{a_y \odot b_z}{a_z \odot b_y}\right)\cdot \overline{i_*} + \log_k\left(\frac{a_z \odot b_x}{a_x \odot b_z}\right)\cdot \overline{j_*} + \log_k\left(\frac{a_x \odot b_y}{a_y \odot b_x}\right)\cdot \overline{k_*}}.$$

Whence (4.4)

$$W_* = k^{\left(\log_k a_y \cdot \log_k b_z - \log_k a_z \cdot \log_k b_y\right)\cdot \overline{i_*} + \left(\log_k a_z \cdot \log_k b_x - \right.}$$

$$\left. - \log_k a_x \cdot \log_k b_z\right)\cdot \overline{j_*} + \left(\log_k a_x \cdot \log_k b_y - \log_k a_y \cdot \log_k b_x\right)\cdot \overline{k_*}.$$

Similarly it is possible to receive ω-images of any objects of a vector analysis.

§ 4.3. ω-image of a field theory

Let's consider ω-images of basic concepts of a field theory.

4.3.1. Gradient

Let in space ω_1 the vector $\overline{a}$ as a vector-function is given: $\overline{a} = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \cdot \overline{e_i}$, where $f = f(x_1, x_2, \dots, x_n)$, and $\overline{e_i}$ – basis vectors (single vectors, i.e. $|e_i| = 1$).

Let's reflection a vector a in space ω_0:

$$\overline{a} = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \cdot \overline{e_i} \setminus \omega_1 \to \omega_0 \setminus A_* = \prod_{i=1}^{n}\left(\left(k^{f'_{x_i}}\right)\odot \overline{e_i}\right) =$$

$$= \prod_{i=1}^{n} \left(k^{{}'f_{x_i} \cdot \overline{e_{i*}}} \right), \text{ where } {}'f_{x_i} = \frac{x_i \cdot f'_{x_i}}{f}, \; f'_{x_i} = \frac{\partial f}{\partial x_i}, \; \overline{e_{i*}} = \log_k \overline{e_i}.$$

After a taking the logarithm quasivectorial A_* we shall receive:

$$\log_k A_* = \log_k \prod_{i=1}^{n} \left(k^{{}'f_{x_i} \cdot \overline{e_{i*}}} \right) = \sum_{i=1}^{n} \log_k k^{{}'f_{x_i} \cdot \overline{e_{i*}}} = \sum_{i=1}^{n} {}'f_{x_i} \cdot \overline{e_{i*}}.$$

As it is visible, in this case $\omega -$reflection has reduced to quasivectorial, univalently identified vector $\overline{a}$ $(\overline{a} \setminus \omega_1 \to \omega_0 \setminus k^{a_*}$, where $a_* = \sum_{i=1}^{n} {}'f_{x_i} \cdot \overline{e_{i*}}$).

On a structure the vector function $\overline{a} -$is a gradient, i.e. $\overline{a} \equiv \mathrm{grad}\, f$. Have received the formula, defining *quasigradient* of function f:

$$\mathrm{grad}\, f \setminus \omega_1 \to \omega_0 \setminus G_* = k^{\sum_{i=1}^{n} \left({}'f_{x_i} \right) \overline{e_{i*}}}, \qquad (4.5)$$

where $G_* -$quasigradient f'.

Let's discover ω-image of the module of a gradient:

$$\left| \mathrm{grad}\, f \right| = \sqrt{\sum_{i=1}^{n} \left(f'_{x_i} \right)^2} \quad \setminus \omega_1 \to \omega_0 \setminus \quad B^{\to (\underline{1} \triangle \underline{2})} =$$

$$= \left(\prod_{i=1}^{n} k^{\left({}'f_{x_i} \right)^2} \right)^{\to (\underline{1} \triangle \underline{2})}, \quad \text{as} \quad \left(f'_{x_i} \right)^2 \quad \setminus \omega_1 \to \omega_0 \setminus \quad \setminus \omega_1 \to \omega_0 \setminus$$

$$k^{{}'f_{x_i}} \odot k^{{}'f_{x_i}} = k^{\left({}'f_{x_i} \right)^2}.$$

As the exponent $\to (\underline{1} \triangle \underline{2}) = \to \sqrt{k}$, was established earlier,

$$B^{\to\sqrt{k}} = k^{\sqrt{\log_k B}} = k^{\sqrt{\log_k \prod_{i=1}^{n} k^{\left('f_{x_i}\right)^2}}} = k^{\sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}}.$$

So, $|\text{grad } f| \setminus \omega_1 \to \omega_0 \setminus k^{\sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}}$.

Let's discover ω-image of direction cosines of a gradient.

Let α_i – angles derivated by a gradient with axeses of in space with n measurements. Then

$$\cos\alpha_i = \frac{f'_{x_i}}{|\text{grad } f|} \setminus \omega_1 \to \omega_0 \setminus k^{\cos\log_k \alpha_{i*}} = k^{'f_{x_i}} \triangle k^{\sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}} =$$

$$= \left(k^{'f_{x_i}}\right)^{\frac{1}{\log_k k^{\sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}}}} = k^{'f_{x_i} \Big/ \sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}}.$$

Whence $\cos\log_k \alpha_{i*} = \dfrac{'f_{x_i}}{\sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}}$, and

$$\sum_{i=1}^{n}\left(\cos\log_k \alpha_{i*}\right)^2 = \sum_{i=1}^{n}\left(\frac{'f_{x_i}}{\sqrt{\sum_{i=1}^{n}\left('f_{x_i}\right)^2}}\right)^2 = 1,$$

where α_{i*} – "angles" makes *quasigradient* with axeses of coordinates.

Have received the formula $\quad \sum_{i=1}^{n}\left(\cos\log_k \alpha_{i*}\right)^2 = 1, \qquad (4.6)$

which is similar to the well-known formula $\sum_{i=1}^{n} \cos^2 \alpha_i = 1$.

4.3.2. Derivative with direction

Is known, that $\frac{df}{ds} = \bar{s} \cdot \operatorname{grad} f$, where $\bar{s}$ – direction. Let's discover ω-image of a derivative on direction:

$$\frac{df}{ds} = \bar{s} \cdot \operatorname{grad} f \setminus \omega_1 \to \omega_0 \setminus P_s = S_* \odot k^{\sum_{i=1}^{n} {}'f_{x_i} \cdot \overline{e_{i*}}}$$

$$\log_k P_s = \left(\sum_{i=1}^{n} \left({}'f_{x_i} \right) \cdot \overline{e_{i*}} \right) \cdot \left(\sum_{i=1}^{n} \left(\log_k s_{xi} \right) \cdot \overline{e_{i*}} \right).$$

Have received ω-image of a scalar product of two vectors $\bar{s}$ and $\mathbf{grad}\, f$.

From (4.2) follows:

$$\log_k P_s = \sum_{i=1}^{n} \left({}'f_{x_i} \right) \cdot \log_k s_{x_i},$$

i.e.

$$P_s = k^{\sum_{i=1}^{n} \left({}'f_{x_i} \right) \cdot \log_k s_{x_i}} \tag{4.7}$$

It is known, that in a scalar field the infinite population of derivatives of scalar function f on direction defines a *gradient* of a fields being a *measure of a heterogeneity* of a field f, in a field of vectors the population of derivatives of a vector on direction determines a tensor of the second rank $\nabla \bar{a}$ with components $\frac{\partial a_i}{\partial x_k}$. It consider as a measure of a heterogeneity of a field of vectors $\bar{a}$.

4.3.3. Divergence

Let's discover ω-image of a divergence of a field of vectors. Let in ω_1

the vector $\bar{a} = \sum_{i=1}^{n} a_{x_i} \cdot \bar{e_i}$ is given. Then it the divergence is equal

$$\mathbf{div}\,\bar{a} = \sum_{i=1}^{n} \frac{\partial a_{x_i}}{\partial x_i}.$$

Let's discover it an image in space ω_0:

$$\sum_{i=1}^{n} \frac{\partial a_{x_i}}{\partial x_i} \setminus \omega_1 \rightarrow \omega_0 \setminus D_* = \prod_{i=1}^{n} k^{(a_{x_i})'_{x_i}}.$$

Obtained expression $D_* = \prod_{i=1}^{n} k^{(a_{x_i})'_{x_i}}$ We shall name *quasidivergence*. By taking the logarithm this expression, we shall receive:

$$\log_k D_* = \log_k \prod_{i=1}^{n} k^{(a_{x_i})'_{x_i}} = \sum_{i=1}^{n} \log_k k^{(a_{x_i})'_{x_i}} = \sum_{i=1}^{n} (a_{x_i})'_{x_i}.$$

Whence
$$D_* = k^{\sum_{i=1}^{n} (a_{x_i})'_{x_i}} \tag{4.8}$$

Quasidivergence is an image in ω_0 $\mathbf{div}\,\bar{a}$, noted in ω_1. Quasidivergence (by analogy with quasigradient) is the transformed image of initial object $(\mathbf{div}\,\bar{a})$ concerning operation of a taking the logarithm, as $\log_k D_* \neq \mathbf{div}\,\bar{a}$.

4.3.4. Curl

Let in ω_1 the vector $\bar{a} = a_x \cdot \bar{i} + a_y \cdot \bar{j} + a_z \cdot \bar{k}$ is given. Then

$$\mathbf{rot}\,\bar{a} = \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right) \cdot \bar{i} + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right) \cdot \bar{j} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right) \cdot \bar{k}.$$

Let's reflection $\mathbf{rot}\,\bar{a}$ in space ω_0:

$$\mathbf{rot}\,\overline{a}\setminus\omega_1\to\omega_0\setminus R_*=\left(\frac{k^{{}^{'}(a_z)_y}}{k^{{}^{'}(a_y)_z}}\odot\overline{i}\right)\cdot\left(\frac{k^{{}^{'}(a_x)_z}}{k^{{}^{'}(a_z)_x}}\odot\overline{j}\right)\times$$

$$\times\left(\frac{k^{{}^{'}(a_y)_x}}{k^{{}^{'}(a_x)_y}}\odot\overline{k}\right)=k^{\left({}^{'}(a_z)_y-{}^{'}(a_y)_z\right)\cdot\overline{i_*}+\left({}^{'}(a_x)_z-{}^{'}(a_z)_x\right)\cdot\overline{j_*}+}$$

$$^{+\left({}^{'}(a_y)_x-{}^{'}(a_x)_y\right)\cdot\overline{k_*}}=k^{(\mathbf{rot}\,\overline{a})_0},\qquad(4.9)$$

where $(\mathbf{rot}\,\overline{a})_0$ – new mathematical object similar on the form to an entry to a usual curl.

In an outcome have received quasicurl R_*. Taking the logarithm it, we shall discover

$$\log_k R_*=(\mathbf{rot}\,\overline{a})_0=\left({}^{'}(a_z)_y-{}^{'}(a_y)_z\right)\cdot\overline{i_*}+\left({}^{'}(a_x)_z-{}^{'}(a_z)_x\right)\cdot\overline{j_*}+$$

$$+\left({}^{'}(a_y)_x-{}^{'}(a_x)_y\right)\cdot\overline{k_*}.$$

4.3.5. Basic formulas, describing quasifield

a). We shall discover ω-image of a *solenoidal* field. $\mathbf{div}\,\overline{a}=0\setminus\omega_1\to$ $\to\omega_0\setminus k^{\sum_{i=1}^{n}{}^{'}(a_{xi})_{xi}}=1\;\Rightarrow\;\sum_{i=1}^{n}{}^{'}(a_{xi})_{xi}=0$. The last equality is equivalent $\sum_{i=1}^{n}(a_{xi})'_{xi}=0$, i.e. $\mathbf{div}\,\overline{a}=0$. It is natural, if in space ω_1 there are no in some area Ω_1 radiants (or drains), they will be absent and in appropriate area Ω_i any space ω_i.

b). We shall discover an image ω-image of a *potential* field. $\mathbf{rot}\,\overline{a}=0$ – This condition potentiality of a field. Then $\mathbf{rot}\ \overline{a}=0 \setminus \omega_1 \to \omega_0 \setminus$ $\setminus \omega_1 \to \omega_0 \setminus k^{\left(\mathbf{rot}\,\overline{a}\right)_0}=1 \quad \Rightarrow \quad \left(\mathbf{rot}\,\overline{a}\right)_0=0,$ i.e. $\left({}'\left(a_z\right)_y-\right.$ $\left.-{}'\left(a_y\right)_z\right)\cdot\overline{i_*}+\left({}'\left(a_x\right)_z-{}'\left(a_z\right)_x\right)\cdot\overline{j_*}+\left({}'\left(a_y\right)_x-{}'\left(a_x\right)_y\right)\cdot\overline{k_*}=0.$

This condition potential quasifields.

c). We shall discover ω-image of a condition of a *harmonicity* of function. For this purpose we shall image at first del ∇ (operator of the Hamilton):

$$\nabla \equiv \sum_{i=1}^{n}\left(\frac{\partial}{\partial x_i}\right)\cdot\overline{e_i} \qquad \setminus \omega_1 \to \omega_0 \setminus \qquad \nabla_0=\prod_{i=1}^{n} k^{{}'\left(\partial x_i\right)}\odot\overline{e_i}=$$

$=\prod\limits_{i=1}^{n} k^{{}'\left(\partial x_i\right)\cdot\overline{e_{i*}}}$, where ${}'\left(\partial x_i\right)=x_i\cdot\left(\dfrac{\partial \ln}{\partial x_i}\right)$. Have received *quasivectorial* a *nabla*

$$\nabla_0\left(\nabla_0=k^{\sum\limits_{i=1}^{n}{}'\left(\partial x_i\right)\cdot\overline{e_{i*}}}\right).$$

Let's take the logarithm ∇_0:

$$\log_k \nabla_0=\sum_{i=1}^{n}{}'\left(\partial x_i\right)\cdot\overline{e_{i*}}=\sum_{i=1}^{n}x_i\cdot\left(\frac{\partial \ln}{\partial x_i}\right)\cdot\overline{e_{i*}},$$

i.e. $$\nabla_0=k^{\sum\limits_{i=1}^{n}x_i\cdot\left(\frac{\partial \ln}{\partial x_i}\right)\cdot\overline{e_{i*}}}. \qquad (4.10)$$

Let's remark, that $\nabla f=\mathbf{grad}\,f$ (product of scalar function f on a vector ∇). Then ω-image will be accordingly quasigradient, i.e. $\nabla_0 f=G_*=k^{\sum\limits_{i=1}^{n}\left({}'f_{x_i}\right)\cdot\overline{e_{i*}}}$ (it follows from (4.10)).

ω-image of a divergence (quasidivergence) D_* agrees (4.8) is equal:

$$D_* = k^{\sum_{i=1}^{n}'\left(a_{x_i}\right)_{x_i}} = k^{\sum_{i=1}^{n} x_i \cdot \frac{\partial \ln a_{x_i}}{\partial x_i}}.$$

On the other hand, $D_* = \nabla_0 \odot A_* = \left(k^{\sum_{i=1}^{n} x_i \cdot \left(\frac{\partial \ln}{\partial x_i}\right) \cdot \overline{e_{i*}}} \right) \odot$

$$\odot \left(k^{\sum_{i=1}^{n}\left(\log_k a_{x_i}\right) \cdot \overline{e_{i*}}} \right) = k^{\left(\sum_{i=1}^{n} x_i \cdot \left(\frac{\partial \ln}{\partial x_i}\right) \cdot \overline{e_{i*}}\right) \cdot \left(\sum_{i=1}^{n}\left(\log_k a_{x_i}\right) \cdot \overline{e_{i*}}\right)}.$$

$$\log_k D_* = \left(\sum_{i=1}^{n} x_i \cdot \left(\frac{\partial \ln}{\partial x_i}\right) \cdot \overline{e_{i*}}\right) \cdot \left(\sum_{i=1}^{n}\left(\log_k a_{x_i}\right) \cdot \overline{e_{i*}}\right) =$$

$$= \sum_{i=1}^{n} x_i \cdot \frac{\partial \ln a_{x_i}}{\partial x_i} \left(\log_k D_* = \sum_{i=1}^{n} x_i \cdot \frac{\partial \ln k^{\log_k a_{x_i}}}{\partial x_i} \right), \qquad (4.11)$$

where "$\cdot$" $-$ sign of a scalar product.

Thus, the *scalar product of two vectors in new expression of an aspect* $\left(\sum_{i=1}^{n} x_i \cdot \left(\frac{\partial \ln}{\partial x_i}\right) \cdot \overline{e_{i*}}\right) \cdot \left(\sum_{i=1}^{n}\left(\log_k a_{x_i}\right) \cdot \overline{e_{i*}}\right)$ *Is reconstructed in* $\sum_{i=1}^{n} x_i \cdot \left(\frac{\partial \ln a_{x_i}}{\partial x_i}\right)$.

Similarly, for *quasicurl* R_* the equality will be fair:

$$R_* = \nabla_0 \otimes A_* = k^{\left(\mathbf{rot}\,\overline{a}\right)_0},$$

where $\otimes -$ operation of a new nature being $\omega -$ image (at reflection $\backslash \omega_1 \to \omega_0 \backslash$) of a vector product.

Let's take the logarithm R_*:

$$\log_k R_* = \log_k\left(\nabla_0 \otimes A_*\right) = \left(\mathbf{rot}\,\overline{a}\right)_0.$$

$$\nabla_0 \otimes A_* = \left(k^{\left(\sum_{i=1}^{n} x_i \cdot \left(\frac{\partial \ln}{\partial x_i} \right) \cdot \overline{e_{i*}} \right)} \right) \otimes \left(k^{\left(\sum_{i=1}^{n} \left(\log_k a_{x_i} \right) \cdot \overline{e_{i*}} \right)} \right)$$

For $n = 3$

$$\nabla_0 \otimes A_* = \left(k^{x \cdot \left(\frac{\partial \ln}{\partial x} \right) \cdot \overline{i_*} + y \cdot \left(\frac{\partial \ln}{\partial y} \right) \cdot \overline{j_*} + z \cdot \left(\frac{\partial \ln}{\partial z} \right) \cdot \overline{k_*}} \right) \otimes$$

$$\otimes \left(k^{\left(\log_k a_x \right) \cdot \overline{i_*} + \left(\log_k a_y \right) \cdot \overline{j_*} + \left(\log_k a_z \right) \cdot \overline{k_*}} \right).$$

On the other hand, $R_* = k^{\left({}'(a_z)_y - {}'(a_y)_z \right) \cdot \overline{i_*} + \left({}'(a_x)_z - {}'(a_z)_x \right) \cdot \overline{j_*} +}$

$+\left({}'(a_y)_x - {}'(a_x)_y \right) \cdot \overline{k_*}$, i.e.

$$\log_k \left(\nabla_0 \otimes A_* \right) = \log_k R_* = \begin{vmatrix} \overline{i_*} & \overline{j_*} & \overline{k_*} \\ x \cdot \frac{\partial \ln}{\partial x} & y \cdot \frac{\partial \ln}{\partial y} & z \cdot \frac{\partial \ln}{\partial z} \\ \log_k a_x & \log_k a_y & \log_k a_z \end{vmatrix}, \quad (4.12)$$

i.e. the *operation* $\otimes$ *between quasivectorials at a taking the logarithm is transformed to a vector product of two appropriate vectors in basis quasivector to the form.*

The formula (4.12) will be coordinated with (4.11). Really, $\left(\left(y \cdot \frac{\partial \ln}{\partial y} \right) \bullet \right.$

$$\left. \bullet \log_k a_z - \left(z \cdot \frac{\partial \ln}{\partial z} \right) \bullet \log_k a_y \right) \cdot \overline{i_*} = \left(\left(y \cdot \frac{\partial \ln}{\partial y} \right) \bullet \log_k a_z \right) \cdot \overline{i_*} -$$

$$-\left(\left(z \cdot \frac{\partial \ln}{\partial z} \right) \bullet \log_k a_y \right) \cdot \overline{i_*} = \left(y \cdot \frac{\partial \ln a_z}{\partial y} - z \cdot \frac{\partial \ln a_y}{\partial z} \right) \cdot \overline{i_*}.$$

So, that the product of projections ($y\cdot\frac{\partial\ln}{\partial y}\cdot\log_k a_z$ and others) at shaping a scalar product in *basis quasivector* of a mathematical construction is realized on such rule: $\left(y\cdot\frac{\partial\ln}{\partial y}\right)\cdot\log_k a_z\Rightarrow y\cdot\frac{\partial\ln a_z}{\partial y}$. It is known, that the entry of a scalar product of a del $\nabla=\frac{\partial}{\partial x}\bar{i}+\frac{\partial}{\partial y}\bar{j}+\frac{\partial}{\partial z}\bar{k}$ and vector $\bar{a}=a_x\bar{i}+$ $+a_y\bar{j}+a_z\bar{k}$ was carried out so:

$\nabla\cdot\bar{a}=\frac{\partial a_x}{\partial x}+\frac{\partial a_y}{\partial y}+\frac{\partial a_z}{\partial z}$, and under the formula (4.11) $\left(x\frac{\partial\ln}{\partial x}\overline{i_*}+\right.$

$$\left.+y\frac{\partial\ln}{\partial y}\overline{j_*}+z\frac{\partial\ln}{\partial z}\overline{k_*}\right)\bullet\left(\left(\log_k a_x\right)\cdot\overline{i_*}+\left(\log_k a_y\right)\cdot\overline{j_*}+\left(\log_k a_z\right)\cdot\overline{k_*}\right)=$$

$$=x\cdot\frac{\partial\ln a_x}{\partial x}+y\cdot\frac{\partial\ln a_y}{\partial y}+z\cdot\frac{\partial\ln a_z}{\partial z}.$$

The formula (4.12) on a structure reminds a curl:

$$\mathbf{rot}\bar{a}=\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ a_x&a_y&a_z\end{vmatrix}$$

$$\log_k R_*=\left(\mathbf{rot}\,\bar{a}\right)_0=\begin{vmatrix}\overline{i_*}&\overline{j_*}&\overline{k_*}\\ {}'\partial_x&{}'\partial_y&{}'\partial_z\\ \log_k a_x&\log_k a_y&\log_k a_z\end{vmatrix},$$

where ${}'\partial_x=x\frac{\partial\ln}{\partial x}$, ${}'\partial_y=y\frac{\partial\ln}{\partial y}$, ${}'\partial_z=z\frac{\partial\ln}{\partial z}$.

c). We shall consider ω-images of a *harmonic* function.

It is known, that $\mathbf{div}(\mathbf{grad}\, f) = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i} = \Delta f$, where f, satisfying to the equation of the Laplace, i.e. equation $\Delta f = 0$ is named as a *harmonic* function.

The theorem 4.4. *If the function* $f(x_1, x_2, \dots, x_n)$ *– harmonic in space* ω_1, *in space* ω_0 *she satisfies to the equation* $\sum_{i=1}^{n}\left(x_i \frac{\partial \ln f}{\partial x_i} + x_i^2 \frac{\partial^2 \ln f}{\partial x_i^2} \right) = 0.$

Proof. Let in ω_1 the harmonic function $f(x_1, x_2, \dots, x_n)$, i.e. $\Delta f = 0$ is given.

As $\frac{\partial^2 f}{\partial x^2} \backslash \omega_1 \to \omega_0 \backslash \overset{''}{-}f = k^{\ln k \cdot x\left(\dot{u}_x + x \cdot \ddot{u}_{xx} \right)}$, where $u = \ln f$, and $\dot{u}_x = \frac{\partial u}{\partial x}$, $\ddot{u}_{xx} = \frac{\partial^2 u}{\partial x^2}$, $\Delta f \backslash \omega_1 \to \omega_0 \backslash \Delta_0 f = \prod_{i=1}^{n} \overset{''}{-}f_{x_i x_i} =$

$= \prod_{i=1}^{n} k^{\ln k \cdot x_i \left(\dot{u}_{x_i} + x_i \cdot \ddot{u}_{x_i x_i} \right)}.$

Whence $\ln \Delta_0 f = \ln \prod_{i=1}^{n} \overset{''}{-}f_{x_i x_i} = \sum_{i=1}^{n} \ln \overset{''}{-}f_{x_i x_i} = \ln^2 k \cdot \sum_{i=1}^{n} x_i \times$

$$\times \left(\frac{\partial \ln f}{\partial x_i} + x_i \frac{\partial^2 \ln f}{\partial x_i^2} \right). \tag{4.13}$$

From a condition $\Delta f = 0$ follows $\Delta_0 f = 1$ $\left(0 \backslash \omega_1 \to \omega_0 \backslash k^0 = 1\right)$, i.e. $\ln \Delta_0 f = 0$. Then $\sum_{i=1}^{n}\left(x_i \cdot \frac{\partial \ln f}{\partial x_i} + x_i^2 \cdot \frac{\partial^2 \ln f}{\partial x_i^2} \right) = 0.$

The theorem is proved.

Let's remark, that $\Delta_0 - \omega$-image of a laplacian (quasilaplacian).

The theorem 4.5. *If* f *– harmonic function, is fair equality:*

$$\left(\sum_{i=1}^{n} x_i\left(\frac{\partial \ln}{\partial x_i}\right)\overline{e_{i*}}\right)\cdot\left(\sum_{i=1}^{n} x_i\left(\frac{\partial \ln f}{\partial x_i}\right)\overline{e_{i*}}\right)=$$

$$=\sum_{i=1}^{n}\left(x_i{}^2\cdot\left(\frac{\partial \ln f}{\partial x_i}\right)\cdot\left(\frac{\partial \ln\left(x_i\left(\frac{\partial \ln f}{\partial x_i}\right)\right)}{\partial x_i}\right)\right)^{35} \qquad (4.14)$$

Proof. As $\Delta f = \mathbf{div}(\mathbf{grad}\, f) = \nabla\cdot(\nabla f)$, that

$$\Delta f = \nabla\cdot(\nabla f)\setminus \omega_1 \to \omega_0 \setminus \Delta_0 f = \nabla_0 \odot (\nabla_0 f) =$$

$$=\left(k^{\sum_{i=1}^{n} x_i\left(\frac{\partial \ln}{\partial x_i}\right)\overline{e_{i*}}}\right)\odot\left(k^{\sum_{i=1}^{n}\left({}'f_{x_i}\right)\overline{e_{i*}}}\right)=$$

$$=k^{\left(\left(\sum_{i=1}^{n} x_i\left(\frac{\partial \ln}{\partial x_i}\right)\overline{e_{i*}}\right)\cdot\left(\sum_{i=1}^{n} x_i\left(\frac{\partial \ln f}{\partial x_i}\right)\overline{e_{i*}}\right)\right)}.$$

Let's take the logarithm this expression:

$$\log_k \Delta_0 f = \left(\sum_{i=1}^{n} x_i\left(\frac{\partial \ln}{\partial x_i}\right)\overline{e_{i*}}\right)\cdot\left(\sum_{i=1}^{n} x_i\left(\frac{\partial \ln f}{\partial x_i}\right)\overline{e_{i*}}\right)\cdot \ln k\,.$$

---

[35] The scalar product of vectors ∇ and ∇f, represented in basis quasivector to the form, is reconstructed in scalar function $\sum_{i=1}^{n}\left(x_i{}^2\cdot\left(\frac{\partial \ln f}{\partial x_i}\right)\cdot\left(\frac{\partial \ln\left(x_i\left(\frac{\partial \ln f}{\partial x_i}\right)\right)}{\partial x_i}\right)\right)$. From (4.14) the new rule of transformation of objects of a new nature follows which the vectors (vectorial functions), noted in basis quasivector to the form are. It is clear from equality (4.14) without any explanations.

From the theorem 4.4 follows, that

$$\ln \Delta_0 f = \ln^2 k \cdot \sum_{i=1}^{n} x_i \left(\frac{\partial \ln f}{\partial x_i} + x_i \frac{\partial^2 \ln f}{\partial x_i^{\,2}} \right), \text{ i.e.}$$

$$\frac{\ln(\Delta_0 f)}{\ln^2 k} = \sum_{i=1}^{n} x_i \left(\frac{\partial \ln f}{\partial x_i} + x_i \frac{\partial^2 \ln f}{\partial x_i^{\,2}} \right) =$$

$$= \sum_{i=1}^{n} \left(x_i^{\,2} \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \cdot \frac{1}{x_i \cdot \left(\frac{\partial \ln f}{\partial x_i} \right)} \cdot \frac{\partial \left(x_i \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \right)}{\partial x_i} \right) =$$

$$= \sum_{i=1}^{n} \left(x_i^{\,2} \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \cdot \frac{\partial \ln \left(x_i \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \right)}{\partial x_i} \right).$$

Whence $\left(\sum_{i=1}^{n} x_i \left(\frac{\partial \ln}{\partial x_i} \right) \overline{e_{i*}} \right) \cdot \left(\sum_{i=1}^{n} x_i \left(\frac{\partial \ln f}{\partial x_i} \right) \overline{e_{i*}} \right) =$

$$= \sum_{i=1}^{n} \left(x_i^{\,2} \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \cdot \frac{\partial \ln \left(x_i \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \right)}{\partial x_i} \right),$$

as was to be shown.

The note. $\Delta_0 f = k^{\ln k \cdot \sum_{i=1}^{n} \left(x_i^{2} \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \cdot \frac{\partial \ln \left(x_i \cdot \left(\frac{\partial \ln f}{\partial x_i} \right) \right)}{\partial x_i} \right)}$.

4.3.6 ω-images of some relations of a vector analysis

In connection with a simplicity all proofs reduced below, are explained without detailed explanations.

a) Let in ω_1 is given $\mathbf{grad}(c \cdot f)$, where $c = const$. Then

$$\mathbf{grad}(c \cdot f) \setminus \omega_1 \to \omega_0 \setminus k^{\sum\limits_{i=1}^{n} {}'(c \odot f)_{x_i} \cdot \bar{e}_{i*}}, \qquad (4.15)$$

where $c \odot f = f^{\log_k c} = f^{c_1} \qquad (c_1 = \log_k c), \qquad {}'\left(f^{c_1}\right)_{x_i} =$

$= x_i \cdot \left(\ln f^{c_1}\right)'_{x_i} = x_i \cdot c_1 \cdot (\ln f)'_{x_i} = c_1 \cdot {}'f_{x_i}$, i.e. ω-image $\mathbf{grad}(c \cdot f)$

in ω_0 is equal $k^{c_1 \cdot \sum\limits_{i=1}^{n} {}'f_{x_i} \cdot \bar{e}_{i*}} = G_*^{c1}$. $\qquad (4.16)$

It is known, that, $\mathbf{grad}(c \cdot f) = c \cdot \mathbf{grad} f$, i.e.

$$c \cdot \mathbf{grad} f \setminus \omega_1 \to \omega_0 \setminus c \odot G_* = G_*^{\log_k c} = G_*^{c1}. \qquad (4.17)$$

Actually, we have proved equality

$$G_*(c \odot f) = c \odot G_*, \qquad (4.18)$$

as $G_*(c \odot f) = k^{\sum\limits_{i=1}^{n} {}'(c \odot f)_{x_i} \cdot \bar{e}_{i*}}$.

Similarly, it is possible to prove, that $\mathbf{grad}(c_1 \cdot f_1 + c_2 \cdot f_2) \setminus \omega_1 \to$ $\to \omega_0 \setminus G_{1*}^{\log_k c1} \cdot G_{2*}^{\log_k c2}$, where G_{1*}, G_{2*} – appropriate ω-images $\mathbf{grad} f_1$ and $\mathbf{grad} f_2$.

b). We shall discover ω-image $\mathbf{rot}(c_1 \bar{a}_1 + c_2 \bar{a}_2)$.

$$\log_k R_* = \mathbf{rot}\left(\bar{a}_1^{\log_k c1} \cdot \bar{a}_2^{\log_k c2}\right)_0 = \left(\mathbf{rot}\left(\bar{a}_1^{c_{1*}} \cdot \bar{a}_2^{c_{2*}}\right)\right)_0 =$$

$$= \left({}'\left(a_{1z}{}^{c_{1*}} \cdot a_{2z}{}^{c_{2*}}\right)_y - {}'\left(a_{1y}{}^{c_{1*}} \cdot a_{2y}{}^{c_{2*}}\right)_z\right)\bar{i}_* +$$

$$+\left({}'\left(a_{1x}^{\;c_{1*}}\cdot a_{2x}^{\;c_{2*}}\right)_z-{}'\left(a_{1z}^{\;c_{1*}}\cdot a_{2z}^{\;c_{2*}}\right)_x\right)\overline{j_*}+$$

$$+\left({}'\left(a_{1y}^{\;c_{1*}}\cdot a_{2y}^{\;c_{2*}}\right)_x-{}'\left(a_{1x}^{\;c_{1*}}\cdot a_{2x}^{\;c_{2*}}\right)_y\right)\overline{k_*}.$$

Let's transform ${}'\left(a_{1z}^{\;c_{1*}}\cdot a_{2z}^{\;c_{2*}}\right)_y$:

$$y\left(\ln a_{1z}^{\;c_{1*}}\cdot a_{2z}^{\;c_{2*}}\right)'_y=y\left(\ln a_{1z}\right)'_y\cdot c_{1*}+y\left(\ln a_{2z}\right)'_y\cdot c_{2*}=$$

$$=c_{1*}\cdot{}'\left(a_{1z}\right)_y+c_{2*}\cdot{}'\left(a_{2z}\right)_y.$$

Then, ${}'\left(a_{1z}^{\;c_{1*}}\cdot a_{2z}^{\;c_{2*}}\right)_y-{}'\left(a_{1y}^{\;c_{1*}}\cdot a_{2y}^{\;c_{2*}}\right)_z=c_{1*}\times$

$\times\left({}'\left(a_{1z}\right)_y-{}'\left(a_{1y}\right)_z\right)+c_{2*}\cdot\left({}'\left(a_{2z}\right)_y-{}'\left(a_{2y}\right)_z\right)$, i.e. $\log_k R_*=$

$$=c_{1*}\cdot\left[\left({}'\left(a_{1z}\right)_y-{}'\left(a_{1y}\right)_z\right)\cdot\overline{i_*}+\left({}'\left(a_{1x}\right)_z-{}'\left(a_{1z}\right)_x\right)\cdot\overline{j_*}+\right.$$

$$\left.+\left({}'\left(a_{1y}\right)_x-{}'\left(a_{1x}\right)_y\right)\cdot\overline{k_*}\right]+c_{2*}\cdot\left[\left({}'\left(a_{2z}\right)_y-{}'\left(a_{2y}\right)_z\right)\cdot\overline{i_*}+\right.$$

$$\left.+\left({}'\left(a_{2x}\right)_z-{}'\left(a_{2z}\right)_x\right)\cdot\overline{j_*}+\left({}'\left(a_{2y}\right)_x-{}'\left(a_{2x}\right)_y\right)\cdot\overline{k_*}\right]=$$

$$=c_{1*}\cdot\left(\mathbf{rot}\,a_1\right)_0+c_{2*}\cdot\left(\mathbf{rot}\,a_2\right)_0.$$

Whence, $R_*=\left(k^{\left(\mathbf{rot}\,a_1\right)_0}\right)^{c_{1*}}\cdot\left(k^{\left(\mathbf{rot}\,a_2\right)_0}\right)^{c_{2*}}=R_{1*}^{c_{1*}}\cdot R_{2*}^{c_{2*}}$, where R_{1*}, R_{2*}, R_* – accordingly ω-images $\mathbf{rot}\,\overline{a_1}$, $\mathbf{rot}\,\overline{a_2}$, $\mathbf{rot}\left(c_1\overline{a_1}+c_2\overline{a_2}\right)$, obtained by reflection $\backslash\,\omega_1\to\omega_0\,\backslash$.

c). We shall discover ω-image $\mathbf{div}\left(\mathbf{grad}\,f\right)$.

$$\mathbf{div}\left(\mathbf{grad}\, f\right) = \nabla \cdot \nabla f \setminus \omega_1 \rightarrow \omega_0 \setminus D_*\left(G(f)\right) = \nabla_0 \odot \nabla_0 f =$$

$$= k^{\sum\limits_{i=1}^{n} x_i \cdot \frac{\partial \ln}{\partial x_i} \overline{e_{i*}}} \odot k^{\sum\limits_{i=1}^{n} {'}(f)_{x_i} \overline{e_{i*}}} = k^{\sum\limits_{i=1}^{n} \left(x_i^2 \cdot \frac{\partial \ln f}{\partial x_i} \cdot \frac{\partial \ln \left(x_i \cdot \frac{\partial \ln f}{\partial x_i} \right)}{\partial x_i} \right)} =$$

$= \Delta_0 f$. (According to the theorem 4.5 and formula (4.14)), where $\Delta_0 f - \omega$-image of laplacian.

d). We shall prove, that $R_*\left(G(f)\right) = 1$. As $G = k^{\sum\limits_{i=1}^{n} {'}(f)_{x_i} \overline{e_{i*}}}$, for $n = 3$ we shall receive $\left(\mathbf{rot}\, G\right)_0 = \left({'}(G_z)_y - {'}(G_y)_z \right) \overline{i_*} + \left({'}(G_x)_z - {'}(G_z)_x \right) \overline{j_*} + \left({'}(G_y)_x - {'}(G_x)_y \right) \overline{k_*}$, $G_x = k^{\frac{x \cdot \partial \ln f}{\partial x}}$, $G_y = = k^{\frac{y \cdot \partial \ln f}{\partial y}}$,

$$G_z = k^{\frac{z \cdot \partial \ln f}{\partial z}}, \quad {'}(G_z)_y = \frac{z \cdot \partial \ln f}{\partial z} \cdot \ln k \cdot {'}\left(\frac{z \cdot \partial \ln f}{\partial z} \right)_y =$$

$$= \frac{z \cdot \partial \ln f}{\partial z} \cdot \ln k \cdot {'}\left(\frac{\partial \ln f}{\partial z} \right)_y = \frac{\frac{z \cdot \partial \ln f}{\partial z} \cdot \ln k \cdot y \cdot \frac{\partial^2 \ln f}{\partial z \partial y}}{\frac{\partial \ln f}{\partial z}} =$$

$= z \cdot y \cdot \ln k \cdot \frac{\partial^2 \ln f}{\partial z \partial y}$. In this case ${'}(G_z)_y$ is discovered, using a table of a not reduced image of a derivative of function:

$${'}\left(a^f\right) = f \cdot \ln a \cdot {'}f, \quad {'}(u \cdot v) = {'}u + {'}v, \quad {'}f\left(\varphi(x)\right) = {'}f_{\varphi} \cdot {'}\varphi_x$$

$$\left({}^{'}\!\left(\frac{z\cdot\partial\ln f}{\partial z}\right)_y = {}^{'}z_y + {}^{'}\!\left(\frac{\partial\ln f}{\partial z}\right)_y = \frac{0 + y\cdot {}'\!\left(\frac{\partial\ln f}{\partial z}\right)_y}{\frac{\partial\ln f}{\partial z}} = \frac{y\cdot\frac{\partial^2\ln f}{\partial z\partial y}}{\frac{\partial\ln f}{\partial z}} \right)$$

Similarly, $${}^{'}(G_y)_z = y\cdot\frac{\partial\ln f}{\partial y}\cdot\ln k\cdot {}^{'}\!\left(y\cdot\frac{\partial\ln f}{\partial y}\right)_z = y\cdot z\times$$

$\times\ln k\cdot\frac{\partial^2\ln f}{\partial y\partial z}$. That $\frac{\partial^2\ln f}{\partial z\partial y} = \frac{\partial^2\ln f}{\partial y\partial z}$, we shall receive ${}^{'}(G_z)_y = {}^{'}(G_y)_z$. Precisely equalities ${}^{'}(G_x)_z = {}^{'}(G_z)_x$ and ${}^{'}(G_y)_x = {}^{'}(G_x)_y$, i.e. $(\mathbf{rot}\,G)_0 = 0$ also are proved. Whence $R_*(G(f)) = k^{(\mathbf{rot}\,G)_0} = 1$. Have received, that

$$\mathbf{rot}(\mathbf{grad}(f)) \setminus \omega_1 \to \omega_0 \setminus R_*(G(f)) = 1$$

Really, $\mathbf{rot}(\mathbf{grad}\,f) = 0$, i.e. $0 \setminus \omega_1 \to \omega_0 \setminus k^0 = 1$.

e). We shall discover ω-image of expression $\nabla\cdot(\nabla\times\overline{a}) = \mathbf{div}\,\mathbf{rot}\,\overline{a}$.

$\nabla\cdot(\nabla\times\overline{a}) \setminus \omega_1 \to \omega_0 \setminus \nabla_0 \odot (\nabla_0 \otimes A_*) = \nabla_0 \odot R_* = D_*(R_*(\overline{a}))$, i.e. $\nabla_0 \odot (\nabla_0 \otimes A_*) = D_*(R_*(\overline{a}))$, where $D_*(R_*(\overline{a}))-$quasidivergence from quasicurl of a vector $\overline{a}$.

For $n = 3$

$$\log_k R_*(\overline{a}) = (\mathbf{rot}\,\overline{a})_0 = \left({}^{'}(a_z)_y - {}^{'}(a_y)_z\right)\overline{i_*} +$$

$$+\left({}^{'}(a_x)_z - {}^{'}(a_z)_x\right)\overline{j_*} + \left({}^{'}(a_y)_x - {}^{'}(a_x)_y\right)\overline{k_*}.$$

Then

$$\log_k D_*\left(R_*\left(\overline{a}\right)\right) = {}'\left(\left(\mathbf{rot}\,\overline{a}\right)_{0x}\right)_x + {}'\left(\left(\mathbf{rot}\,\overline{a}\right)_{0y}\right)_y + {}'\left(\left(\mathbf{rot}\,\overline{a}\right)_{0z}\right)_z,$$

where ${}'\left(\left(\mathbf{rot}\,\overline{a}\right)_{0x}\right)_x$, ${}'\left(\left(\mathbf{rot}\,\overline{a}\right)_{0y}\right)_y$, ${}'\left(\left(\mathbf{rot}\,\overline{a}\right)_{0z}\right)_z$ $-$projection $\left(\mathbf{rot}\,\overline{a}\right)_0$, i.e. $\log_k R_*$, on appropriate to an axes of coordinates ($\left(\mathbf{rot}\,\overline{a}\right)_{0x}$ $-$on OX etc.)

By analogy to the previous conclusion it is possible to show, that

$$D_*\left(R_*\left(\overline{a}\right)\right) = 1.$$

(It is well-known, that $\mathbf{div}\left(\mathbf{rot}\,\overline{a}\right) = 0$, i.e. $D_*\left(R_*\left(\overline{a}\right)\right) = k^0 = 1$).

Similarly, it is possible to find ω-images and other relations of a vector analysis. For example,

$$\overline{a} \times \mathbf{grad}\, f = \mathbf{rot}\left(f\,\overline{a}\right) \backslash \omega_1 \to \omega_0 \backslash V_* = A_* \otimes G =$$

$$= k^{\sum\limits_{i=1}^{n}\left(\log_k a_{x_i}\right)\overline{e_{i*}}} \otimes k^{\sum\limits_{i=1}^{n}\left({}'f_{x_i}\right)\cdot\overline{e_{i*}}} = R_*\left(f \odot \overline{a}\right).$$

The note. For $n = 3$:

$$\log_k V_* = \begin{vmatrix} \overline{i_*} & \overline{j_*} & \overline{k_*} \\ \dfrac{x \cdot \partial \ln}{\partial x} & \dfrac{y \cdot \partial \ln}{\partial y} & \dfrac{z \cdot \partial \ln}{\partial z} \\ \log_k\left(a_x \odot f\right) & \log_k\left(a_y \odot f\right) & \log_k\left(a_z \odot f\right) \end{vmatrix}$$

The rule of disclosure of such continuant is reduced above (see 4.3.5).

§4.4 Examples quasifields

4.4.1. Field of a tensor of the 2-nd rank

Let's consider a field of a tensor of the 2-nd rank $\mathrm{T}(r)$, having components $T_{ik} = T_{ik}(r)$. For an example of such field can be a field of voltages in an elastic medium.

The stream tensor of a field through a surface is a surface integral taken from a scalar product of a tensor T on a vector perpendicular $\overline{n}$:

$$\overline{W} = \iint\limits_{\sigma} T \cdot \overline{n} \cdot d\sigma .$$

$\overline{W}$ – it is a vector (as against a stream of a field of vectors). The components of a stream tensor of a field are equal:

$$W_i = \iint\limits_{\sigma} T_{ik} \cdot n_k \cdot d\sigma = \iint\limits_{\sigma} \left(T_{i1} \cdot n_1 + T_{i2} \cdot n_2 + T_{i3} \cdot n_3\right) \cdot d\sigma .$$

The procedure of a contraction on defined (for example, first) indexes is known:

$$W_i = \iint\limits_{\sigma} T_{ki} \cdot n_k \cdot d\sigma .$$

Let $T_{ik} = p_{ik}$ – tensor of voltages in an elastic skew field. Selecting in this skew field some surface σ and defining equal in effect $\overline{F}$ all forces of voltage affixed on this surface:

$$\overline{F} = \iint\limits_{\sigma} p_n \cdot d\sigma ,$$

where p_n – voltage at the element $d\sigma$ with perpendicularly $\overline{n}$ ($F_k = \iint\limits_{\sigma} p_{nk} d\sigma$ – component $\overline{F}$), agrees $p_{nk} = p_{ik} \cdot n_i$ (law of a transformation of coordinates at a modification of a frame), we shall receive $F_k = \iint\limits_{\sigma} p_{ik} \cdot n_i \cdot d\sigma$, i.e. the stream of a tensor of voltages through a surface taken in an elastic medium, is equal of equal in effect all forces of voltages affixed on this surface.

Divergence of a tensor:

$$\overline{\mathbf{div}\,\mathrm{T}} = \nabla \cdot \mathrm{T},$$

and derivative with direction $\frac{dT}{ds} = \bar{s} \cdot \nabla T$, $\frac{d\,T_{ik}}{ds} = s_m \frac{\partial T_{ik}}{\partial x_m}$ etc.

Let's discover ω-images of some of the above-stated magnitudes.

It is known, that $\overline{W} = \overline{I_\sigma} = \iint\limits_\sigma T \cdot \bar{n} \cdot d\sigma = I_{xy} + I_{xz} + I_{yz}$, where $I_{xy} = \iint\limits_{D_{xy}} T\,dxdy$, $I_{xz} = \iint\limits_{D_{xz}} T\,dxdz$, $I_{yz} = \iint\limits_{D_{yz}} T\,dydz$; D_{xy}, D_{xz}, D_{yz} − projection of a surface σ on coordinate planes XOY, XOZ, YOZ.

Then $\overline{W} \setminus \omega_1 \rightarrow \omega_0 \setminus W_* = \left(I_{yz*} \odot \bar{i}\right) \cdot \left(I_{xz*} \odot \bar{j}\right) \cdot \left(I_{xy*} \odot \bar{k}\right) \quad \Rightarrow$

$$\log_k W_* = \left(\log_k I_{yz*}\right) \cdot \overline{i_*} + \left(\log_k I_{xz*}\right) \cdot \overline{j_*} +$$

$$+ \left(\log_k I_{xy*}\right) \cdot \overline{k_*} = \left(\log_k w_x\right) \cdot \overline{i_*} + \left(\log_k w_y\right) \cdot \overline{j_*} + \left(\log_k w_z\right) \cdot \overline{k_*},$$ where

w_x, w_y, w_z − projection quasivectorial W_* on an axes of coordinates; I_{xy*}, I_{xz*}, I_{yz*} − ω-images of appropriate double integrals I_{xy}, I_{xz}, I_{yz}:

$$I_{xy} \setminus \omega_1 \rightarrow \omega_0 \setminus I_{xy*} = \exp\left(\int\limits_{a1}^{b1} \log_k \left(\exp\left(\int\limits_{\varphi_1(x)}^{\varphi_2(x)} \frac{\log_k T_z}{y} \cdot dy \right) \right) \frac{dx}{x} \right)$$ [36];

$$I_{xz} \setminus \omega_1 \rightarrow \omega_0 \setminus I_{xz*} = \exp\left(\int\limits_{a2}^{b2} \log_k \left(\exp\left(\int\limits_{\psi_1(x)}^{\psi_2(x)} \frac{\log_k T_y}{z} \cdot dz \right) \right) \frac{dx}{x} \right);$$

[36] The limits of an integration are noted in the correspondence with initial area of an integration, i.e. the ω − reflections are saved same, as in a repeated integral up to.

$$I_{yz} \backslash \omega_1 \to \omega_0 \backslash I_{yz*} = \exp\left(\int\limits_{a3}^{b3} \log_k \left(\exp\left(\int\limits_{\chi_1(y)}^{\chi_2(y)} \frac{\log_k T_x}{z} \cdot dz \right)\right) \frac{dy}{y} \right);$$

$$w_x = I_{yz*},\ w_y = I_{xz*},\ w_z = I_{xy*}. \tag{4.19}$$

Let's remark, that (4.19) it is uneasy to prove if to take into account, that $\log_k\left(T_z \odot \overline{k}\right) = \left(\log_k T_z\right) \cdot \overline{k_*}$,[37] $\log_k\left(T_y \odot \overline{j}\right) = \left(\log_k T_y\right) \cdot \overline{j_*}$, $\log_k\left(T_x \odot \overline{i}\right) = \left(\log_k T_x\right) \cdot \overline{i_*}$.

Then, for example,

$$I_{xy} \backslash \omega_1 \to \omega_0 \backslash \overline{I_{xy_*}} = \exp\left(\int\limits_{a1}^{b1} \log_k \left(\exp\left(\int\limits_{\varphi_1(x)}^{\varphi_2(x)} \frac{\log_k \left(T_z \odot \overline{k}\right)}{y} \cdot dy \right)\right) \frac{dx}{x} \right) =$$

$$= \exp\left(\int\limits_{a1}^{b1} \log_k \left(\exp\left(\int\limits_{\varphi_1(x)}^{\varphi_2(x)} \frac{\left(\log_k T_z\right) \cdot \overline{k_*}}{y} \cdot dy \right)\right) \frac{dx}{x} \right) \Rightarrow \log_k \overline{I_{xy_*}} =$$

$$= \frac{1}{\ln k} \left(\int\limits_{a1}^{b1} \log_k \left(\exp\left(\int\limits_{\varphi_1(x)}^{\varphi_2(x)} \frac{\log_k I_z}{y} \cdot dy \right)\right) \frac{dx}{x} \right) \cdot \overline{k_*} \Rightarrow \overline{I_{xy_*}} =$$

$= I_{xy_*} \cdot \overline{k_*}$ etc.

$$\overline{W} \backslash \omega_1 \to \omega_0 \backslash I_{yz_*} \cdot \overline{i_*} + I_{xz*} \cdot \overline{j_*} + I_{xy_*} \cdot \overline{k_*}$$

Under the formula of a Ostrogradskii-Gauss ω-image of a surface integral can be replaced ω-image of a triple integral, i.e.

[37] Let's remark, that in this case foundation and single vector are designated by one letter k.

$$W_{**} = \exp\left(\int_a^b \log_k\left(\exp\left(\int_{p_1(x)}^{p_2(x)} \log_k\left(\exp\left(\int_{m_1(x,y)}^{m_2(x,y)} \frac{\log_k f_x}{z} \cdot dz\right)\right) \cdot \frac{dy}{y}\right)\right) \cdot \frac{dx}{x}\right)^{38},$$

where W_{**}, f_x – scalar projections on an axes OX quasivectorial W_* and $f = f(x,y,z) \equiv \overline{\mathbf{div}\, T(x,y,z)} = \nabla \cdot T(x,y,z)$.

Let's reflection $\overline{\mathbf{div}\, T}$, $\dfrac{dT}{ds}$:

$$\overline{\mathbf{div}\, T} \setminus \omega_1 \to \omega_0 \setminus \overline{D_*} = k^{\sum_{i=1}^{n} {}'\left(T_{x_i}\right)_{x_i} \cdot \overline{e_{i*}}}; \ \log_k \overline{D_*} = \sum_{i=1}^{n} \left(\log_k \overline{D_{*x_i}}\right) \cdot \overline{e_{i*}},$$

$$\frac{dT}{ds} \setminus \omega_1 \to \omega_0 \setminus \overline{P_*} = S_* \odot \overline{\nabla_0 T} = k^{\sum_{i=1}^{n} \left(\log_k s_{x_i}\right) \cdot \overline{e_{i*}}} \odot k^{\sum_{i=1}^{n} {}' T_{x_i} \cdot \overline{e_{i*}}},$$

$$\log_k \overline{P_*} = \left(\sum_{i=1}^{n} \left(\log_k s_{x_i}\right) \cdot \overline{e_{i*}}\right) \cdot \left(\sum_{i=1}^{n} {}' T_{x_i} \cdot \overline{e_{i*}}\right) = \sum_{i=1}^{n} {}' T_{x_i} \cdot \log_k s_{x_i} \Rightarrow$$

$$\overline{P_*} = k^{\sum_{i=1}^{n} {}' T_{x_i} \cdot \log_k s_{x_i}}$$

$$\frac{d\,T_{ik}}{ds} \setminus \omega_1 \to \omega_0 \setminus P_{*_i} = S_{*m} \odot \left(k^{x_m \cdot \frac{\partial \ln T_{ik}}{\partial x_m} \cdot \overline{e_{i*}}}\right) =$$

$$= k^{\sum_{i=1}^{n} \left(\log_k s_{mx_i}\right) \cdot \overline{e_{i*}}} \odot k^{\sum_{i=1}^{n} {}' \left(T_{ik}\right)_{x_m} \cdot \overline{e_{i*}}};$$

---

[38] The limits of an integration are noted in the correspondence with area of an integration in an

$$\log_k P_{*_i} = \left(\sum_{i=1}^{n} \left(\log_k s_{mxi}\right) \cdot \overline{e_{i*}} \right) \cdot \left(\sum_{i=1}^{n} {}' \left(T_{ik}\right)_{xm} \cdot \overline{e_{i*}} \right) =$$

$$= \sum_{i=1}^{n} {}' \left(T_{ik}\right)_{xm} \cdot \log_k s_{mxi} \Rightarrow P_{*_i} = k^{\sum_{i=1}^{n} {}' \left(T_{ik}\right)_{xm} \cdot \left(\log_k s_{mxi}\right)},$$

where P_{*_i} – component quasivectorial $\overline{P_*}$.

$\overline{W_*}$, $\overline{D_*}$, $\overline{P_*}$ – quasivectorial, describing quasitensor a field. $\log_k \overline{W_*}$, $\log_k \overline{D_*}$, $\log_k \overline{P_*}$ – vectorial functions.

At reflection $\backslash \omega_1 \to \omega_0 \backslash$ tensor of a field we shall receive *quasitensor* a field. Tensor (and, special case it, vectorial) the exposition of physical magnitudes is a mode of representation of a defined objective reality. Similarly, quasitensor (quasivectorial) the exposition of these magnitudes is a mathematical modeling of a *possible* objective reality, which is not necessary exists in the present instant and it is not necessary can be detected by available tools of an identification and research.

4.4.2. Quasilaplacian a field

As is known, the field of vectors $\overline{a}(r)$ is named laplacian, if in anyone it to a point is fulfilled equalities $\mathbf{rot}\,\overline{a} = 0$ and $\mathbf{div}\,\overline{a} = 0$, i.e. the laplacian field is simultaneously both potential and solenoidal, and in a symply connected region completely is defined by a scalar potential φ. And, $\Delta\varphi = 0$ (as, if $\mathbf{rot}\,\overline{a} = 0$ and $\overline{a} = \nabla\varphi$ for a symply connected region, $\mathbf{div}\,\overline{a} = \mathbf{div}\,\nabla\varphi = \Delta\varphi = 0$). The research of a potential φ is based on the following properties of potential functions:

– if in area G, limited σ, function everywhere harmonic, $\iint\limits_{\sigma} \frac{\partial\varphi}{\partial n} \cdot d\sigma = 0$;

initial triple integral.

– if φ and ψ – potential functions everywhere in area v limited surface σ, $\iint_{\sigma} \varphi \cdot \frac{\partial \psi}{\partial n} \cdot d\sigma = \iint_{\sigma} \psi \cdot \frac{\partial \varphi}{\partial n} \cdot d\sigma$;

– the function φ, harmonic inside G, can be found in any point G (on values φ and $\frac{\partial \varphi}{\partial n}$ on the boundary σ area G) under the formula

$$4\pi\rho(M_0) = \iint_{\sigma} \left(\varphi \frac{\partial}{\partial n}\left(\frac{1}{r}\right) - \frac{1}{r}\frac{\partial \varphi}{\partial n} \right) d\sigma;$$

– the equation $\Delta\varphi = 0$ has a unique value in area G, if on the boundary (σ) φ accepts the given values.

Let φ – potential of an irrotational flow of an incompressible liquid with a denseness ρ, i.e.

$$\overline{V} = \nabla\varphi, \ \mathbf{div}\,\overline{V} = 0, \ \Delta\varphi = 0, \ \mathbf{rot}\,\overline{V} = 0,$$

where $\overline{V}$ – velocity.

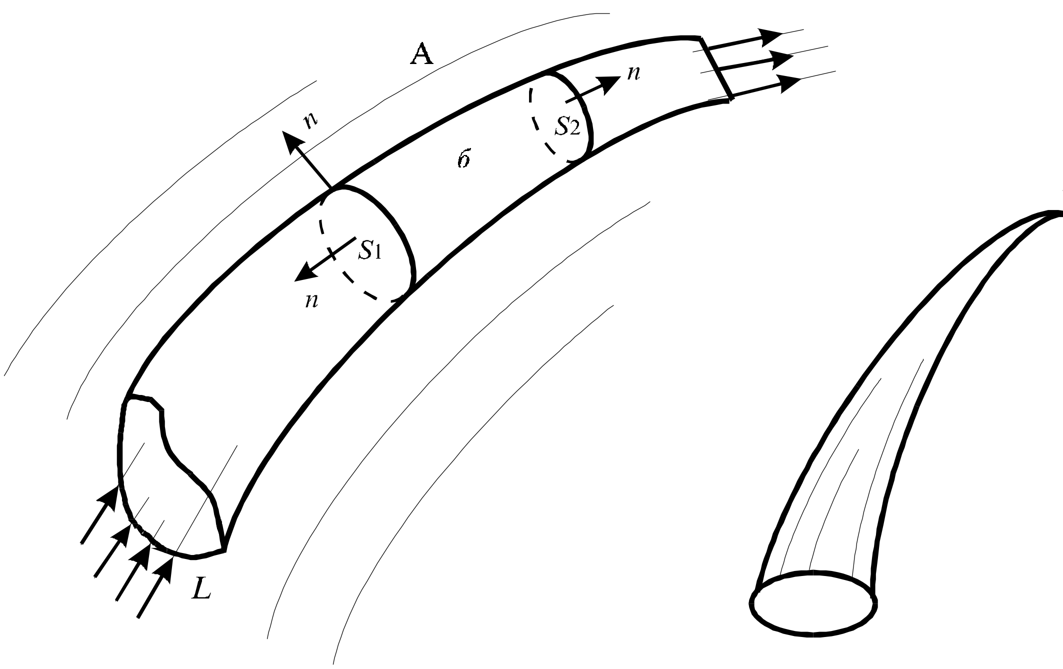


Fig. 5. The intensity of a vectorial handset of a solenoidal field is fixed along all handset. The vectorial handset of a solenoidal field cannot terminate or to begin in a field.

Then from a property of potential functions follows, that the *intensity* of handsets of a current or vectorial handsets (Fig. 5), i.e. field, derivated by vectorial lines, $\overline{V}$, is *fixed* on length of handsets; in a symply connected region limited to rigid walls, on which $\frac{\partial\varphi}{\partial n}=0$, there cannot be a continuous irrotational flow; the kinetic energy of some volume of a liquid G, limited by a surface σ, can be calculated under the formula

$$W_k=\frac{\rho}{2}\iint_{\sigma}\varphi\frac{\partial\varphi}{\partial n}d\sigma,$$

that follows from the definition

$$W_k=\frac{\rho}{2}\iiint_{\sigma}V^2dG$$

and equality $\overline{V}=\nabla\varphi$ $\left(W_k=\frac{\rho}{2}\iiint_{\sigma}(\nabla\varphi)^2dG\right)$.

Let's reflection $\overline{V}$, $\mathbf{div}\overline{V}$, $\Delta\varphi$, $\mathbf{rot}\overline{V}$, W_k from ω_1 in ω_0:

$$\overline{V}=\nabla\varphi=\mathbf{grad}\varphi\setminus\omega_1\to\omega_0\setminus V_*=k^{\sum\limits_{i=1}^{n}\varphi'_{x_i}\cdot\overline{e_{i*}}};$$

$$\mathbf{div}\overline{V}\setminus\omega_1\to\omega_0\setminus D_*=k^{\sum\limits_{i=1}^{n}\left(V_{x_i}\right)'_{x_i}};\ \Delta\varphi\setminus\omega_1\to\omega_0\setminus\Delta_0\varphi,$$

i.e. $$\ln\Delta_0\varphi=\ln k\cdot\sum_{i=1}^{n}x_i\cdot\left(\frac{\partial\ln\varphi}{\partial x_i}+x_i\cdot\frac{\partial^2\ln\varphi}{\partial x_i^2}\right);\qquad(4.20)$$

$$\mathbf{rot}\overline{V}\setminus\omega_1\to\omega_0\setminus R_*=k^{\left(\mathbf{rot}\overline{V}\right)_0},$$ i.e.

$$\log_k R_* = \left({}'(V_z)_y - {}'(V_y)_z \right) \cdot \overline{i_*} + \left({}'(V_x)_z - {}'(V_z)_x \right) \cdot \overline{j_*} +$$

$$+ \left({}'(V_y)_x - {}'(V_x)_y \right) \cdot \overline{k_*};$$

$$W_k \setminus \omega_1 \to \omega_0 \setminus W_{k*} =$$

$$= \exp\left(\int_a^b \log_k \left(\exp\left(\int_{\varphi_1(x)}^{\varphi_2(x)} \log_k \left(\exp\left(\int_{\psi_1(x,y)}^{\psi_2(x,y)} \frac{\log_k \frac{\rho \cdot V \cdot V}{2}}{z} \cdot dz \right) \right) \times \right.\right.\right.$$

$$\left.\left.\left. \times \frac{dy}{y} \right) \right) \cdot \frac{dx}{x} \right), \qquad (4.21)$$

where $V = V(x,y,z)$, $a,b,\varphi_1,\varphi_2,\psi_1,\psi_2$ – limits of an integration, which are at reviewing area G integration in firstimage, i.e. in expression $W_k = \iiint\limits_G \frac{\rho V^2}{2} dx\,dy\,dz$.

Quasivectorial (V_*, R_*) and quasiscalar $(D_*, \Delta_0 \varphi, W_{k*})$ the magnitudes are performances quasifield of a velocity of an irrotational flow of an incompressible liquid.

The mathematical model quasifield (4.20) has the identified physical sense consisting in a possibility of presence at any physical performance of a field of analog magnitudes (infinite of a spectrum of identifiers) of a new nature with modified dimensionalities.

Alternate to the classical monofigurative judgment about physical subservers the infinite-spectral exposition last promotes more global understanding of an objective reality.

The problem of polyfigurative physical thinking is connected as to difficulty of overcoming of a barrier between customary and qualitatively by new representation about objects, and lack of a unique experimental material on the given problem. And, nevertheless, just this problem creates fundamental premises for existence of the concept of a set of identical objects.

4.4.3. Basic theorem quasivectorial of the analysis

Anyone continuous quasivectorial the field $A_*(r)$, *given in all space and vanishing on infinity together with quasidivergence and quasicurl can be represented by a unique image as a product quasipotential* $A_{1_*}(r)$, *and quasisolenoidal* $A_{2_*}(r)$ *fields* $\left(A_*(r) = A_{1_*}(r) \cdot A_{2_*}(r)\right)$.[39]

Thus a basic condition of a potential field is the equality $\mathbf{rot}\,\overline{a_1}(r) = 0$, and quasipotential expresss so: $\left(\mathbf{rot}\,\overline{a_1}(r)\right)_0 = 0$, i.e. $\left({}'(a_{1z})_y - {}'(a_{1y})_z\right) \cdot \overline{i_*} +$
$+\left({}'(a_{1x})_z - {}'(a_{1z})_x\right) \cdot \overline{j_*} + \left({}'(a_{1y})_x - {}'(a_{1x})_y\right) \cdot \overline{k_*} = 0$.

Condition quasisolenoidal the following:

$$\sum_{i=1}^{n} {}'\left(a_{2x_i}\right)_{x_i} = 0.$$

Not stopping on a proof of this theorem, we shall remark, that the outcome is possible to receive by a way ω-reflection $\backslash \omega_1 \to \omega_0 \backslash$ basic theorem of a vector analysis.

4.4.4. ω-image of an electromagnetic field

Let $\overline{E}$ and $\overline{H}$ – vectors of strength of electrical and magnetic fields being functions of a point and time; $\rho = \rho(r,t)$ – density function of charges; $\overline{j} = \overline{j}(r,t)$ – vectorial function a current density $\overline{j} = \overline{j}(r,t)$. Let's consider an electromagnetic field in vacuum.

The connection $\overline{E}$ and $\overline{H}$ is known:

$$\frac{1}{c} \cdot \frac{\partial}{\partial t} \iint_{\sigma L} \overline{H} \cdot \overline{n} \cdot d\sigma = -\int_L \overline{E} \cdot dl, \tag{4.22}$$

[39] Superposition of logs of fields, i.e in this case is realized.

$$\log_k A_*(r) = \log_k A_1(r) \cdot \log_k A_2(r).$$

$$\iint\limits_{\sigma L} \overline{H} \cdot \overline{n} \cdot d\sigma = 0, \tag{4.23}$$

where $c \approx 3{,}28 \cdot 10^8 \dfrac{\text{yd}}{\text{s}}$ $\left(c \approx 3 \cdot 10^8 \dfrac{\text{m}}{\text{s}}\right)$. The equation (4.22) states, that the modification in time of a stream of a magnetic field $\iint\limits_{\sigma L} \overline{H} \cdot \overline{n} \cdot d\sigma$ through a surface σ, leaning on an outline L, is equal to circulation of an electrical field $\varepsilon = \int\limits_{L} \overline{E} \cdot dl$ along an outline L ($\varepsilon -$ electromotive force and the first equation is a law of an electromagnetic induction M. Faraday). The equation (4.23) shows, that the stream of a magnetic field through a closed surface σ_C Arbitrary form is always equal to zero. Applying the formula Stokes, we shall receive $\iint\limits_{\sigma L} \left(\dfrac{1}{c} \cdot \dfrac{\partial \overline{H}}{\partial t} + \textbf{rot}\,\overline{E}\right) \cdot \overline{n} \cdot d\sigma = 0$ and by virtue of an arbitrary σ_L,

$$\frac{\partial \overline{H}}{\partial t} = -c \cdot \textbf{rot}\,\overline{E}\,. \tag{4.24}$$

Under the formula of the Ostrogradskii $\iint\limits_{\sigma V} \overline{H} \cdot \overline{n} \cdot d\sigma = \iiint\limits_{V} \textbf{div}\,\overline{H} \cdot dV = 0$,

whence, by virtue of an arbitrary of volume V, $\textbf{div}\,\overline{H} = 0$. (4.25)

The equations (4.22, 4.23) and (4.24, 4.25) are named as a *homogeneous* pair of the equations of the Maxwell accordingly in the integrated and differential forms.

The connection of vectors $\overline{E}$ and $\overline{H}$ with a density function of charges ρ And current $\overline{j}$ Is defined by an *inhomogeneous* pair of the equations of the Maxwell:

$$\frac{\partial}{\partial t} \iint\limits_{\sigma L} \overline{E} \cdot \overline{n} \cdot d\sigma = c \cdot \int\limits_{L} \overline{H} \cdot dl - 4\pi \iint\limits_{\sigma L} \overline{j} \cdot \overline{n} \cdot d\sigma \tag{4.26}$$

$$\iint\limits_{\sigma L} \overline{E} \cdot \overline{n} \cdot d\sigma = 4\pi \iiint\limits_{V} \rho \cdot dV \tag{4.27}$$

Applying to it is possible to write (4.26) formula Stokes,

$$\frac{\partial \overline{E}}{\partial t} = c \cdot \textbf{rot}\,\overline{H} - 4\pi \overline{j} \tag{4.28}$$

To (4.27) applying the formula of the Ostrogradskii, we shall receive (by virtue of an arbitrary of volume V):

$$\mathbf{div}\,\overline{E} = 4\pi\rho \tag{4.29}$$

The equations (4.28) and (4.29) are named as an *inhomogeneous pair* of the equation of the Maxwell in the differential form, and the systems (4.22, 4.23, 4.26, 4.27) and (4.24, 4.25, 4.28, 4.29) represent a set of equations of the Maxwell accordingly in the integrated and differential forms. This system describes an electromagnetic field in vacuum.

The reflection of a system from ω_1 in ω_0 is carried out by analogy with above-stated: a) all vectorial performances (grad, div, rot etc.) are reflections under the known already formulas; b) ω-images of integrals (multiple, surface etc.) are best for noting as ω-images of repeated integrals, sometimes using thus ω-images of the formulas of a Ostrogradskogo-Gauss, Stokes and Green; c) differential objects are reflections transformed ω-images of a derivative; d) the constants are easier for introducing under a sign of integro-differential objects – fundamental principles to not make an error at an operation in terms ω-transformations etc.

As an example we shall reduce a little ω-images of the equations:

$$\frac{\partial\overline{H}}{\partial\tau} + c\cdot\mathbf{rot}\,\overline{E} = 0 \setminus \omega_1 \to \omega_0 \setminus k^{{}'\left(\overline{H}\right)_\tau}\cdot\left(c\odot k^{\left(\mathbf{rot}\,\overline{E}\right)_0}\right) = 1 \quad\Rightarrow$$

$$\Rightarrow {}'\left(\overline{H}\right)_\tau + \log_k c\left(\left({}'\left(E_z\right)_y - {}'\left(E_y\right)_z\right)\cdot\overline{i_*} + \left({}'\left(E_x\right)_z - {}'\left(E_z\right)_x\right)\cdot\overline{j_*} + \right.$$

$$\left. + \left({}'\left(E_y\right)_x - {}'\left(E_x\right)_y\right)\cdot\overline{k_*} = 0; \right. \tag{4.30}$$

$$\mathbf{div}\,\overline{H} = 0 \setminus \omega_1 \to \omega_0 \setminus k^{\sum\limits_{i=1}^{n}{}'\left(H_{x_i}\right)_{x_i}} = 1 \tag{4.31}$$

$$\left(\overline{H} = \sum_{i=1}^{n} H_{x_i}\cdot\overline{e_i}\right);$$

$$\iint\limits_{\sigma V}\overline{H}\cdot\overline{n}\cdot d\sigma = 0 \quad \setminus \omega_1 \to$$

$$\to \omega_0 \setminus \exp\left(\int_a^b \log_k\left(\exp\left(\int_{\varphi_1(x)}^{\varphi_2(x)} \log_k\left(\exp\left(\int_{\psi_1(x,y)}^{\psi_2(x,y)} \frac{\log_k f(x,y,z)}{z}\cdot dz\right)\right)\times\right.\right.\right.$$

$$\left.\left.\left.\times\frac{dy}{y}\right)\right)\cdot\frac{dx}{x}\right), \tag{4.32}$$

where $f(x,y,z)=\mathbf{div}\,\overline{H}$, and the limits of an integration a, b, φ_1, φ_2, ψ_1, ψ_2 turn out at reviewing area of an integration V, limited surface σ_V.

Other equations of an initial system are similarly reflections also. For example,

$$\iint_{\sigma V} \overline{E}\cdot\overline{n}\cdot d\sigma = 4\pi\iiint_V \rho\cdot dV \setminus \omega_1 \to \omega_0 \setminus I_{xy*}\cdot I_{xz*}\cdot I_{yz*} =$$

$$=\exp\left(\int_a^b \log_k \exp\left(\int_{\varphi_1(x)}^{\varphi_2(x)} \log_k\left(\exp\left(\int_{z_1(x,y)}^{z_2(x,y)} \frac{\log_k(4\pi\cdot\rho(x,y,z))}{z}\cdot dz\right)\right)\times\right.\right.$$

$$\left.\left.\times\frac{dy}{y}\right)\cdot\frac{dx}{x}\right), \tag{4.33}$$

where
$$I_{xy*}=\exp\left(\int_{a1}^{b1}\log_k\left(\exp\left(\int_{\varphi_1(x)}^{\varphi_2(x)}\frac{\log_k E_z}{y}dy\right)\right)\cdot\frac{dx}{x}\right),$$

$$I_{xz*}=\exp\left(\int_{a2}^{b2}\log_k\left(\exp\left(\int_{\psi_1(x)}^{\psi_2(x)}\frac{\log_k E_y}{z}dz\right)\right)\cdot\frac{dx}{x}\right),$$

$$I_{yz*}=\exp\left(\int_{a3}^{b3}\log_k\left(\exp\left(\int_{\chi_1(y)}^{\chi_2(y)}\frac{\log_k E_x}{z}dz\right)\right)\cdot\frac{dy}{y}\right);$$

$$\frac{\partial E}{\partial t} = c\cdot\mathbf{rot}\,\overline{H} - 4\pi\overline{j}\setminus\omega_1 \to \omega_0 \setminus k^{'E_t} = \frac{c\odot k^{(\mathbf{rot}\,\overline{H})_0}}{k^{4\pi}\odot\overline{\delta}} \quad\Rightarrow$$

$$\Rightarrow t\frac{\partial E}{\partial t} = \log_k c\cdot\left(\left({}'(H_z)_y - {}'(H_y)_z\right)\cdot\overline{i_*} + \left({}'(H_x)_z - {}'(H_z)_x\right)\times\right.$$

$$\left.\times\overline{j_*} + \left({}'(H_y)_x - {}'(H_x)_y\right)\cdot\overline{k_*}\right) - 4\pi\overline{\delta_*}, \tag{4.34}$$

where $\overline{\delta_*} = \log_k\overline{\delta}$, and $\overline{\delta} \equiv \overline{j}$ – current density (the new label $\overline{\delta}$ is entered to not confuse a current density with a single vector $\overline{j}$).

$$\mathbf{div}\,\overline{E} = 4\pi\cdot\rho\setminus\omega_1 \to \omega_0\setminus k^{\sum\limits_{i=1}^{n}{}'(E_{xi})_{xi}} = k^{4\pi\rho_0} \quad\Rightarrow$$

$$\Rightarrow \quad \sum_{i=1}^{n}{}'(E_{xi})_{xi} = 4\pi\rho_0, \tag{4.35}$$

where ρ_0 – ω-image ρ.

The equations (4.30…4.35) describe electromagnetic quasifield in vacuum and are ω-image of this electromagnetic field.

4.4.5. ω-images of scalar and vectorial potentials electromagnetic fields

As is known ([35], page 236-237) in an electrodynamics the important role is played by auxiliary functions: scalar $\varphi = \varphi(r,t)$ and vectorial $\overline{a} = \overline{a}(r,t)$ potentials. Let's enter them so that they satisfied to a homogeneous pair of the equations of the Maxwell.

$$\frac{\partial\overline{H}}{\partial t} + c\cdot\mathbf{rot}\,\overline{E} = 0 \tag{4.24}$$

$$\mathbf{div}\,\overline{H} = 0 \tag{4.25}$$

As $\mathbf{div}\,\mathbf{rot}\,\overline{H} = 0$, $\overline{H}(r,t) = \mathbf{rot}\,\overline{a}(r,t)$ and after a substitution in (4.24) we shall receive $\mathbf{rot}\left(\frac{1}{c}\frac{\partial\overline{a}}{\partial t} + \overline{E}\right) = 0$ (4.36), whence $\overline{E} + \frac{1}{c}\frac{\partial\overline{a}}{\partial t} = -\nabla\varphi$ (4.37), where $\varphi-$arbitrary function from r and t (in this case used equality $\mathbf{rot}\,\mathbf{grad}\,\varphi=0$).

Replacement $\bar{a}$ on $a+\nabla f$ and simultaneously replacement φ on $\varphi-\frac{1}{c}\frac{\partial f}{\partial t}$ do not change vectors $\bar{E}$ and $\bar{H}$ $\left(\bar{H}=\mathbf{rot}\,\bar{a}=\right.$ $\left.=\mathbf{rot}\left(\bar{a}+\nabla f\right)=\mathbf{rot}\,a+\mathbf{rot}\,\mathbf{grad}\,\varphi\right.$ and

$$\bar{E}=-\frac{1}{c}\cdot\frac{\partial\bar{a}}{\partial t}-\nabla\varphi=-\frac{1}{c}\cdot\frac{\partial\left(\bar{a}+\nabla f\right)}{\partial t}-\nabla\left(\varphi-\frac{1}{c}\frac{\partial f}{\partial t}\right)=$$

$$\left.=-\frac{1}{c}\cdot\frac{\partial\bar{a}}{\partial t}-\frac{1}{c}\nabla\frac{\partial f}{\partial t}-\nabla\varphi+\frac{1}{c}\nabla\frac{\partial f}{\partial t}\right).$$

Substituting (4.36) and (4.37) in an inhomogeneous pair of the equations of the Maxwell (4.28), (4.29), we shall receive the equations, with which should satisfy scalar φ and vectorial $\bar{a}$ potentials:

$$\frac{\partial}{\partial t}\left(-\frac{1}{c}\cdot\frac{\partial\bar{a}}{\partial t}-\nabla\varphi\right)=c\cdot\mathbf{rot}\,\mathbf{rot}\,\bar{a}-4\pi\cdot\bar{j},\tag{4.38}$$

$$\mathbf{div}\left(-\frac{1}{c}\cdot\frac{\partial\bar{a}}{\partial t}-\nabla\varphi\right)=-4\pi\cdot\rho.\tag{4.39}$$

It is possible, obviously, not limiting a generality, to enter arbitrary function f and to impose on φ and $\bar{a}$ a side condition (Lorenz):

$$\mathbf{div}\,\bar{a}+\frac{1}{c}\cdot\frac{\partial\varphi}{\partial t}=0\tag{4.40}$$

Whence $\mathbf{div}\left(a+\nabla f\right)+\frac{1}{c}\cdot\frac{\partial\varphi}{\partial t}=0$ (4.41), and the arbitrary function ∇f is rendered concrete and is determined from this equation:

$$\Delta f=-\frac{1}{c}\cdot\frac{\partial\varphi}{\partial t}-\mathbf{div}\,\bar{a}.\tag{4.42}$$

Using known equality $\mathbf{rot}\,\mathbf{rot}\,\bar{a}=\mathbf{grad}\,\mathbf{div}\,\bar{a}-\Delta\bar{a}$ and relation (4.40) and (4.42), we shall receive from (4.38) and (4.39) equations for the definition $\bar{a}$ and φ, and consequently $\bar{E}$ and $\bar{H}$:

$$\Delta\bar{a}-\frac{1}{c^2}\cdot\frac{\partial^2\bar{a}}{\partial t^2}=-\frac{4\pi}{c}\bar{j} \qquad (4.43)$$

$$\Delta\varphi-\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2}=-4\pi\cdot\rho\,. \qquad (4.44)$$

In this case introduce an operator d'Alembert:

$$\Box\equiv\Delta-\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\,. \qquad (4.45)$$

Then (4.43) and (4.44) will accept an aspect:

$$\Box\bar{a}=-\frac{4\pi}{c}\cdot\bar{j} \qquad (4.46)$$

$$\Box\varphi=-4\pi\cdot\rho \qquad (4.47)$$

It is wave equations for potentials.

Let's discover ω- Images $\Box$, $\Box\bar{a}$, $\Box\varphi$.

$$\ln\Box_*=\ln^2 k\cdot\left(\left(\sum_{i=1}^{n}\left(x_i\frac{\partial\ln}{\partial x_i}+x_i{}^2\frac{\partial^2\ln}{\partial x_i{}^2}\right)\right)-\frac{1}{c^2}t\left(\frac{\partial\ln}{\partial t}+t\frac{\partial^2\ln}{\partial t^2}\right)\right).$$

Let's explain this expression.

$$\Box\equiv\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\backslash\omega_1\to\omega_0\backslash\Box_*=\frac{\Delta_0}{\left(k\triangle k^{c^2}\right)\odot\,\overset{''}{-}\left(\partial^2\right)_{t^2}}=$$

$$=\frac{\Delta_0}{\sqrt[c^2]{k}\odot k^{\ln k\cdot t\cdot\left(\frac{\partial\ln}{\partial t}+t\cdot\frac{\partial^2\ln}{\partial t^2}\right)}},$$

where $\Box_*$ $-$*quasioperator d'Alembert,* Δ_0 $-$*quasilaplacian,* $\overset{''}{-}\left(\partial^2\right)_{t^2}=$ $=k^{\ln k\cdot t\cdot\left(\frac{\partial\ln}{\partial t}+t\cdot\frac{\partial^2\ln}{\partial t^2}\right)}$ $-\ \omega$- image of a derivative $\dfrac{\partial^2}{\partial t^2}$.

Whence

$$\square_* = k^{\ln k \cdot \left(\left(\sum_{i=1}^{n} \left(x_i \frac{\partial \ln}{\partial x_i} + x_i{}^2 \frac{\partial^2 \ln}{\partial x_i{}^2} \right) \right) - \frac{1}{c^2} t \left(\frac{\partial \ln}{\partial t} + t \frac{\partial^2 \ln}{\partial t^2} \right) \right)}. \quad (4.48)$$

Let's image $\square \bar{a}$ And $\square \varphi$

$$\square \bar{a} \setminus \omega_1 \to \omega_0 \setminus \square_* A_*,$$

$$\square \bar{a} + \frac{4\pi}{c} \bar{j} = 0 \setminus \omega_1 \to \omega_0 \setminus B_* = (\square_* A_*) \cdot \left(k^{\frac{4\pi}{c}} \odot \bar{\delta} \right) = 1 \quad \Rightarrow$$

$\Rightarrow \quad \ln B_* = 0$, i.e.

$$\ln k \cdot \left(\left(\sum_{i=1}^{n} \left(x_i \frac{\partial \ln a_{x_i}}{\partial x_i} + x_i{}^2 \frac{\partial^2 \ln a_{x_i}}{\partial x_i{}^2} \right) \right) - \frac{1}{c^2} t \left(\frac{\partial \ln a_{x_i}}{\partial t} + \right.\right.$$

$$\left.\left. + t \frac{\partial \ln a_{x_i}}{\partial t} \right) \right) + \frac{4\pi}{c} \cdot \overline{\delta_*} = 0, \quad (4.49)$$

where $\bar{\delta} \equiv \bar{j} -$ current density;

$A_* = k^{\sum_{i=1}^{n} (\log_k a_{x_i}) \cdot \overline{e_{i*}}}$, and $\square_* A_*$ is formed from primary image $\bar{a}$ quasivectorial A_* way entries in ω-images of partial derivatives on a variable x_i appropriate projections of a vector $\bar{a}$.

At last,

$$\ln C_* = \ln\left((\square_* \Phi_*) \cdot k^{4\pi \cdot \rho} \right) = 0 \quad \Rightarrow \quad \ln C_* =$$

$$= \ln k \cdot \left(\left(\sum_{i=1}^{n} \left(x_i \frac{\partial \ln \varphi}{\partial x_i} + x_i{}^2 \frac{\partial^2 \ln \varphi}{\partial x_i{}^2} \right) \right) - \frac{1}{c^2} t \left(\frac{\partial \ln \varphi}{\partial t} + t \frac{\partial^2 \ln \varphi}{\partial t^2} \right) \right) +$$

$$+ 4\pi \cdot \rho = 0, \quad (4.50)$$

I.e. $\left(\sum_{i=1}^{n}\left(x_i\frac{\partial \ln\varphi}{\partial x_i}+x_i{}^2\frac{\partial^2\ln\varphi}{\partial x_i{}^2}\right)\right)-\frac{1}{c^2}t\left(\frac{\partial \ln\varphi}{\partial t}+t\frac{\partial^2\ln\varphi}{\partial t^2}\right)+\ +\frac{4\pi\rho}{\ln k}=0,$

(4.51)

where Φ_* – *quasipotential* (ω-image of a potential φ $(\backslash\omega_1\rightarrow\omega_0\backslash)$), C_* – ω-image $(\backslash\omega_1\rightarrow\omega_0\backslash)$ expression $\Box\varphi+4\pi\rho=0$, i.e. $C_*=1$.

(4.49) and (4.50) - wave equations for quasipotentials electromagnetic quasifield. The equations (4.30…4.35) and (4.49, 4.51), practically, completely characterize electromagnetic quasifield in vacuum. Similarly, it would be possible to find and additional performances – quasienergy and quasivectorial electromagnetic quasifield.

4.4.6. ω-image of thermodynamic magnitudes

In a thermodynamics are known ([45], page 376-379) ten basic magnitudes: *pressure* (p), *volume* (v), *temperature* (T), *entropy* (S), *interior energy* (U), *enthalpy* (H), *free energy* (F), *thermodynamic potential* (Z), *amount of heat* (Q) and *work*, made by a thermodynamic system, (A). Each can be deduced(removed) from these magnitudes as function two others. For an entry of these relations the concept of a *jacobian* is introduced.

If two functions $u=u(x,y)$ and $v(x,y)$ are given, the jacobian $(J(u,v))$ is equal:

$$J(u,v)=\frac{D(u,v)}{D(x,y)}=\begin{vmatrix}\left(\frac{\partial u}{\partial x}\right) & \left(\frac{\partial u}{\partial y}\right)\\ \left(\frac{\partial v}{\partial x}\right) & \left(\frac{\partial v}{\partial y}\right)\end{vmatrix} \tag{4.52}$$

(The index below means, that the partial derivative undertakes, when this magnitude a constant, i.e. $\left(\frac{\partial u}{\partial x}\right)_y$ – means, that $y=const$).

The formula $J(x,y)\cdot J(z,w)+J(y,z)\cdot J(x,w)+J(z,x)\cdot J(y,w)=0$, (4.53) is known connecting any thermodynamic variables x,y,z,w.

Let's discover ω-image of the formula (4.52):

$$J(u,v)=\left(\frac{\partial u}{\partial x}\right)\cdot\left(\frac{\partial v}{\partial y}\right)-\left(\frac{\partial u}{\partial y}\right)\cdot\left(\frac{\partial v}{\partial x}\right)\backslash\,\omega_1\to\omega_0\,\backslash\,J_*=$$

$$=\frac{k^{\left({}'u_x\right)}\odot k^{\left({}'v_y\right)}}{k^{\left({}'u_y\right)}\odot k^{\left({}'v_x\right)}}=k^{\left({}'u_x\right)\cdot\left({}'v_y\right)-\left({}'u_y\right)\cdot\left({}'v_x\right)} \qquad (4.54)$$

Then $\log_k J_*=\left({}'u_x\right)\cdot\left({}'v_y\right)-\left({}'u_y\right)\cdot\left({}'v_x\right)$, where J_*- *quasijacobian.*

Let's discover ω-image of the formula (4.53):

$$J(x,y)\cdot J(z,w)+J(y,z)\cdot J(x,w)+J(z,x)\cdot J(y,w)=0\,\backslash\,\omega_1\to$$

$$\to\omega_0\,\backslash\left(k^{J(\log_k x,\log_k y)}\odot k^{J(\log_k z,\log_k w)}\right)\times$$

$$\times\left(k^{J(\log_k y,\log_k z)}\odot k^{J(\log_k x,\log_k w)}\right)\times$$

$$\times\left(k^{J(\log_k z,\log_k x)}\odot k^{J(\log_k y,\log_k w)}\right)=$$

$$=k^{J(x_*,y_*)\cdot J(z_*,w_*)+J(y_*,z_*)\cdot J(x_*,w_*)+J(z_*,x_*)\cdot J(y_*,w_*)},$$

where $x_*=\log_k x$, $y_*=\log_k y$ etc.

Whence

$$J(x_*,y_*)\cdot J(z_*,w_*)+J(y_*,z_*)\cdot J(x_*,w_*)+$$

$$+J(z_*,x_*)\cdot J(y_*,w_*)=0. \qquad (4.55)$$

The formulas (4.54) and (4.55) allow completely to describe infinite a spectrum thermodynamic quasifield.

§ 4.5. The concept of a reality quasifield

In *chapter* 2 of the present monography the possibility of the extension of a field of real numbers by a way ω-reflections was shown. For example, from *fractional* numbers R_f is uneasy to receive *negative* R_- $\left(a_1 = (:a) \setminus \omega_0 \to \omega_1 \setminus \log_k (:a) = a_2 \in R_-,\right.$ where $(a,k) \in R_+$, $(k,a) > 1$, $R_f = \{:a\} \setminus \omega_0 \to \omega_1 \setminus R_- = \left.\{\log_k (:a)\}\right)$; from negative – Δ-*number* R_Δ $\left(a_2 \setminus \omega_0 \to \omega_1 \setminus \log_k a_2 = a_3 \in R_\Delta,\ k \neq 1,\ k \in R,\right.$ $a_2 \in R_-$; $R_- = \{a_2\} = \{\log_k (:a)\} \setminus \omega_0 \to$ $\to \omega_1 \setminus R_\Delta = \{\log_k a_2\} = \left.\{\log_k \log_k (:a)\}\right)$; from Δ-numbers – $\mathbb{\Delta}$ – *number* $R_{\mathbb{\Delta}}$ $\left(a_3 \setminus \omega_0 \to \omega_1 \setminus \log_k a_3 = a_4 \in R_{\mathbb{\Delta}},\ k \neq 1,\ k \in R,\right.$ $a_3 \in R_\Delta$;

$$R_\Delta \setminus \omega_0 \to \omega_1 \setminus R_{\mathbb{\Delta}} = \{\log_k a_2\} = \left.\{\log_k \log_k \log_k (:a)\}\right), \text{ etc.}$$

Infinite a spectrum of fields various Δ-*numbers* $(R_\Delta, R_{\mathbb{\Delta}}, R_{\Delta i}, \ldots)$, constructed by reflexive reflection of a *field* real number $\mathbf{R}$ rather $(-\infty)$, *overfields* (for example, overfield $\mathbf{R}_0 = \Delta_0 \cup \mathbf{R}$ rather $(\Delta\infty)$); *images of functions* generated on sets Δ-arguments) etc. means, that a certain object similar to this line-up of objects. For example, *quasifield* it is expedient to perceive as the *possibility* of existence infinite. In connection with infinite by the extension of fields is inevitable infinite of images of any functional or correlation assotiation, i.e. the physical object made in the mathematical form, has infinite a spectrum quasi-physical ω-images).

In *chapter* 3 the same situation was illustrated with integro-differential objects. The derivative has a uncountable set ω-images in space ω_0, i.e. exists infinite a spectrum ω-images of any differential equation, and consequently, and infinite a spectrum it of solutions, each of which simulates some objective reality. Last, obviously, does not exist as the forms of a substance, in our representation about her. Principal a moment in understanding of a substance in case of transformation it in quasisubstance[41] at ω-reflections is the statement, that *everyone*

[41] Under *quasisubstance* an outcome ω – reflections material here is understood. In extended representation in quasisubstance all abstract objects are included.

logically valid mathematical expression including physical magnitudes, identifies some physical reality.

Let's consider the most *elementary* example. Let in space ω_1 the segment AB, divided by a point C on two parts is given. Let's reflection it in space ω_0, if $|AC| = a_1$, $|CB| = a_2$, $|AB| = a$ $(a = a_1 + a_2)$, and the outcome of reflection ω_1 in ω_0 length of a segment $|AB|$ will be square of a rectangle CAA_1B (fig. 6): $a_1 + a_2 \setminus \omega_1 \to \omega_0 \setminus k^{a_1+a_2} = k^{a_1} \cdot k^{a_2} = a_1' \cdot a_2'$, where $a_1' = k^{a_1}$, and $a_2' = k^{a_2}$.

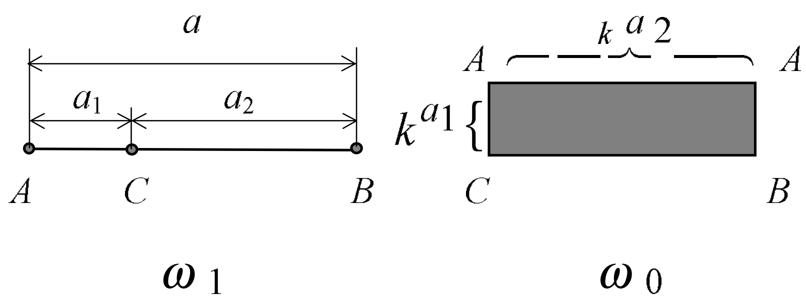


Fig. 6. Component ω-reflection $(\setminus \omega_1 \to \omega_0 \setminus)$ segment AB, divided on *two* part, from space ω_1 in space ω_0 (in case of a partition of a segment AB on *two* parts a_1 and a_2 $(|ab| = a = a_1 + a_2)$).

The segment AB on n equal parts is separable: $|AC_1| = |C_1C_2| = \ldots =$ $= |C_{n-1}B| = \Delta a$, where $\Delta a = a/n$. Let's reflection a segment AB from ω_1 in ω_0 under condition of a partition it in space ω_1. It is uneasy to show, that at division of a segment AB $(|AB| = a)$ on three and more parts we shall receive a skew field (at $n = 3$) and *hypervolume* (at $n > 3$) with identical legs located on axeses of coordinates. Accepting length of an elementary segment tending to zero $(\Delta a \to 0)$, and total number of segments to infinity $(n \to \infty)$, we obtain length of each displayed segment to equal unit $(k^0 = 1)$. So the projection on a plane XOY will represent guadrate, and in three-dimensional space - cube with a leg of an edge equal to unit (square of a figure, being projection гипертел, is equal 1).

Let's remark, that, reflection in ω_0 from ω_1 “segment” of zero length $\left(|AB| = 0\right)$, we shall receive single ω-image, *i.e. at passage from* ω_i $(i \neq 0)$ *in* ω_0 *geometric objects and, obviously, the physical magnitude equal to zero, gain final really “felt” (“appreciable”), i.e. not equal to zero, value (for example, at reflection* $\backslash \omega_1 \to \omega_0 \backslash$ *this value - unit) and, on the contrary, final can be transformed in* ***zero***.

Incidentally we shall remark, that the dimensionality of space is included into understanding last from a position ω-reflection. Thus the *space can have any dimensionality from zero ad infinitum*. And, the concrete dimensionality it depends, on a device intellectual - psyhological of means of the observer, i.e. from it of ability of *vision* (perception) of a real …

So, summarizing, it is possible to mark the following hypothetical aspects (postulates) of the concept ω-images of a reality.

1. The real space supposes existence anyone ω-space, i.e. it can be considered as infinite a spectrum combined (inserted) ω-spaces.

2. Everyone ω-space can be considered as ω_0 (arbitrary of a choice of zero), as all ω-spaces are related on interior connections.

3. An outcome ω-reflection of zero can be any number $\left(0 \backslash \omega_i \to \omega_{i-1} \backslash k^0 = 1 \backslash \omega_{i-1} \to \omega_{i-2} \backslash k\right.$ etc.).

Similarly, at ω-passages of certain physical magnitudes the transformation from zero it of a value to anyone another (is admissible at a modification of its essence), i.e. the apparent emerging from “anything” is incorporated in ω-structure of space and existing regularities of reflections of objects in this space.

Let's remark, however, that the distribution of the theory ω-reflection to physical objects at the given stage while is rather problematic.

§ 4.6. A problematics quasivectorial of the analysis

In connection with an extensiveness of the given problem we shall specify only *basic* problems originating at designing and study quasivectorial of the analysis.

1. It is necessary more carefully to work the mechanism ω-reflections of scalar and vectorial functions. Thus it is necessary to *prove establishing moment*, i.e., by considering various ω-images of vectors and scalars (in adjacent ω-space and space of different ranks) to prove the common theorems quasivectorial of the analysis.

2. To realize ω-reflections tensor of the analysis, to generate *quasitensor* the analysis and to lead all proofs according to the recommendations, datas in item 1. And, it is desirable to receive the formulas for n-dimensional of space (in the present work the author was limited sometimes only to three-dimensional space).

3. To give the geometric interpretation to ω-transformation of objects noted as the formulas. To investigate ω-reflections of various integrated objects from vectorial functions and their combinations.

4. To increase number of examples such as examples reduced in item 4.4. To find ω-images of a means of a quantum mechanics. All conclusions are necessary for giving in quasivectorial (or quasitensor) nomenclatures. *To explain physical essence (nature) quasiobjects.* More precisely to justify infinite of images of objects at the expense of the extension of a field of real numbers (see item 4.5.).

5. To lead ω-reflections of transformations of vectors at various modifications of frames. To investigate ω-transformation of vectors in curvilinear frames. To apply quasivectorials of transformation in the identified quasimatrix form.

6. To find a rational entry (symbolics) quasitensor of magnitudes at various ω-reflections.

7. To develop the justified theory n-dimensional of physical objects and quasiobjects (ω-images).

8. To generate the theory quasiobjects in the theory of the Cartan.

9. To find ω-reflections of basic concepts of differential geometry. To study a structure of global space as a superposition every possible physical ω-spaces.

10. To give mathematical exposition infinite спекторов spectrum and superspectrum quasifield with the help of wave functions, taking into account a continuity of spectra on an indication k and discretization them on an indication i, and also presence of the kernel (basic form of an entry of object).

To give global mathematical exposition on the given item in view of various functions of connection. Thus to enter generalizing positions (performance and criterions).

11. To take apart problems of a space temporal continuum in light of representations ω-images *common* and *special* of relativity theories.

CHAPTER 5. A MISCELLANY (APPLICATION)

§ 5.1. The general provisions

In the present chapter the examples of applications of a material explained in the previous chapters are considered. The purpose it to supplement by the last facts permitting deeper to understand a problematics of the monography. The range of considered here problems is rather wide. In connection with a fragmentariness of a research the material of this chapter was solved to select under a title "Miscellany". The subjects of the separate paragraphs is selected so that in chapter 5 alongside with a solution of various problems in a uniform key ω-reflection the personalising of the paragraphs was realized, each of which can be investigated separately without acquaintance with others.

§ 5.2. About division second inversing

In common infinite a set of algebraic operations, about which the speech went above, division, as well as any inverse operation, should be ambiguous. Let's consider some aspects of this new operation. Let's designate by its double forward slash "//" $\left(b = a//c \quad \Rightarrow \quad b \cdot c = a\right)$. An example of this operation can be operation between exponents at a taking the root: $\sqrt[c]{d^b} = d^{b//c}$.

Lemma 5.1. *The operation between exponents at a taking the root is division second inversing:*

$$\boxed{\sqrt[c]{d^b} = d^{b//c}}, \qquad (5.1)$$

where $\{b,c,d\} \in \mathbf{R}$, $c \neq (0;1)$.

Proof. Let $b//c = x$, $(b,c) \in \mathbf{R}$. Then, according to properties Δ-numbers, at $c \in \mathbf{N}_2$ x has two values, which one of real, and second – is Δ-number. If $b//c = m \in \mathbf{R}$, $\sqrt[c]{d^b} = +l$, and at $m \in \Delta_0$ we shall receive $d^m = -l$, as $d^{\Delta p} = -\left(d^p\right)$[42] on properties Δ-numbers $\left(p \in \mathbf{R}, m = \Delta p\right)$.

All this proves, that the division $b//c$ is division second inversing. (As against him the division by first inversing has one value).

---

[42] The equality $d^{\Delta p} = -\left(d^p\right)$ follows from a property Δ-numbers. $\log_k(-d) = = \Delta \log_k d$, where $k, d \in R_+$, $k \neq 1$.

The note. It is possible to reduce little bit other proof of a lemma. Let's designate $\sqrt[c]{a^b} = x$. Then $a^b = x^c$. After a taking the logarithm we shall receive $\log_a\left(a^b\right) = \log_a\left(x^c\right)$. Whence $b = \left(\log_a x\right) \cdot c$[43], i.e. $\log_a x = b//c$ and $x = a^{b//c}$.

Lemma 5.2. *For* $a, b, c \in \mathbf{R}$ *has a place equality*

$$a \cdot (b//c) = (a \cdot b)//c \tag{5.2}$$

Proof. From (5.1) we shall receive

$$\left(d^a\right)^{b//c} = \sqrt[c]{\left(d^a\right)^b} = \sqrt[c]{d^{a \cdot b}} = d^{(a \cdot b)//c}, \text{ i.e.}$$

$$a \cdot (b//c) = (a \cdot b)//c.$$

Corollary. From (5.2) follows

$$a \cdot \Delta b = \Delta(a \cdot b), \tag{5.3}$$

that met already in properties Δ-numbers.

The theorem 5.1. *For* $(a, b) \in \mathbf{R}$ *the equalities take place*

a) $$a \cdot \frac{1}{b} = a//b, \tag{5.4}$$

b) $$\frac{b}{c} \cdot a = \frac{b \cdot a}{c} \tag{5.5}$$

Proof. a). Let $a \cdot \frac{1}{b} = x_1 \Rightarrow a = x_1//(1/b)$. Agrees (5.2) $a = x_1//(1/b) = (x_1 \cdot 1)//(1/b) = x_1 \cdot (1//(1/b))$. Let's designate $1//(1/b) = y$. Then $1 = y \cdot \frac{1}{b} \Rightarrow \frac{1}{y} = \frac{1}{b} \Rightarrow y \equiv b$, i.e. $a = x_1 \cdot y = = x_1 \cdot b \Rightarrow x_1 = a//b \Rightarrow$

[43] The exponent is carried out to the right: $\log\left(x^n\right) = (\log x) \cdot n$, as $\log\left(x^n\right) = \log(x \cdot x \cdot \ldots \cdot x) = \log x + \log x + \ldots + \log x = (\log x) \cdot n$.

$a \cdot \frac{1}{b} = a//b$, as was to be shown.

b). We shall designate $\frac{b}{c} \cdot a = x_2$. Then $\frac{b}{c} = x_2//a \Rightarrow$ $b = c \cdot (x_2//a)$. Agrees (5.2) we shall note $b = c \cdot (x_2//a) = (c \cdot x_2)//a$, i.e. $b \cdot a = c \cdot x_2$ and $x_2 = \frac{b \cdot a}{c}$, as was to be shown.

Corollary. $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$. This equality follows from item b) of the given theorem, i.e.

$$\frac{b}{c} \cdot a = \underbrace{\frac{b}{c} + \frac{b}{c} + \ldots + \frac{b}{c}}_{a} = \frac{\overbrace{b + b + \ldots + b}^{a}}{c} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{b}{c} \cdot k + \frac{b}{c} \cdot m = \frac{b \cdot k}{c} + \frac{b \cdot m}{c} = \frac{b \cdot k + b \cdot m}{c},$$

where $k + m = a$.

Changing factors b, k and m, we can receive equalities $b \cdot k = a$, $b \cdot m = b$, where the variables, standing in a right member are taken from the formula $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.

The theorem 5.2 For all $a \in \mathbf{N}_1$ and $b \in \left]\Delta\infty, +\infty\right[$, $b \notin \{\Delta 0, \theta, 0\}$ the equalities take place:

$$\frac{a}{b} = a//b \quad [44] \qquad (5.6)$$

And $a \cdot b = b \cdot a$.

Proof. From (5.6), i.e., if the first part of the theorem 5.2 is correct, follows: $a = b(a//b)$. According to a lemma 5.2 $a = (b \cdot a)//b \Rightarrow$ $\Rightarrow a \cdot b = b \cdot a$. This equality has a place for anyone odd a $(a \in \mathbf{N}_1)$,

[44] It is easy to prove equality (5.6) from (5.4).

$b \in]\Delta\infty, +\infty[$ and $b \notin \{\Delta 0, \theta, 0\}$. Really, let $b \in \mathbf{R}$. Then $a \cdot b = b \cdot a$ By virtue of a commutability of operation of multiplication of real numbers $(\{a, b\} \in \mathbf{R})$. Let $b \in \Delta_0$, i.e. $b = \Delta b_1$, where $b_1 \in \mathbf{R}$. Then $a\Delta b_1 = \Delta(a \cdot b_1)$, and $\Delta b_1 \cdot a = \Delta(b_1 \cdot a) = \Delta(a \cdot b_1)$ at $a \in \mathbf{N}_1$, i.e. $a \cdot \Delta b_1 = \Delta b_1 \cdot a$ and $a \cdot b = b \cdot a$. The theorem is proved.

Corollary. As $\dfrac{b}{a} = b//a$ under the mentioned above conditions, under these conditions the equality $(a+b)//c = a//c + b//c$ is correct, which immediately follows from a lemma 5.1.

$$d^{(a+b)//c} = \sqrt[c]{d^{(a+b)}} = \sqrt[c]{d^a \cdot d^b} = \sqrt[c]{d^a} \cdot \sqrt[c]{d^b} = d^{a//c} \cdot d^{b//c} =$$

$$= d^{a//c + b//c} \quad \Rightarrow \quad (a+b)//c = a//c + b//c.$$

Certainly, the above mentioned proofs do not give exhausting exposition of division second inversing. Nevertheless, the common representation about this operation from above-stated can be generated. The basic positions are those:

a) The division second inversing is connected to violation of a commutability of operation of multiplication $(a \cdot b \neq b \cdot a)$, i.e. from $a = b//c \Rightarrow a \cdot c = b$ (but not $c \cdot a = b$!), and from $a = \dfrac{b}{c} \Rightarrow c \cdot a = b$;

b) The division should be applied second inversing at the extension of a field of real numbers up to a set R_0 $(R_0 = \mathbf{R} \cup \Delta_0)$;

c) Division second inversing explains emerging two values at the radical of an even degree: $\sqrt[c]{d} = \pm m$ at $c \in \mathbf{N}_2$.

In summary, we shall mark, that there is a series of the theorems and rules, defining nature of this or that operation, it of a property and corollary.

For example:

a). *At commutative operation both inverses to her the operations coincide on a value.*

b). *The noncommutative operation has various inverses of operation (conditionally we shall name them is "analog of the radical" and "analog of a log").*

§ 5.3 Examples ω-images of objects of a calculus

It is uneasy to receive various ω-images of plants of a calculus.

1. For example, modified formulas Taylor, which we shall understand various ω-images of this formula look so (for $x_0 = a$):

a) At reflection $\backslash \omega_1 \to \omega_0 \backslash$:

$$f(x) = \prod_{n=0}^{\infty} \left({}^{(n)}f(a) \right)^{\frac{\left(\log_k\left(\frac{x}{a}\right)\right)^n}{n!}}, \qquad k > 1, \qquad (5.7)$$

where ${}^{(n)}f(a) - \omega$-image $\left(\backslash \omega_1 \to \omega_0 \backslash\right)$ n by a derivative;

b) at image $\backslash \omega_{-1} \to \omega_0 \backslash$

$$f(x) = \log_k \sum_{n=0}^{\infty} k^{\binom{\text{o}}{\underline{n}} f(a) + n \cdot \log_k\left(k^x - k^a\right) - \log_k n!}, \; k > 1, \quad (5.8)$$

where ${}^{\binom{\text{o}}{\underline{n}}} f(a) - \omega$-image $\left(\backslash \omega_{-1} \to \omega_0 \backslash\right)$ n by a derivative;

c) for $\backslash \omega_0' \to \omega_0 \backslash$

$$f(x) = \sum_{n=0}^{\infty} \sqrt[p]{\frac{\left(f_{k2}^{(n)}(a)\right)^p \cdot \left(x^p - a^p\right)}{n!}}, \; p \neq 0,1, \qquad (5.9)$$

where $f_{k2}^{(n)}(a) - \omega$-image $\left(\backslash \omega_0' \to \omega_0 \backslash\right)$ n by a derivative;

It is possible to find infinite a spectrum ω-images of the formula Taylor.

For approximate evaluations rationally to use the following formula, which we shall receive for three terms ω-image of the formula Taylor, space, not reduced in a scale, ω_0:

$$f(a\cdot h)\approx f(a)\cdot h^{'f}\cdot\left(h^{\ln h}\right)^{\frac{'f\cdot''f}{2!}}\cdot\left(h^{(\ln h)^2}\right)^{\frac{'f\cdot\left(''f\right)^2+'f\cdot''f\cdot'''f}{3!}},$$

Where $'f='f(a)$, $''f=''f(a)$, $'''f='''f(a)$, $a=x_0$, $h=\dfrac{x}{x_0}$. (5.10)

Let's prove this formula. Let in a neighbourhood of some point x_0 function $f(x)$ differentiable three times. Shall present it as:

$$f(x)\approx a_0\cdot\left(\frac{x}{x_0}\right)^{a1}\cdot\left(\left(\frac{x}{x_0}\right)^{\ln\left(\frac{x}{x_0}\right)}\right)^{a2}\cdot\left(\left(\frac{x}{x_0}\right)^{\ln^2\left(\frac{x}{x_0}\right)}\right)^{a3}$$

Let's discover $'f(x_0)$. $'f=x(\ln f)'$, but $\ln f(x)\approx\ln a_0+a_1\times$

$$\times\ln\left(\frac{x}{x_0}\right)+a_2\cdot\ln^2\left(\frac{x}{x_0}\right)+a_3\cdot\ln^3\left(\frac{x}{x_0}\right)$$

$$(\ln f)'\approx a_1\cdot\left(\frac{1}{x}\right)+2a_2\cdot\ln\left(\frac{x}{x_0}\right)\cdot\frac{1}{x}+3a_3\cdot\ln^2\left(\frac{x}{x_0}\right)\cdot\frac{1}{x}$$

$$'f=x\cdot(\ln f)'\approx a_1+2a_2\cdot\ln\left(\frac{x}{x_0}\right)+3a_3\cdot\ln^2\left(\frac{x}{x_0}\right)$$

Whence, $a_1='f(x_0)$.

Let's discover $'f(x_0)\cdot''f(x_0)$. $'f\cdot''f=x(\ln f)'+x^2(\ln f)''$.

Then, $${}'f(x_0)\cdot{}''f(x_0)\approx x_0\left(a_1\cdot\left(\frac{1}{x_0}\right)+2a_2\cdot\ln\left(\frac{x}{x_0}\right)\cdot\frac{1}{x_0}+\right.$$

$$\left.+3a_3\cdot\ln^2\left(\frac{x}{x_0}\right)\cdot\frac{1}{x_0}\right)+x_0^2\left(-a_1\left(\frac{1}{x_0^2}\right)+\frac{1}{x_0^2}\cdot 2a_2-3a_3\frac{1}{x_0^2}\times\right.$$

$$\left.\times\ln\left(\frac{x}{x_0}\right)\right)=a_1-a_1+2a_2 \quad\Rightarrow\quad a_2=\frac{{}'f\cdot{}''f}{2!}.$$

Similarly we shall discover ${}'f(x_0)\cdot\left({}''f(x_0)\right)^2+{}'f(x_0)\cdot{}''f(x_0)\cdot{}'''f(x_0)$.

Let's designate $\ln f=\varphi$. Then ${}'f=x\cdot\varphi'$, ${}''f={}'(x\cdot\varphi')=1+\dfrac{x\cdot\varphi''}{\varphi'}$,

$${}'f\cdot{}''f=x\cdot\varphi'+x^2\cdot\varphi'', \qquad {}'''f={}'\left(1+\frac{x\cdot\varphi''}{\varphi'}\right)=\frac{x\cdot\varphi'}{\varphi'+x\cdot\varphi''}\times$$

$$\times\frac{(x\cdot\varphi'')'\cdot\varphi'-(\varphi'')^2\cdot x}{(\varphi')^2}.$$ Let's explain: $$\left(\left(\frac{x\cdot\varphi''}{\varphi'}\right)'=\right.$$

$$\left.=\frac{(x\cdot\varphi'')'\cdot\varphi'-\varphi''\cdot x\cdot\varphi''}{(\varphi')^2}=\frac{x\cdot\left(\varphi'\cdot\varphi''+x\cdot\varphi'\cdot\varphi'''-x\cdot(\varphi'')^2\right)}{(\varphi')^2+x\cdot\varphi'\cdot\varphi''}\right).$$

Then $${}'f\cdot{}''f\cdot{}'''f=\left(x\cdot\varphi'+x^2\cdot\varphi''\right)\cdot{}'''f=\frac{x^2}{\varphi'}(\varphi'\cdot\varphi''+x\cdot\varphi'\cdot\varphi'''-$$

$$\left.-x\cdot(\varphi'')^2\right).$$

Let's discover $'f\cdot\left(''f\right)^2$. $'f\cdot\left(''f\right)^2=\left(x\cdot\varphi'+x^2\cdot\varphi''\right)\cdot\left(1+\frac{x\cdot\varphi''}{\varphi'}\right)=$

$$=\frac{x}{\varphi'}\left(\left(\varphi'\right)^2+2x\varphi'\cdot\varphi''+x^2\left(\varphi''\right)^2\right).$$

Then

$$'f\left(''f\right)^2+'f\cdot{}''f\cdot{}'''f=\frac{x}{\varphi'}\left(\left(\varphi'\right)^2+2x\varphi'\cdot\varphi''+x^2\left(\varphi''\right)^2\right)+$$

$$+\frac{x^2}{\varphi'}\left(\varphi'\cdot\varphi''+x\cdot\varphi'\cdot\varphi'''-x\cdot\left(\varphi''\right)^2\right)=x\cdot\varphi'+3x^2\varphi''+x^3\varphi'''=$$

$$=x\left(\ln f\right)'+3x^2\left(\ln f\right)''+x^3\left(\ln f\right)'''.$$

$$\left(\ln f\right)'\Big|_{x=x_0}=\frac{a_1}{x_0},\qquad \left(\ln f\right)''\Big|_{x=x_0}=-\frac{a_1}{x_0^{\,2}}+\frac{2a_2}{x_0^{\,2}},$$

$$\left(\ln f\right)'''\Big|_{x=x_0}=\frac{2a_1}{x_0^{\,3}}-\frac{1\cdot2\cdot3\cdot a_2}{x_0^{\,3}}+\frac{1\cdot2\cdot3\cdot a_3}{x_0^{\,3}},$$

$$x_0^{\,3}\left(\ln f\right)'''\Big|_{x=x_0}=2a_1-3!a_2+3!a_3.$$

At $x=x_0$ $'f\left(''f\right)^2+'f\cdot{}''f\cdot{}'''f=\frac{x_0\cdot a_1}{x_0}+3x_0^2\cdot\left(-\frac{a_1}{x_0^{\,2}}+\right.$

$$\left.+\frac{2a_2}{x_0^{\,2}}+x_0^{\,3}\cdot\left(\frac{2a_1}{x_0^{\,3}}-\frac{3!\cdot a_2}{x_0^{\,3}}+\frac{3!\cdot a_3}{x_0^{\,3}}\right)\right)\Rightarrow$$

$$\Rightarrow\quad a_3=\frac{'f\left(''f\right)^2+'f\cdot{}''f\cdot{}'''f}{3!}.$$

The note. ω-images of differentials will be:

$$\delta f=(\delta x)^{'f},\quad \delta(\delta f)=\delta^2 f=\underset{\delta x}{'}\left((\delta x)^{'f}\right)=\left((\delta x)^{\ln\delta x}\right)^{''f\cdot 'f},$$

$$\delta^3 f=\delta\left(\delta^2 f\right)=\left((\delta x)^{\ln^2\delta x}\right)^{'f\cdot ''f\cdot\left(''f+'''f\right)},\qquad (5.11)$$

$$\delta^4 f=\left((\delta x)^{\ln^3\delta x}\right)^{'f\cdot ''f\cdot\left(\left(''f+'''f\right)^2+''f\cdot '''f+'''f\cdot {}^{IV}f\right)}\text{ etc.}$$

Not stopping on ω-images of various objects of a calculus, we shall remark, that they are simple for receiving by analogy to appropriate objects of the well-known analysis.

2. For example, for anyones $a,b,c\in\mathbf{R}$ the equality is fair:

$$\int_a^b(\delta x)^{f(x)}=\int_a^c(\delta x)^{f(x)}\cdot\int_c^b(\delta x)^{f(x)}.$$

Let $a<c<b$. Then

$$\prod_{i=a}^{b}(\Delta x_i)^{f(\xi_i)}=\prod_{i=a}^{c}(\Delta x_i)^{f(\xi_i)}\cdot\prod_{i=c}^{b}(\Delta x_i)^{f(\xi_i)}.$$

Passing to a limit, we shall receive:

$$\int_a^b(\delta x)^{f(x)}=\int_a^c(\delta x)^{f(x)}\cdot\int_c^b(\delta x)^{f(x)}.$$

Let $a<b<c$,

$$\int_a^c (\delta x)^{f(x)} = \int_a^b (\delta x)^{f(x)} \cdot \int_b^c (\delta x)^{f(x)} \quad \Rightarrow \quad \int_a^b (\delta x)^{f(x)} = \frac{\int_a^c (\delta x)^{f(x)}}{\int_b^c (\delta x)^{f(x)}}.$$

But $\quad \int_c^b (\delta x)^{f(x)} = \frac{1}{\int_b^c (\delta x)^{f(x)}}, \qquad$ i.e. $\quad \int_a^b (\delta x)^{f(x)} = \int_a^c (\delta x)^{f(x)} \times$

$\times \int_c^b (\delta x)^{f(x)}$, as was to be shown.

3. We shall prove the formula ${}^{\underline{p}}\left(u^v\right) = {}^{\underline{p}}v \cdot \ln v + {}^{\underline{p}}u$.

$$ {}^{\underline{p}}\left(u(x)^{v(x)}\right) = \lim_{\delta x \to 1} \log_{\delta x} \log_{u(x)^{v(x)}}^{u\left(x^{\delta x}\right)^{v\left(x^{\delta x}\right)}} =$$

$$= \lim_{\delta x \to 1} \log_{\delta x} \left(\frac{v\left(x^{\delta x}\right)}{v(x)} \cdot \log_{u(x)}^{u\left(x^{\delta x}\right)} \right) =$$

$$= \lim_{\delta x \to 1} \log_{\delta x} \frac{v\left(x^{\delta x}\right)}{v(x)} + \lim_{\delta x \to 1} \log_{\delta x} \log_{u(x)}^{u\left(x^{\delta x}\right)} =$$

$$= \lim_{\delta x \to 1} \frac{{}'\left(v\left(x^{\delta x}\right)\right) - {}'\left(v(x)\right)}{{}'(\delta x)} + {}^{\underline{p}}u(x) =$$

$$= \lim_{\delta x \to 1} \frac{{}'\left(v\left(x^{\delta x}\right)\right) - 0}{1} + {}^{\underline{p}}u(x) = \lim_{\delta x \to 1} {}'\left(v\left(x^{\delta x}\right)\right) + {}^{\underline{p}}u(x) =$$

$$= \lim_{\delta x \to 1} \log_{\delta_*(\delta x)} \delta_* v\left(x^{\delta x}\right) + \underline{\mathrm{p}}u(x) =$$

$$= \lim_{\delta x \to 1} \log_{\delta_*\left(x^{\delta x}\right)} \delta_* v\left(x^{\delta x}\right) \cdot \log_{\delta_*(\delta x)} \delta_*\left(x^{\delta x}\right) + \underline{\mathrm{p}}u(x) =$$

$$= \log_{\delta_* x} \delta_* v(x) \cdot \lim_{\delta x \to 1} \delta x \cdot \ln x + \underline{\mathrm{p}}u(x) = {}'v(x) \cdot \ln x + \underline{\mathrm{p}}u(x) =$$

$$= \underline{\mathrm{p}}v(x) \cdot \frac{\ln v(x)}{\ln x} \cdot \ln x + \underline{\mathrm{p}}u(x) = \underline{\mathrm{p}}v(x) \cdot \ln v(x) + \underline{\mathrm{p}}u(x),$$

as was to be shown.

Let's remark also, that *by analogy* to the formula ${}'(u+v) = \dfrac{u \cdot {}'u + v \cdot {}'v}{u+v}$ has a place equality:

$$\underline{\mathrm{p}}(u \cdot v) = \log_{u \cdot v}\left(u^{\underline{\mathrm{p}}u} \cdot v^{\underline{\mathrm{p}}v}\right), \qquad (5.14)$$

which it is simple to prove. Really, $\underline{\mathrm{p}}(u \cdot v) = \left({}'u + {}'v\right)\log_{u \cdot v} x =$

$$= \left(\underline{\mathrm{p}}u \cdot \log_x u + \underline{\mathrm{p}}v \cdot \log_x v\right) \cdot \log_{u \cdot v} x = \underline{\mathrm{p}}u \cdot \log_{u \cdot v} x^{\log_x u} +$$

$$+ \underline{\mathrm{p}}v \cdot \log_{u \cdot v} x^{\log_x v} = \underline{\mathrm{p}}u \cdot \log_{u \cdot v} u + \underline{\mathrm{p}}v \cdot \log_{u \cdot v} v =$$

$$= \frac{\underline{\mathrm{p}}u \cdot \ln u + \underline{\mathrm{p}}v \cdot \ln v}{\ln u \cdot v} = \frac{\ln u^{\underline{\mathrm{p}}u} + \ln v^{\underline{\mathrm{p}}v}}{\ln u \cdot v} = \log_{u \cdot v}\left(u^{\underline{\mathrm{p}}u} \cdot v^{\underline{\mathrm{p}}v}\right),$$

as was to be shown.

Let's prove, that $\underline{\mathrm{p}}(u+v)=\dfrac{\log\left({}^{2}u^{\underline{\mathrm{p}}u}\cdot{}^{2}v^{\underline{\mathrm{p}}v}\right)}{\log\left({}^{2}(u+v)\right)}$. (5.15)

Really, $\underline{\mathrm{p}}(u+v)=(u+v)'\cdot\log_{{}^{2}(u+v)}{}^{2}x=\left(\underline{\mathrm{p}}u\cdot\log_{{}^{2}x}{}^{2}u+\right.$

$$\left.+\underline{\mathrm{p}}v\cdot\log_{{}^{2}x}{}^{2}v\right)\cdot\log_{{}^{2}(u+v)}{}^{2}x=\frac{\log\left({}^{2}u^{\underline{\mathrm{p}}u}\cdot{}^{2}v^{\underline{\mathrm{p}}v}\right)}{\log\left({}^{2}(u+v)\right)}.$$

Selecting *any object* (formula, the theorem etc.) calculus, is possible is to *found* it ω-image from *any space.*

For example, take a mode of tangents (mode of a Newton) for an approximate evaluation of the radicals of the equations:

$$y-f(x_2)=f'(x_2)(x-x_2),\ f(x_2)>0,\ x_2-\text{given final value}$$

of argument.

By analogy $\left(\dfrac{x}{x_2}\right)^{'f(x_2)}=\dfrac{y}{f(x_2)}$. At $x=a_1$, $y=1$, (as $0\backslash\omega_1\to\omega_0\backslash 1$, $y=0\backslash\omega_1\to\omega_0\backslash y=1$), i.e.

$$\left(\frac{a_1}{x_2}\right)^{'f(x_2)}\cdot f(x_2)=1\ \Rightarrow\ a_1=x_2\cdot\sqrt[{'f(x_2)}]{:f(x_2)}\ \text{etc.}$$

The note. Let's prove also formula, meeting in the text:

$$\int(\delta x)^{f(x)}=\exp\left(\int\frac{f(x)}{x}dx\right).$$

$$ {}^{'}f(x)=\frac{x\cdot f'(x)}{f(x)}, \qquad \log_{\delta x}\delta f(x)=\frac{x\cdot df(x)}{f(x)\cdot dx} \quad \Rightarrow $$

$$ \Rightarrow \quad \int\frac{\log_{\delta x}\delta f(x)}{x}dx=\int\frac{df(x)}{f(x)}=\ln f(x) $$

$$ f(x)=\exp\left(\int\frac{\log_{\delta x}\delta f(x)}{x}dx\right)=\exp\left(\int\frac{f(x)}{x}dx\right). $$

Or otherwise:

$$ \int(\delta x)^{f(x)}=F(x) \quad \Rightarrow \quad {}^{'}F(x)=f(x) \quad \Rightarrow $$

$$ \Rightarrow \quad \frac{x\cdot F'(x)}{F(x)}=f(x) \quad \Rightarrow \quad (\ln F(x))'=\frac{f(x)}{x} \quad \Rightarrow $$

$$ \Rightarrow \quad \ln F(x)=\int\frac{f(x)}{x}dx \quad \Rightarrow \quad F(x)=\exp\left(\int\frac{f(x)}{x}dx\right), $$

i.e. $\int(\delta x)^{f(x)}=\exp\left(\int\frac{f(x)}{x}dx\right)$, as was to be shown.

In summary we shall note expressions for some ω- *images of differentials.*

As in ω_0 $df(x)=f'(x)\cdot dx$, i.e. $f'(x)=\frac{df(x)}{dx}$, that, by analogy to it,

$d f(x) = f'(x) \cdot dx \setminus \omega_{-1} \to \omega_0 \setminus \nu f(x) = \overset{\bullet}{f}(x) + \nu x$, and $d f(x) \setminus \omega_1 \to$[45]

$\to \omega_0 \setminus \delta f = (\delta x)^{'f} \Rightarrow {}^{'}f(x) = \log_{\delta x} \delta f(x)$.

Then "*supersuperdifferential*"

$$\delta x = {}^{s f}(\delta x) \quad \Rightarrow \quad {}^{s}f(x) = \operatorname{slog}_{\delta x} \delta f(x),$$

where ${}^{s}f(x)$ – "*supersuperderivative*".

If for a basis to take ω-image of a derivative ${}^{\underline{p}}f(x)$, obtained by reflection:

$$f'(x) \setminus \omega_1 \to \omega_0 \setminus k^{'f(x)} = {}^{\underline{p}}f(x),$$

and ${}^{\underline{p}}f(x)$, as it was specified, it is an image of a derivative reduced in a scale ω_0, $\delta f = {}^{\underline{p}}f(x) \odot \delta x \Rightarrow {}^{\underline{p}}f(x) = \delta f(x) \mathbin{\triangle} \delta x$, where $d f \setminus \omega_1 \to \omega_0 \setminus \delta f$.

Similarly, ω-image of a derivative ${}^{p}f$, where ${}^{p}f(x)$ – ω-image obtained by pceudo-reflection $f'(x) \setminus \omega_2 \to \omega_0 \setminus_* {}^{p}f(x)$, i.e. ${}^{p}f(x)$ – is not reduced in a scale ω_0, has a differential

$$\delta f = (\delta x)^{{}^{p}f(x)} \quad \Rightarrow \quad {}^{p}f(x) = \log_{\delta x} \delta f(x).$$

Obviously, $\nu f = \overset{\bullet}{f} + \nu x \Rightarrow \overset{\bullet}{f}(x) = \nu f(x) - \nu x$, where $\overset{\bullet}{f}$ – image of a derivative obtained $\setminus \omega_{-1} \to \omega_0 \setminus$ and reduced to ω_0 etc.

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[45] ${}^{'}f$ – an image of a derivative not reduced in a scale ω_0 and obtained by pceudo-reflection $f' \setminus \omega_1 \to \omega_0 \setminus_* {}^{'}f$, where $\setminus \omega_1 \to \omega_0 \setminus_*$ means, that the outcome ω – reflection is not scaled ω_0.

§ 5.4. ω- image of imaginary unit

At first we shall consider reflection $\backslash\, \omega_1 \to \omega_0 \,\backslash$ complex number.

Let in ω_1 the complex number $z = a + bi$ is given. Let's reflection it in space ω_0. For this purpose we shall define operation of a *reflexive taking the root*. As the reflexive exponentation is defined as $a^{\to b} = \underbrace{a \odot a \odot a \odot \ldots \odot a}_{\log_k b \in \mathbf{Z},\, k \neq 1}$, the inverse operation, i.e. reflexive taking the root, will be noted so:

$$\sqrt[b\leftarrow]{a} = x \quad \Rightarrow \quad a = x^{\to b} = \underbrace{x \odot x \odot x \odot \ldots \odot x}_{\log_k b \in \mathbf{Z},\, k \neq 1} =$$

$$= k^{\left(\log_k x\right)^{\log_k b}} \quad \Rightarrow \quad \log_k a = \left(\log_k x\right)^{\log_k b} \quad \Rightarrow$$

$$\Rightarrow \quad \left(\log_k a\right)^{1/\log_k b} = \log_k x \quad \Rightarrow \quad x = k^{\sqrt[\log_k b]{\log_k a}},$$

i.e.

$$\sqrt[b\leftarrow]{a} = k^{\sqrt[\log_k b]{\log_k a}} \tag{5.18}$$

Then $i = \sqrt{-1} \,\backslash\, \omega_1 \to \omega_0 \,\backslash = k^{\sqrt{-1}} = k^i$

$$\left(a = -1 \,\backslash\, \omega_1 \to \omega_0 \,\backslash\, a_* = k^{-1} = \frac{1}{k} \quad \Rightarrow \quad \log_k a_* = -1,\right.$$

$$\left. b = 2 \,\backslash\, \omega_1 \to \omega_0 \,\backslash\, b_* = k^2 \quad \Rightarrow \quad \log_k b_* = \log_k k^2 = 2\right),$$

where $a_*, b_* - \omega$- images a and b in ω_0 from ω_1.

$$z = a + bi \,\backslash\, \omega_1 \to \omega_0 \,\backslash\, k^a \cdot \left(k^b \odot k^i\right) = k^{a+bi}.$$

If $k = e^m$, $k^{a+bi} = e^{ma+mbi} = x + iy \quad \Rightarrow \quad x = k^a \times \times \cos(b \cdot \ln k)$, $y = k^a \cdot \sin(b \cdot \ln k)$, i.e. $x + iy = k^a \times$

$$\times \left(\cos(b \cdot \ln k) + i \cdot \sin(b \cdot \ln k)\right) \tag{5.19}$$

So, ω-*image of a complex number at reflection* $\backslash\omega_1 \to \omega_0\backslash$ *is too complex number.*

In an outcome, as well as it was necessary to expect, we did not manage to receive numbers of a new nature (all operations and the operands are well-known).

Let's try now to find analog of number i in area Δ-numbers. For this purpose we shall consider a structure of imaginary unit: $i = \pm\sqrt{-1} = (-1)^{1//2}$, i.e. imaginary unit i was derivated from negative and positive numbers in an outcome of introduction of operation of division second inversing. At reflection $\backslash\omega_0 \to \omega_1\backslash$ operand -1 and operation of division second inversing we shall receive: $-1\backslash\omega_0 \to \omega_1\backslash \log_k(-1) = \Delta 0$, if $\log_k(-1) \in \Delta_0$, $//\backslash\omega_0 \to$ $\to \omega_1\backslash \lozenge$, i.e. division second inversing to be imageed *in reflexive division second inversing*. Obviously,

$$k^{\Delta m} \lozenge k^n = k^{\Delta m//n} \backslash\omega_0 \to \omega_1\backslash \Delta m//n, \text{ where}$$

In our case $\sqrt[n]{k^{\Delta m}} = \sqrt{-1}$, i.e. $k^{\Delta m} \equiv -1$. Whence $\Delta m \equiv \Delta 0$, and $n = 2$, i.e. $i\backslash\omega_0 \to \omega_1\backslash j = \Delta 0//2$, where $j-$imaginary Δ-unit is number of a *new* nature. Because of numbers j can be constructed the theory of functions j-variable. This theory will be ω-image TFCV (the theory of function of a complex variable) in area Δ-numbers.

§ 5.5. Examples of a solution of the differential equations with application of a method ω-reflections

The mathematical methods ω-reflections can be used for the extension of a class of the solved differential equations ad infinitum. And, ω-images of the known equations (and they will be more complicated on a structure by the differential equations, which solution is hampered) and their solutions are rather simple without any transformations. The essence above-stated consists in the following:

a) we select *any* known equation with a solution (it means anyone on complexity the equation, which solution already is known);

b) we represent, that this equation and it a solution “are” in some space ω_i or ω_i' (in the present work the elementary case, when $i = 1$) is selected;

c) We reflection the equation and solution in ω_0.

It is natural, that at multiple ω-reflection on “vertical” $(i = \text{var}$, $k = const)$ and on “horizontal” $(i = const,\ k = \text{var})$, and also, combining simultaneously that and others ω-reflection, i.e. $i = \text{var}$, $k = \text{var}$, are possible are to received as much as complicated on a structure by the differential equation with a ready solution. It is necessary to not forget, that in this book the ω-spaces with an *exponential* function k^x of connection are circumscribed *only*. However, realizing ω-reflections with other function of connection, we shall receive, practically, all infinities a set of the every possible differential equations and their solutions.

Before to reduce examples of solutions of the differential equations with application of a means ω-reflections, we shall prove the theorem, which earlier met (theorem 3.15, with 127), but other interpretation of an outcome.

The theorem 5.3. If f *non-negative continuous in some area* D *the function also has in* D n*-derivatives, at* $k \neq 0{,}1$ $k \in \mathbf{R}$ *and* $f \neq 0$ a ratio $\varphi = \ln \underline{{}^{(n)}}f \Big/ \left(\ln \underline{{}^{(n-1)}}f\right)'$ linearly concerning argument $\varphi = cx$ (5.20), $c = \ln k$.

Proof. Let's transform φ: $\ln \underline{{}^{(n)}}f = cx \cdot \left(\ln \underline{{}^{(n-1)}}f\right)'$, i.e.

$$\underline{{}^{(n)}}f = \exp\left(cx \cdot \left(\ln \underline{{}^{(n-1)}}f\right)'\right) = k^{x \cdot \left(\ln \underline{{}^{(n-1)}}f\right)'}.$$

Further proof coincides with a proof т. 3.15.

$$\underline{{}^{''}}f = \underline{{}^{(2)}}f = k^{x \cdot \left(\ln \underline{{}^{'}}f\right)'} = k^{x(y' + x \cdot y'') \cdot \ln k}, \quad y = \ln f \qquad (5.21)$$

Similarly, $\underline{{}^{'''}}f = \underline{{}^{(3)}}f = k^{x \cdot \left(\ln \underline{{}^{(2)}}f\right)'}$

$$\underline{{}^{(3)}}f = k^{\ln^2 k \cdot x \cdot \left(y' + 3xy'' + x^2 y'''\right)} \qquad (5.22)$$

$$\underline{(4)}f = k^{x\cdot\left(\ln \underline{(3)}f\right)'} = k^{\ln^3 k\cdot x\cdot\left(y'+7xy''+6x^2y'''+x^3y^{\mathrm{IV}}\right)};$$

$$\underline{(5)}f = k^{x\cdot\left(\ln \underline{(4)}f\right)'}$$

$$\underline{(5)}f = k^{\ln^4 k\cdot x\cdot\left(y'+15xy''+25x^2y'''+10x^3y^{\mathrm{IV}}+x^4y^{\mathrm{V}}\right)} \text{ etc.}$$

Let equality $\underline{(n)}f = k^{x\cdot\left(\ln \underline{(n-1)}f\right)'}$ correctly. Then $\underline{(n+1)}f = k^{\left(\underline{(n)}f\right)'}$. Let's designate $\underline{(n)}f = Z$, i.e.

$$\underline{(n+1)}f = k^{'z} = k^{x\cdot z'/z} = k^{x\cdot(\ln z)'} = k^{x\cdot\left(\ln \underline{(n)}f\right)'}.$$

The theorem is proved.

Quite often at problem solving of mathematical physics there is a *laplacian* $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$. Reflection it from ω_1 in ω_0, we shall receive a log *quasilaplacian*:

$$\ln\underline{\Delta} = x\cdot\frac{\partial\ln}{\partial x} + y\cdot\frac{\partial\ln}{\partial y} + z\cdot\frac{\partial\ln}{\partial z} + x^2\cdot\frac{\partial^2\ln}{\partial x^2} +$$

$$+ y^2\cdot\frac{\partial^2\ln}{\partial y^2} + z^2\cdot\frac{\partial^2\ln}{\partial z^2} \tag{5.23}$$

Example 1. Let in ω_1 the equation $f' = x^2$ and it a solution $f = \dfrac{x^3}{3}$ is given. To find ω-images of this equation and its solution:

$$f' = x^2 \setminus \omega_1 \to \omega_0 \setminus k^{x\cdot(\ln f)'} = k^{\log_k^2 x} \quad \Rightarrow$$

$$\Rightarrow \quad x\cdot(\ln f)' = \log_k^2 x; \qquad (1)$$

$$\frac{x^3}{3} \backslash \omega_1 \to \omega_0 \backslash\, k^{\log_k^3 x} \triangle\, k^3 = k^{\log_k^3 x / \log_k k^3} = k^{\log_k^3 x/3}$$

$$\log_k f = \frac{\log_k^3 x}{3} \quad \Rightarrow \quad \ln f = \frac{\log_k^3 x}{3}\cdot \ln k\,.$$

Whence $(\ln f)' = \dfrac{3\cdot \log_k^2 x\cdot \ln k}{3}\cdot(\log_k x)' = \dfrac{\log_k^2 x\cdot \ln k}{x\cdot \ln k} = \dfrac{\log_k^2 x}{x}$, that the equation $(\ln f)' = \dfrac{\log_k^2 x}{x}$, and it a solution $f = k^{\log_k^3 x/3}$ has coincided with (1).

Example 2. Let in ω_1 the equation in terms of this space is given:

$$\underline{f''} = \underline{x} \odot \underline{\cos x}$$

Image of this equation in space ω_0:

$$k^{\ln k\cdot x\cdot\left((\ln f)' + x\cdot(\ln f)''\right)} = x^{\cos\log_k x}, \quad k \neq 1$$

as $\underline{\cos x} \backslash \omega_1 \to \omega_0 \backslash\, k^{\cos\log_k x} = k^{\cos(\log x/\log k)}$.

Using one of solutions of the equation $f'' = x\cdot\cos x$ in ω_0 $f = 2\cdot\sin x - x\cdot\cos x$, we shall note in terms ω_1 and then we shall discover an appropriate amount of a solution in ω_0. Let's receive

$$f = k^{\left(2\cdot\sin(\log x/\log k) - (\log x/\log k)\cdot\cos(\log x/\log k)\right)}.$$

The notes. 1. The entry in terms of space ω_1 can be lowered and immediately to note a unknown quantity a solution in ω_0: $f'' = x\cdot\cos x(\omega_1)$, i.e. the equation is given in space ω_1.

2. If the equation $f'' = x\cdot\cos x$ was given in space ω_1' and it would be required to find it a solution in space ω_0, it is necessary to realize passage

$\omega_1' \to \omega_0' \to \omega_0$, or $\omega_1' \to \omega_1 \to \omega_0$. Let's receive the following differential equation [46] :

$$\frac{p}{\ln k} \cdot \left(\frac{f_t''}{f} - \left(\frac{f_t'}{f} \right)^2 \right) = t \cdot \cos t,$$

where $t = \log_k x^p$, $f = f\left(x^p\right)$.

The solution it the equation in ω_0 will be noted so:

$$f = k^{\left(2 \cdot \sin \log_k x^p - \log_k x^p \cdot \cos \log_k x^p\right)},$$

Example 3. Let in ω_1 the equation $\underline{f_{\tau\tau}''} = \underline{a}^2 \odot \underline{f_{xx}''}$, $\underline{f}(\underline{0}, \tau) = \underline{0}$, $\underline{f}(\underline{l}, \tau) = \underline{0}$, $\underline{f}(x, \underline{0}) = \underline{\varphi}(x)$, $\underline{f_\tau'}\Big|_{\tau=0} = \underline{\Psi}(x)$, $(a, l, k) = const$ is given.

In ω_0 this equation looks so:

$$\tau \cdot \left(\chi_\tau' + \tau \cdot \chi_{\tau\tau}'' \right) = a^2 \cdot x \cdot \left(\chi_x' + x \cdot \chi_{xx}'' \right) \qquad (1)$$

where $\chi = \ln f$. Is realizable reflection $\backslash \omega_1 \to \omega_0 \backslash$.

Knowing a solution of the similar equation $\left(f_{\tau\tau}'' = a^2 \cdot f_{xx}'' \right)$ in ω_0 we shall note a solution of the equation:

$$f = \prod_{n=k}^{\infty} \left(C_n^{\cos\left(\frac{an\pi}{l} \cdot \log_k^\tau\right)} \cdot D_n^{\sin\left(\frac{an\pi}{l} \cdot \log_k^\tau\right)} \right)^{\sin\left(\frac{n\pi}{l} \cdot \log_k^x\right)},$$

[46] In an example 5 the brief theory of reflection $\backslash \omega_0' \to \omega_0 \backslash$ is given.

where $$C_n = \exp\left(\int_1^{k^l} \frac{\varphi\left(\log_k^x\right)\cdot\sin\left(\frac{n\pi}{l}\cdot\log_k^x\right)}{x}\cdot dx\right)^{\frac{2}{l}},$$

$$D_n = \exp\left(\int_1^{k^l} \frac{\psi\left(\log_k^x\right)\cdot\sin\left(\frac{n\pi}{l}\cdot\log_k^x\right)}{x}\cdot dx\right)^{\frac{2}{an\pi}}.$$

Example 4. Let in ω_1 in terms of this space the equation is noted:

$$\underline{\underline{\Delta}}\underline{f} = \underline{-1}\odot\underline{m} \qquad (m = const,\ k = e)$$

The solution of the equation $\Delta f = -m$ in ω_0 is known ($\Delta -$laplacian). By virtue of interior identity of spaces this solution will be also solution of the similar equation "inside" ω_1. Let's reflection the equation and solution in ω_0.

The log quasilaplacian will be noted so:

$$\ln\underline{\Delta} = x\cdot\frac{\partial\ln}{\partial x} + y\cdot\frac{\partial\ln}{\partial y} + z\cdot\frac{\partial\ln}{\partial z} + x^2\cdot\frac{\partial^2\ln}{\partial x^2} + y^2\cdot\frac{\partial^2\ln}{\partial y^2} +$$

$$+ z^2\cdot\frac{\partial^2\ln}{\partial z^2} = \frac{\partial\ln}{\partial r} + r\cdot\left(\frac{\partial\ln}{\partial r} + r\cdot\frac{\partial^2\ln}{\partial r^2}\right) + z\cdot\left(\frac{\partial\ln}{\partial z} + z\cdot\frac{\partial^2\ln}{\partial z^2}\right)$$

$$\left|\underline{u}(\underline{r},\underline{z})\right| < \infty, \qquad \underline{u}(\underline{r},\underline{0}) = u(\underline{r},\underline{l}) = \underline{u}(\underline{R},\underline{z}) = \underline{0}, \qquad \underline{0} \le \underline{r} < \underline{R},$$

$$\underline{0} < \underline{z} < \underline{l}, \qquad (R,l) = const.$$

Then

$$\ln\underline{\Delta f} = -m$$

and $$f(r,z)=\exp\left\{\frac{m\cdot\left(R^2-\ln^2 r\right)}{4}+m\cdot R^2\times\right.$$

$$\times\sum_{n=1}^{\infty}\left(\frac{J_2(\mu_n)}{\mu_n^2\cdot J_1^2(\mu_n)\cdot sh\left(\frac{\mu_n\cdot l}{R}\right)}\right)\cdot\left[\left(ch\left(\frac{\mu_n\cdot l}{R}\right)-1\right)\times\right.$$

$$\left.\left.\times sh\left(\frac{\mu_n\cdot\ln z}{R}\right)-sh\left(\frac{\mu_n\cdot l}{R}\right)\times ch\left(\frac{\mu_n\cdot\ln z}{R}\right)\right]\cdot J_0\left(\frac{\mu_n\cdot\ln z}{R}\right)\right\}.$$

Example 5. Let in ω_1 the equation $f^{\mathrm{V}}=\sin x$ is given. Let's discover ω-images of this equation and it a solution in ω_0:

$$f^{\mathrm{V}}=\sin x\setminus\omega_1\to\omega_0\setminus f^*=k^{\ln^4 k\cdot x\cdot\left(y'+15xy''+25x^2y'''+\right.}$$

$$\left.+10x^3y^{\mathrm{IV}}+x^4y^{\mathrm{V}}\right)=k^{\sin\log_k x}\quad\Rightarrow\quad\ln^4 k\cdot x\times$$

$$\times\left(y'+15xy''+25x^2y'''+10x^3y^{\mathrm{IV}}+x^4y^{\mathrm{V}}\right)=\sin\log_k x,\qquad(1)$$

where $y=\ln f$, (1) is an image in ω_0 input equation. As the solution $f^{\mathrm{V}}=\sin x$ is known: $f=-\cos x$, $f^*=k^{-\cos\log_k x}$ – this solution of the equation (1), in what easily is possible to be convinced.

So, knowing the equation and it is uneasy at once to note a solution in ω_1, in ω_0 both equation and solution.

The differential equations displayed from adjacent spaces ω_0' are similarly solved.

Let's note ω-images of derivatives at reflection from adjacent space in well-known ω_0:

$$f'\setminus\omega_0'\to\omega_0\setminus f'_{k2}=\frac{f}{x}\sqrt[p]{\frac{x\cdot f'}{f}},$$

where $p=\log_{k_1}k_2$; k_1,k_2 – accordingly parameters of function of connection k^x spaces ω_0 and ω_0';

$$f''\setminus\omega_0' \to \omega_0\setminus f''_{k_2}=\left(\frac{\left(f^p\right)''}{p^2\cdot x^{2(p-1)}}-\frac{(p-1)\cdot\left(f^p\right)'}{p^2\cdot x^{2p-1}}\right)^{\frac{1}{p}} \quad \text{etc.}$$

The note. Images of derivatives of the order more second has a rather complicated structure.

Example 6. Let in ω_0' the equation $f'=x^2$ is given. One of it of solutions $f=\frac{x^3}{3}$. Let's discover ω-images of the equation and its solution in ω_0:

$$f'=x^2\setminus\omega_0' \to \omega_0\setminus\left(\frac{\left(f^p\right)'}{p\cdot x^{p-1}}\right)^{\frac{1}{p}}=\left(x^{2p}\right)^{\frac{1}{p}} \Rightarrow$$

$$\Rightarrow \left(f^p\right)'=p\cdot x^{p-1}\cdot x^{2p}=p\cdot x^{3p-1} \qquad (1)$$

$$\frac{x^3}{3}\setminus\omega_0' \to \omega_0\setminus\left(\frac{x^{3p}}{3}\right)^{\frac{1}{p}} \Rightarrow f^p=\frac{x^{3p}}{3},$$ where $f-\omega_0$ - image $\frac{x^3}{3}$ or image of a solution of an input equation. Then $\left(f^p\right)'=p\cdot x^{3p-1}$, that satisfies (1).

Example 7. The equation $f'=\cos x$ in space ω_0' let is given. To find ω-images of this equation and its solution in space ω_0.

Solution. One of solutions of the equation $f'=\cos x$ in ω_0' will be $f=\sin x$. Then

$$f'=\cos x\setminus\omega_0' \to \omega_0\setminus f'_{k_1}=\sqrt[p]{\cos x^p}, \qquad (1)$$

where $p=\log_{k_1}k_2$; k_1, k_2 – accordingly parameters of function of connection k^x spaces ω_0 and ω_0'; $f'_{k_1}=\frac{f}{x}\cdot\left(x\cdot\frac{f'_{k_2}}{f}\right)^{\frac{1}{p}}$; $f'_{k_2}\equiv f'$. An image of a solution $f=\sin x \setminus \omega_0' \to \omega_0 \setminus f_{k_1}=\sqrt[p]{\sin x^p}$.

So, in ω_0 the equation $f'=\cos x$ looks so:

$$\frac{f}{x}\cdot\left(\frac{x\cdot f'}{f}\right)^{\frac{1}{p}}=\sqrt[p]{\cos x^p} \quad \Rightarrow \quad f'\cdot f^{p-1}=x^{p-1}\cdot\cos x^p, \qquad (2)$$

and it a solution $f=\sqrt[p]{\sin x^p}$ (3). Actually, by solving the elementary equation $y'=\cos x$, we have found a solution more complicated on a structure of the equation $f'\cdot f^{p-1}=x^{p-1}\cdot\cos x^p$.

Check: $f=\left(\sin x^p\right)^{\frac{1}{p}} \quad \Rightarrow \quad f'=x^{p-1}\cdot\cos x^p\times\left(\sin x^p\right)^{\frac{1}{p}-1}$,

$f^{p-1}=\left(\sin x^p\right)^{1-\frac{1}{p}}$, and $f'\cdot f^{p-1}=x^{p-1}\cdot\cos x^p$, i.e. the solution $f=\sqrt[p]{\sin x^p}$ is real correctly.

The note. The equation $f'\cdot f^{p-1}=x^{p-1}\cdot\cos x^p$, certainly, simple to solve by a replacement $x^p=t$, i.e.

$$\int f^{p-1}\cdot df=\int x^{p-1}\cdot\cos x^p\cdot dx \Rightarrow f=\sqrt[p]{p\cdot\int x^{p-1}\cdot\cos x^p\cdot dx}$$

and $f=\sqrt[p]{\int\cos t\cdot dt}=\sqrt[p]{\sin x^p}$.

In a common case, if $f'_{k_1}=\frac{f}{x}\left(\frac{x\cdot f'}{f}\right)^{\frac{1}{p}}=\varphi\left(x^m\right)$,

$$f=\sqrt[p]{\int \varphi^p(t)\cdot dt}.$$

For example, $f'=\operatorname{ctg} x$. A solution of this equation $f=\ln\sin x$. Then $f'_{k1}=\sqrt[p]{\operatorname{ctg} x^p}$, i.e. $\left(\frac{f}{x}\right)^{1-\frac{1}{p}}\cdot(f')^{\frac{1}{p}}=\sqrt[p]{\operatorname{ctg} x^p}$, and solution of this equation will be $f=\sqrt[p]{\ln\sin x^p}$ $\left(f=\sqrt[p]{\int\left(\sqrt[p]{\operatorname{ctg} t}\right)^p dt}\right.$, where $\left.\varphi(t)=\sqrt[p]{\operatorname{ctg} t}\right)$. It is possible again to reflection the equation (2):

$$\frac{f}{x}\left(x\cdot\frac{f'}{f}\right)^{\frac{1}{p}}\cdot f^{p-1}=\sqrt[p]{x^{p\cdot(p-1)}\cdot\cos x^{p^2}}.$$

It the solution will be formed of a solution of the equation (3):

$$f=\sqrt[p]{\sqrt[p]{\sin x^{p^2}}}=\left(\sin x^{p^2}\right)^{\frac{1}{p^2}} \text{ etc.}$$

Example 8. Let in ω_1 the equation $f''+f'=x$ is given to find it ω-image and it a solution in space ω_0.

Solution. In ω_1 space the input equation has a solution

$$f=\frac{x^2}{2}-x.$$

Let's reflection the equation and solution from ω_1 in ω_0:

$$f''+f'=x\setminus\omega_1\to\omega_0\setminus k^{\ln k\cdot x\cdot(y'+x\cdot y'')}\cdot k^{x\cdot y'}=k^{\log_k x}\ \left(y=\ln f\right), \text{ i.e.}$$

$$(\ln k+1)\cdot xy'+\ln k\cdot x^2\cdot y''=\log_k x,$$

and the solution it in ω_0 will be:

$$\frac{x^2}{2}-x\setminus\omega_1\to\omega_0\setminus f^*=k^{\frac{1}{2}\log_k^2 x-\log_k x}=$$

$$=k^{\log_k x\cdot\left(\frac{1}{2}\log_k x-1\right)}=x^{\log_k\sqrt{x}-1}.$$

All reduced examples illustrate a solution only of *inverse* task: under the ready equation and it to a solution we construct infinite a spectrum similar on ω-factor, but more complicated on a structure of the equations and their solutions. Not stopping on a proof we shall remark, that there is *a ω-space, where any given differential equation can be solved analytically, and also there is an algorithm of searching such ω-space.*

It is natural, that not always solution can be represented by a finite number of known functions … That in the given book one function of connection $\left(k^x\right)$ is investigated only and there is no depth of study of a problem about the differential equations, solution of the *direct* task (solution of *any* given differential equation, applying a method ω-reflections), within the framework of the above-stated material, practically, it is impossible.

In summary, we shall remark, that ω-images of derivatives can be used not only for a solution of the differential equations, but also for a wider circle of other tasks. Examples of their elementary application are rationalization of a determination of a local extremum or raise of an exactitude of an evaluation of very large numbers.

Let there is a density function of probability of a logarithmically normal aleatory variable:

$$p(x)=\prod_{i=1}^{k}\chi_i=\prod_{i=1}^{k}\left(\frac{\log e}{\sigma_i x_i\cdot\sqrt{2\pi}}\right)\cdot\exp\left(\frac{-\left(\log_{x_i}-a_i\right)^2}{2\cdot\sigma_i^{\,2}}\right),$$

$$x_i>0,\ -\infty<a_i<\infty,\ \sigma_i^{\,2}>0,\ \left(x_i\le 0\quad\Rightarrow\quad p(x)=0\right).$$

For a determination of an extremum $p(x)$ we shall take advantage of a condition ${}^{'}p(x)=0$:

$$'p(x)=\bigcup_{1}^{k}\left\{'P_{x_i}(x)\right\}=0, \qquad 'P_{x_i}(x)=\sum_{i=1}^{k}{}'(\chi_i)_{x_i}=$$

$$=\sum_{i=1}^{k}\left({}'\left(\frac{\log e}{\sigma_i\sqrt{2\pi}}\right)_{x_i}+{}'\left(\exp\left(\frac{-(\log x_i-a_i)^2}{2\cdot\sigma_i^{\,2}}\right)\right)_{x_i}-{}'(x_i)_{x_i}=\right.$$

$$=\sum_{i=1}^{k}\left(-(\log x_i-a_i)\cdot\frac{\log e}{\sigma_i^{\,2}}-1\right)=0 \;\Rightarrow\; x_i=\exp\left(\frac{a_i-\dfrac{\sigma_i^{\,2}}{\log e}}{\log e}\right),$$

$$i=\in(1,\ldots,k) \text{ etc.}$$

Such solution is easier well-known. A key to an exact evaluation of large numbers sit in the formula:

$$\exp(f-x+\ln f'+\Delta x)\approx \exp f\left(\ln\left(e^{x}+e^{\Delta x}\right)\right)-\exp f(x), \quad (5.24)$$

which is uneasy for receiving from a derivative

$$\overset{\mathrm{o}}{f}=f-x+\ln f', \; \overset{\mathrm{o}}{f}=\lim_{\Delta x\to(-\infty)}\left[\ln\left(e^{f\left(\ln\left(e^{x}+e^{\Delta x}\right)\right)}-e^{f}\right)-\Delta x\right]. \quad (5.25)$$

For large Δx:

$$\overset{\mathrm{o}}{f}=f-x+\ln f'\approx \ln\left(e^{f\left(\ln\left(e^{x}+e^{\Delta x}\right)\right)}-e^{f}\right)-\Delta x.$$

Selecting fast-growing it is possible to receive function and setting any values x and very large values Δx, technique of an evaluation with a large exactitude of rather large numbers.

In approximate evaluations it is expedient to use invariant concerning an image of a derivative ${}^{\mathrm{p}}_{i}f$ the formula (${}^{\mathrm{p}}_{i}f$ $-$image in ω_0 derivative noted in ω_i-space):

$$f\left({}_{j}^{i}\Re_{x0}^{\delta x}\right) \approx {}_{j}^{i}\Re_{f(x0)}^{{}_{j}^{i+1}\Re_{\delta x}^{{}_{i}^{\mathrm{p}}f(x0)}} \tag{5.26}$$

The note. In different situations it is necessary to apply ω-images from different spaces.

Let's reduce the elementary example:

$$\sin 20^\circ \approx \frac{1}{2} + \cos 30^\circ \cdot \frac{10\pi}{180^\circ} \approx 0{,}651$$

$$\sin 20^\circ \approx \frac{1}{2} \cdot \left(\frac{20}{30}\right)^{30\pi \cdot \mathrm{ctg}\left(\frac{30^\circ}{180^\circ}\right)} \approx 0{,}346,$$

i.e. even not reduced to a scale ω_0 derivative ${}^{'}f$ gives in this case outcome close to true (0,342), as against application of a usual derivative in the formula of small increments $\left(f(x+\Delta x) \approx f(x) + f'(x) \cdot \Delta x\right)$, where the outcome in 1,9 times exceeds true. (Though in other cases the formula can appear not suitable).

Let's remark also, that at a solution of the applied tasks (in particular economic, hydrodynamic, chemical and others, processes, connected to optimization, and apparatus of this or that process engineering), frequently it is expedient a mathematical model to represent as ω-images of the differential equations and to apply side conditions of an extremum, following from ω-images of a derivative.

§ 5.6. ω-images of the numerical methods of a solution of differential and integral equations

One of the most important practical applications ω-images is the modernizing of the numerical methods of a solution differential and integral equations. Such application, to some extent, is compensated in development of a computer engineering. Nevertheless, the problem this deserves attention as, from a point of view, saving of time of maintenance of the computer, and in a plane of more deep understanding of effects originating at ω-reflections. The examples of the most elementary situations understand the present book. So, most interesting is the conclusion about a passage to the limit of adjacent spaces ω_0' in space of a higher rank ω_1. (The space ω_1 according to the theorem 3.15 is an asymptotics of adjacent spaces ω_0').

5.6.1. ω-image of a numerical solution of the differential equation

As is known, in a method of the Euler $i+1-$a value of functions (y_{i+1}) discover under the formula.

$$y_{i+1} = y_i + (x_{i+1} - x_i) \cdot f(x_i, y_i) \tag{5.27}$$

By noting (5.27) in terms ω_1-spaces and then by reflection obtained expression in ω_0, we shall discover:

$$y_{i+1} = y_i \cdot \left(\frac{x_{i+1}}{x_i}\right)^{x_i \cdot \frac{f(x_i, y_i)}{y_i}} \tag{5.28}$$

Really, ${}'y = \log\left(\frac{y_{i+1}}{y_i}\right) \Big/ \log\left(\frac{x_{i+1}}{x_i}\right)$, where ${}'y = x_i \cdot (\ln y)'$.

Then $f(x_i, y_i) \approx \frac{y_i}{x_i} \cdot \log\left(\frac{y_{i+1}}{y_i}\right) \Big/ \log\left(\frac{x_{i+1}}{x_i}\right)$. Whence we shall receive the formula (5.28).

Similarly, in case of image from adjacent space it is uneasy to receive (see theorem 3.14):

$$y'_{k2} = \lim_{\Delta x \to 0} \frac{\sqrt[p]{f^p \cdot \left(\sqrt[p]{x^p + (\Delta x)^p}\right) - f^p(x)}}{\Delta x},$$

$$y'_{k2} \quad \to \quad \frac{\sqrt[p]{y_{i+1}^p - y_i^p}}{\sqrt[p]{x_{i+1}^p - x_i^p}} = \sqrt[p]{\frac{y_{i+1}^p - y_i^p}{x_{i+1}^p - x_i^p}},$$

$$\left(f'_{k2} = \frac{f}{x} \cdot \sqrt[p]{\frac{f' \cdot x}{f}} \quad \to \quad \left(\frac{x \cdot f'_{k2}}{f} \right)^p = \frac{f' \cdot x}{f} \quad \to \right.$$

$$\left. \to \quad f' = \frac{f}{x} \cdot \left(\frac{x \cdot f'_{k2}}{f} \right)^p = \left(\frac{x}{f} \right)^{p-1} \cdot \left(f'_{k2} \right)^p \right)$$

$$f(x_i, y_i) = y' = \left(\frac{x_i}{y_i} \right)^{p-1} \cdot \frac{y_{i+1}^p - y_i^p}{x_{i+1}^p - x_i^p} \quad \to$$

$$\to \quad \left(\frac{y_i}{x_i} \right)^{p-1} \cdot f(x_i, y_i) = \frac{y_{i+1}^p - y_i^p}{x_{i+1}^p - x_i^p}.$$

$$y_{i+1} = \sqrt[p]{\left(x_{i+1}^p - x_i^p \right) \cdot \left(\frac{y_i}{x_i} \right)^{p-1} \cdot f(x_i, y_i) + y_i^p} \qquad (5.29)$$

Example. In a table the outcomes of a solution of the equation $y' = x \cdot \sqrt{y}$ for the entry conditions $x_0 = 1$, $y_0 = 1$, $h = 0.1$, $N = 5$ are represented.

Table 1. Outcomes of a numerical solution of the equation $y' = x \cdot \sqrt{y}$ ($x_0 = 1$, $y_0 = 1$, $h = 0.1$, $N = 5$).

x	**Euler**	**S-Euler**	**P-Euler**	**True value**
1.1	1.1	1.1	1.1	1.10776
1.2	1.21537	1.21616	1.21576	1.23210
1.3	1.34766	1.35015	1.34888	1.37476
1.4	1.49858	1.50380	1.50114	1.53760
1.5	1.66996	1.67912	1.67444	1.72266

The note. S-Euler, P-Euler – upgraded method of the Euler obtained by reflection accordingly \ $\omega_1 \to \omega_0$ \ and \ $\omega_0' \to \omega_0$ \.

Let's estimate an error of evaluations. In a well-known method of the Euler a summarized error:

$\beta_0 = \varepsilon_0 = \frac{1}{2!} \cdot |f''(\xi)| \cdot h^2$, $x_i < \xi < x_{i+1}$, $\varepsilon_0 -$error in a method of the Euler. The error ε_1 is equal a upgraded method of the Euler by a way $\backslash \omega_1 \to \omega_0 \backslash$ transformations $(\varepsilon_1, \beta_1 -$images in ω_0 summarized error of a method of the Euler from ω_1, not reduced (ε_1) and reduced (β_1) to a scale $\omega_0)$:

$$\varepsilon_1 = \left(\overset{''}{f}(\xi)\right)^{\frac{\ln^2 H}{2!}} = C^{\frac{\ln^2 H}{2!}} \qquad (C = const),$$

$$\ln \varepsilon_1 \sim \ln^2 H \quad \Rightarrow \quad \varepsilon_1 \sim e^{\ln^2 H} = \left(1 + \frac{h}{x_i}\right)^{\ln\left(1 + \frac{h}{x_i}\right)},$$

$$\varepsilon_1 \sim \left(1 + \frac{h}{x_i}\right) \odot \left(1 + \frac{h}{x_i}\right) \quad \Rightarrow \quad \varepsilon_1 \sim H \odot H, \qquad (5.30)$$

where $H - \omega -$image of a pitch $h\ (h \backslash \omega_1 \to \omega_0 \backslash H)$.

So, $\varepsilon_1 \sim H \odot H = H^{\to 2}$, i.e. $\beta_1 = \ln \varepsilon_1 \sim \ln^2 \left(1 + \frac{h}{x_i}\right)$.

Whence $\beta_1 \sim \frac{h^2}{x_i^2} \qquad \left(\ln\left(1 + \frac{h}{x_i}\right) \approx \frac{h}{x_i}\right)$.

In an outcome:

$$\frac{\beta_0}{\beta_1} \sim h^2 : \frac{h^2}{x_i^2} = x_i^2, \qquad (5.31)$$

i.e. at $|x_i| > 1$ the upgraded method of the Euler is more effective, than usual. At magnification $|x_i|$ and $|x_i| > 1$ the exactitude of evaluations fast grows. It is uneasy to show, that in Runge-Kutta method the ratio $\frac{\beta_0}{\beta_1} \sim x_i^{\,4}$, i.e. at growth x_i rather fast is possible to receive, actually, true value.

Let's give the additional explanations. If $f(x)-$function of passage between $\omega-$spaces, the absolute error $\Delta_{\text{com.}}$ is proportional $\left(f^{-1}(h+x_i)-f^{-1}(x_i)\right)^2$.

Really, let $h = x_{i+1} - x_i$, and

$$h \setminus \omega_1 \to \omega_0 \setminus H.$$

Then $H = f\left(f^{-1}(x_{i+1}) - f^{-1}(x_i)\right) = f\left(f^{-1}(h + x_i) - f^{-1}(x_i)\right)$,

The not reduced absolute error

$$\varepsilon \sim f\left(f^{-1}(H) \cdot f^{-1}(H)\right),$$

and the absolute error $\omega-$image of a method of the Euler in ω_0, reduced to a scale ω_0 is equal:

$$\Delta_{\text{com}} \sim f^{-1}\left(f\left(f^{-1}(H) \cdot f^{-1}(H)\right)\right) = \left(f^{-1}(H)\right)^2 =$$

$$= \left(f^{-1}\left(f\left(f^{-1}(h+x_i) - f^{-1}(x_i)\right)\right)\right)^2 = \left(f^{-1}(h+x_i) - f^{-1}(x_i)\right)^2,$$

as was to be shown.

If to increase a rank $\omega-$space and to put $f(x) = e^x$,

$$h \setminus \omega_2 \to \omega_0 \setminus H_s = x_{i+1} \triangle x_i = (h+x_i)^{1/\ln x_i}.$$

Then $\Delta_{H_s} = \left(\ln\ln (h+x_i)^{1/\ln x_i}\right)^2 = \ln^2\left(\log_{x_i}(h+x_i)\right)$.

By designating $\Delta_{\text{com}} \equiv \Delta_s$, and $\Delta_{H_s} \equiv \Delta_{2s}$, we shall receive

$$\Delta_s \sim \ln^2\left(1+\frac{h}{x_i}\right) \approx \left(\frac{h}{x_i}\right)^2, \qquad \Delta_{2s} \sim\sim \ln^2 \frac{\ln(h+x_i)}{\ln x_i} =$$

$$= \ln^2 \frac{\left(\left(\frac{h}{x_i}+1\right)\cdot x_i\right)}{\ln x_i} \approx \ln^2\left(\frac{\frac{h}{x_i}+\ln x_i}{\ln x_i}\right) = \ln^2\left(\frac{h}{x_i \cdot \ln x_i}+1\right) \approx$$

$\approx \left(\frac{h}{x_i \cdot \ln x_i}\right)^2$, i.e. error in a well-known method of the Euler $\Delta_0 \sim h^2$, and in upgraded methods: $\Delta_s \sim \left(\frac{h}{x_i}\right)^2$, $\Delta_{2s} \sim \left(\frac{h}{x_i \cdot \ln x_i}\right)^2$ etc.

By taking a ratio $\frac{\Delta_0}{\Delta_s}$ and $\frac{\Delta_0}{\Delta_{2s}}$, we shall discover:

$$\frac{\Delta_0}{\Delta_s} \sim \frac{h^2}{h^2 / x_i^2} = x_i^2,$$

and $$\frac{\Delta_0}{\Delta_{2s}} \sim \frac{h^2}{h^2 / \left(x_i^2 \cdot \ln^2 x_i\right)} = \left(x_i \cdot \ln x_i\right)^2,$$

i.e. the upgraded methods are exacter than a usual method of the Euler accordingly in x_i^2 time and in $\left(x_i \cdot \ln x_i\right)^2$ time.

In summary we shall discover a value of argument x_i, when the upgraded method of the Euler (ω- image \ $\omega_1 \to \omega_0$ \) on an exactitude is adequate to a Runge-Kutta method:

$$\frac{h^4}{h^2 / x_i^2} = 1 \quad \Rightarrow \quad x_i = \frac{1}{h}.$$

5.6.2. ω-image of a solution of an integral equation

ω-modernizing the numerical methods of a solution of integral equations (in.eq.) reduces also in a drop of an error of these methods. Besides using, ω-images in.eq., we shall receive in.eq. a more complicated structure with a ready solution.

Let's consider elementary one-dimensional in.eq. the Fredholm of the 2-nd sort

$$\varphi(x) = \lambda \cdot \int_a^b R(x,s) \cdot \varphi(s) \cdot ds + f(x), \qquad (5.32)$$

where $f(x)$ is continuous on $[a,b]$, λ – numerical parameter, $R(x,s)$ – kernel, continuous on $a \le x$, $s \le b$, $\int_a^b \int_a^b |R(x,s)|^2 \cdot dx\,ds < \infty$.

Let's reflection (5.32) of ω_1 in ω_0:

$$k^{\varphi(\log_k x)} = k^{\lambda} \odot \exp\left(\int_a^b \frac{\log_k R(x,s) \cdot \varphi(s) \cdot ds}{s} \right) \cdot k^{f(\log_k x)} \quad \Rightarrow$$

$$\Rightarrow \varphi(\log_k(x)) = \lambda \cdot \left(\frac{1}{\ln k} \cdot \left(\int_a^b \frac{\log_k R(x,s) \cdot \varphi(s) \cdot ds}{s} \right) + f(\log_k x) \right),$$

where $f(\log_k x)$ is continuous on $[a,b]$, $a \le x$, $s \le b$,

$$\exp\left(\int_a^b \log_k \left(\exp\left(\int_a^b \frac{\log_k |R(x,s)|^2}{s} ds \right) \right) \frac{dx}{x} \right) < \infty,$$

$$\int_a^b \log_k \left(\exp\left(\int_a^b \frac{\log_k |R(x,s)|^2}{s} ds \right) \right) \frac{dx}{x} < \infty$$

$$\text{or} \int_a^b \left(\int_a^b \frac{\log_k \left| R(x,s) \right|^2}{s} ds \right) \cdot \frac{dx}{x} < \infty .$$

The approximate solution (5.32) is noted so:

$$\varphi_n^{\sim}(x) = f(x) + \lambda \cdot \sum_{j=1}^{n} c_j \cdot R\left(x, x_j\right). \tag{5.33}$$

Let's reflection $\varphi_n^{\sim}(x)$ from ω_1 in ω_0:

$$\varphi_n^{\sim}(x) \setminus \omega_1 \to \omega_0 \setminus k^{\varphi_n^{\sim}(\log_k x)},$$

$$R\left(x, x_j\right) \setminus \omega_1 \to \omega_0 \setminus k^{R\left(\log_k x, \log_k x_j\right)},$$

$$\varphi_n^{\sim}(x) = f(x) + \lambda \cdot \sum_{j=1}^{n}{}' c_j \cdot R\left(x, x_j\right) \setminus \omega_1 \to \omega_0 \setminus k^{\varphi_n^{\sim}(\log_k x)} =$$

$$= k^{f(\log_k x)} \cdot k^{\lambda} \odot \prod_{j=1}^{n} k^{c_j} \odot k^{R\left(\log_k x, \log_k x_j\right)} \Rightarrow$$

$$\Rightarrow \quad k^{f(\log_k x) + \lambda \cdot \sum_{j=1}^{n} c_j \cdot R\left(\log_k x, \log_k x_j\right)} \Rightarrow$$

$$\Rightarrow \quad \varphi_n^{\sim}\left(\log_k x\right) = f\left(\log_k x\right) + \lambda \cdot \sum_{j=1}^{n} c_j \cdot R\left(\log_k x, \log_k x_j\right) \tag{5.34}$$

Not stopping on examples of a solution of integral equations by a way ω-reflections, we shall remark, that the positive effect is reached only at appropriate reflection that or other method of a solution in.eq. in whole. For this purpose it is enough to take from the literature [42] any such methods and to realize ω-reflection.

§ 5.7 Examples ω-image of a vector of adjacent space

The problem, affected in chapter 4, about quasivectorial the analysis can receive the identified elementary development if to enter other concept quasivectorial. For example, let in adjacent space ω_0' the vector $\bar{a}$ is given. Let's reflection it in space ω_0 in two acts:

– we shall reflection $\bar{a}$ in ω_1;

– we shall reflection obtained an ω-image $\bar{a}$ from ω_1 in ω_0.

Let's receive:

$$\bar{a} \setminus \omega_0' \to \omega_1 \setminus \log_{k_2} \bar{a} \setminus \omega_1 \to \omega_0 \setminus k_1^{\log_{k_2} \bar{a}} = \overline{a_*},$$

where $\overline{a_*}$ – image of a vector $\bar{a}$ in space ω_0, if it is given originally in adjacent space ω_0'; k_1 and k_2 – factors in function of connection k^x between spaces $\omega_1 \to \omega_0(k_1)$ and $\omega_1 \to \omega_0'(k_2)$.

From equality $\overline{a_*} = k_1^{\log_{k_2} \bar{a}} = \left(k_1^{\frac{1}{\ln k_2}} \right)^{\ln \bar{a}} = \left(\bar{a}\right)^p$, where $p = \log_{k_2} k_1$, follows, that $\overline{a_*} = \bar{a}^p$, and $\dfrac{\ln \overline{a_*}}{\ln \bar{a}} = p_*$ and $\dfrac{\overline{a_*}}{\bar{a}} = \dfrac{k_1}{k_2}$,

$\overline{a_*} = \dfrac{k_1}{k_2} \cdot \bar{a} = \lambda \cdot \bar{a}$, i.e. $\overline{a_*}$ both $\bar{a}$ – collinear vectors and $\left|\overline{a_*}\right| = \lambda \cdot \left|\bar{a}\right|$.

It is quite natural, that by virtue of a property of a contiguity of spaces ω_0 and ω_0' any vector $\bar{a} = \sum_{i=1}^{n} a_i \cdot \overline{e_i}$ has an image $\overline{a_*} = \sum_{i=1}^{n} a_{*_i} \cdot \overline{e_i}$, distinguished from $\bar{a}$ only modulo, i.e. $\overline{a_*}$ – *ordinary* vector oriented in space ω_0 the same as and a vector $\bar{a}$ in space ω_0'.

However, in this case, there was also new mathematical object $\ln \bar{a}$, which is specific *quasivectorial* or *quasivectorial of the second type*. Let's designate $\overline{\overline{a_k}} = \log_k \bar{a}$. Let vector $\bar{a}$ is given in space ω_{-1}.

Let's reflection it in space ω_0, slanting, that the conversion factor $\omega_{-1} \to \omega_0$ is equal k:

$$\bar{a} \setminus \omega_{-1} \to \omega_0 \setminus \log_k \bar{a} = \bar{\bar{a}}_k.$$

Similarly,

$$\bar{a} \setminus \omega_0 \to \omega_1 \setminus \log_{k_1} \bar{a},$$

$$\bar{a} \setminus \omega_0' \to \omega_1 \setminus \log_{k_2} \bar{a}.$$

In case the projections of a vector $\bar{a}$ are functions, that, knowing the formulas of reflection of functions and integro-differential objects in ω-spaces, it is uneasy to construct *quasivectorial the analysis of the second type*. Let's remark, that the principal element of this analysis is the object $\bar{\bar{a}}_k = \log_k \bar{a}$ (or simple $\bar{\bar{a}}$).

§ 5.8. Some additional facts about numbers of a new nature

In chapter 2 the attempt is made to expand a field of real numbers. In particular, the set R_Δ Δ-numbers is represented which can be received from operation " $\circ$ " (easier additions) and reflexive reflection of a set $\mathbf{R}$ real numbers. Is shown, that the field Δ-numbers turns out also ω-reflection of a set $\mathbf{R}_-$ negative numbers:

$$\mathbf{R}_- \setminus \omega_0 \to \omega_1 \setminus \mathbf{R}_\Delta.$$

Is established, that $\mathbf{R}_\Delta = \{\log(-a)\}$, where $a \in R_+$.

Outcome of similar reflection Δ-numbers will be a set $R_{\underline{\Delta}}$ $\underline{\Delta}$ $-$numbers:

$$R_\Delta \setminus \omega_0 \to \omega_1 \setminus R_{\underline{\Delta}}.$$

The process of such reflections can be continued ad infinitum. If to designate a $R_{\Delta i}$ $-$set of numbers generated by a way of ω_i-reflection $\left(R_{\Delta i-1} \setminus \omega_0 \to \omega_1 \setminus R_{\Delta i}\right)$, it is possible to note, that we deal with infinite by a spectrum $\{R_{\Delta i}\}$ infinite of *sets of numbers of a new nature*. For example, $R_\Delta \equiv R_{\Delta 0}$, $R_{\underline{\Delta}} \equiv R_{\Delta -1}$ etc. Besides it is not necessary to forget, that each of sets $\mathrm{R}_{\Delta i}$ $\Delta i-$numbers is generated by introduction of new algebraic operation. So, the set R_{Δ} Δ-numbers arises at reviewing operation " $\circ$ ", i.e. than the first operation the additions are easier. Similarly, the set $\mathrm{R}_{\triangle}$ (or $\mathrm{R}_{\Delta-1}$) is an outcome of the following operation easier than addition (operation easier than operation " $\circ$ "). So is constructed infinite a spectrum $\left\{\mathrm{R}_{\Delta i}\right\}$, where $i-$number of an operation is easier than addition if to consider, that the well-known addition has a value of an index $i=1$. In a fig. 1 is represented numerical direct, supplemented by Δ-numbers. It is natural, that this direct can have an *infinite prolongation to the left* (but not to the right!).

It is uneasy to establish also other classes of numbers of a new nature. For example, in §5.4 present chapters were shown, that an ω-image of imaginary unit is the number $j=\frac{\Delta 0}{2}$ $\left(i \setminus \omega_0 \to \omega_1 \setminus j\right)$, which because of is constructed ω-image of the theory of functions of a complex variable (ω-TFCV). Besides there are "single" elements: $r=\sqrt{\Delta 1}$, $t=\sqrt{-(\Delta 1)}$. All combinations from real and complex numbers (various sets $\mathrm{R}_{\Delta i}$) are numbers of a new nature.

Without a proof[47] we shall write out some equalities $((a,b,c)\in \mathrm{R}$, $a\in \mathrm{R}_{+}$; in a series of cases $(a,b,c)\in \mathrm{Z})$:

$(\Delta 1)\cdot i=i\cdot(\Delta 1)=\Delta i \quad \left(i=\sqrt{-1}\right)$; $\quad (\Delta a)\cdot i=\Delta(a\cdot i)$; $\quad (\Delta i)\cdot a=i\cdot a$

$(a\in \mathbf{N}_2)$; $\quad i\cdot(\Delta a)=\Delta(i\cdot a)$; $\quad (\Delta i)\cdot a=\Delta(i\cdot a) \quad (a\in \mathbf{N}_1)$; $\quad i\cdot\Delta i=$

$=(\Delta i)^2=-\Delta 1 \quad \Rightarrow \quad \frac{\Delta 1}{i}=\Delta i$; $\quad \Delta 1=(\Delta i)\cdot i$; $\quad \frac{\Delta i}{i}=\Delta 1$; $\quad \frac{\Delta i}{i}=\Delta 1$;

$\frac{\Delta i}{i}=\frac{\Delta i}{i}$; $\quad a^{\Delta 0}=-1$; $\quad a^{\triangle 0}=\Delta 1$; $\quad \ln(\Delta i)=\frac{\Delta 0}{2}+\triangle 0=j+\triangle 0$;

$\ln(\Delta a)=\ln a+\triangle 0=r\cdot a$; $\quad r\cdot a-r\cdot b=\ln\frac{a}{b}=\ln\frac{\Delta a}{\Delta b}$; $\quad \ln\Delta 1=\triangle 0$

$(\ln(-1)=\Delta 0)$; $\quad$ (at $\quad a\in \mathrm{R}_{+}$ $\quad \ln(:a)=-\ln a+0\in \mathbf{R}_{-}$ $\quad$;

[47] The separate equalities already met in the text. To prove offered equalities the reader can independent.

$\ln(-a) = \ln a + \Delta 0 = \Delta \ln a \in \mathrm{R}_{\Delta}$; $\ln \Delta a = \ln a + \triangle 0 = \triangle \ln a \in \mathrm{R}_{\triangle}$ etc.);

$\ln\left(-\left(\sqrt{-\Delta 1}\right)\right) = \Delta 0 + \frac{\Delta 0}{2} + \frac{\triangle 0}{2}$; (at $a, b \in \mathbf{R}$, $a + \Delta b = \Delta(a+b) \in \mathrm{R}_{\Delta}$;

$a + \triangle b = \triangle(a+b) \in \mathrm{R}_{\triangle}$; $a + \triangle(\Delta b) = \Delta(\triangle(a+b))$; $\Delta a + \Delta b = a + b$;

$\Delta a + \triangle b = \Delta(\triangle(a+b))$; $\Delta a + \triangle(\Delta b) = \triangle(a+b) \in \mathrm{R}_{\triangle}$; $\mathbf{R} + \mathbf{R} = \mathbf{R}$,

$\mathbf{R}_{\Delta} + \mathbf{R} = \mathbf{R}_{\Delta}$, $\mathbf{R}_{\triangle} + \mathbf{R} = \mathbf{R}_{\triangle}$ etc.); $\triangle \ln a = \ln \Delta a$ $(\Delta \ln a = \ln(-a))$;

$\left(a^{-b} =: \left(a^{b}\right) \in \mathbf{R}_{f}\right.$; $a^{\Delta b} = -\left(a^{b}\right) \in \mathbf{R}_{-}$; $a^{\triangle b} = \Delta\left(a^{b}\right) \in \mathbf{R}_{\Delta}$ etc.);

$(\Delta a)^{b} = \Delta\left(a^{b}\right)$, but $(\Delta a)^{b} = a^{b}$, if $a \in \mathbf{N}_1$, $b \in \mathbf{N}_2$;

$$(-a)^{\Delta b} = \begin{cases} a^{b}, \ b \in \mathbf{N}_1 \\ (-a)^{\Delta b} = a^{\Delta b}, \ b \in \mathbf{N}_2 \end{cases} \text{ by analogy with}$$

$$(-a)^{-b} = \begin{cases} -\left(:\left(a^{b}\right)\right), \ b \in \mathbf{N}_1 \\ \left(:a^{b}\right), \ b \in \mathbf{N}_2 \end{cases};$$

$(-a)^{\triangle b} = (-1)^{b} \cdot a^{b}$, $b \in \mathbf{N}_2$, $(-a)^{\triangle b} = (-1)^{b} \cdot \Delta\left(a^{b}\right)$, $a^{b} \in \mathbf{N}_1$; $i = k^{j}$

$(k \neq 0)$; $(\Delta a)^{-b} = \begin{cases} a^{-b}, \ (a,b) \in \mathbf{N}_2 \\ a^{-b} \cdot \Delta 1, \ a^{b} \in \mathbf{N}_1 \end{cases}$ etc.; $(-a)^{\triangle b} = \begin{cases} (-1)^{b} \cdot a^{b}, \ a^{b} \in \mathbf{N}_2 \\ (-1) \cdot \Delta\left(a^{b}\right), \ a^{b} \in \mathbf{N}_1 \end{cases}$;

$\Delta a + j = a + \Delta j$; $\sqrt[\Delta a]{\Delta b} = \sqrt[a]{b} \cdot \sqrt[a]{-\Delta 1}$ and others.

Example of numbers of a *new* nature are also numbers of a type $\alpha \Delta \beta$ at $\alpha \notin \mathbf{Z}$. The modification α on $\pm|\gamma|$ will call a shift of a point representing number on direct accordingly to the right or to the left (in an assotiation from a sign at $|\gamma|$, saving a sense of number. *It is possible*, that numbers ξ_k

$\left(\xi_k = \alpha_k \Delta \beta_k\right)$ in this case turn out, are located about $(-\infty)$, but not being the Δ-numbers (observe some analogy with hypernumbers, known of the non-standart analysis) …

Does not call doupts that any *new* operation "gives rise" to numbers of a *new* nature. Using combinations of new numbers and various operations, it is possible to receive difference various new variations of numbers similarly to complex.

In summary we shall mark, that in light of new representations about numbers the area of admissible values of functions extends. (The known functions can be considered on sets of *new* numbers).

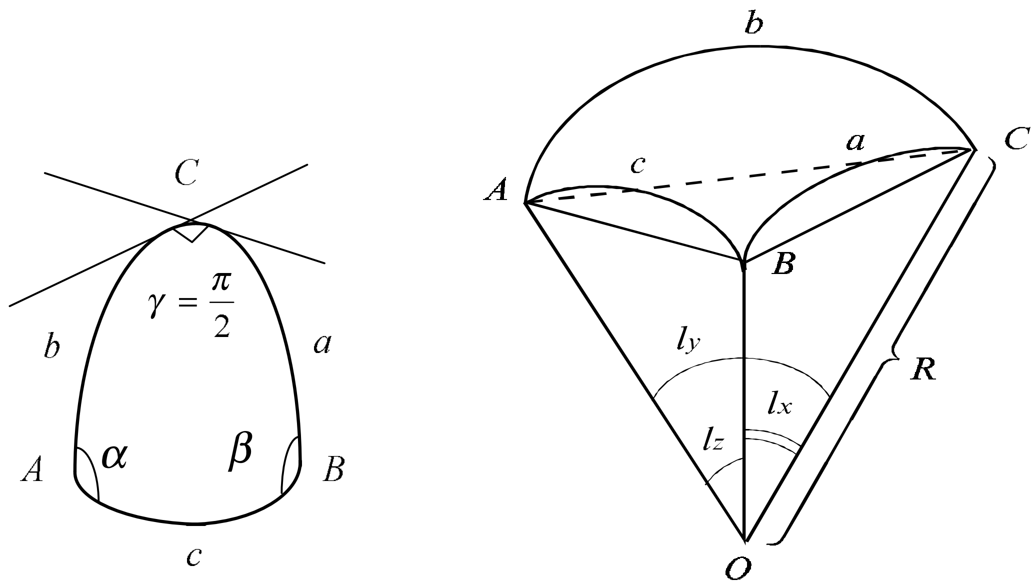

Fig. 7. An image of a rectangular spherical triangle ABC.

§ 5.9. An example of application ω- reflections in spherical trigonometry

Let on an orb the rectangular triangle ABC is given. Let's design it on a plane, i.e. we shall discover flat reflection of this triangle or triangle, on which the spherical triangle (fig. 7) leans.

It is possible to note the following relations:

$$\alpha = \arcsin\left(\frac{\sin l_x}{\sin l_z}\right),$$

$$\beta = \arcsin\left(\frac{\sin l_y}{\sin l_z}\right),$$

$$\gamma = \frac{\pi}{2}, \qquad \varepsilon = \alpha + \beta - \frac{\pi}{2}, \quad s_1 = R^2 \cdot \varepsilon,$$

where l_x, l_y, l_z – angles COB, COA, BOA; ε – spherical kurtosis; s_1 – square of a spherical triangle ABC. Then $k = \frac{s_1}{s_2}$, where s_2 – square of a flat triangle ABC ($p = \frac{x+y+z}{2}$, $s_2 = \sqrt{p(p-x)(p-y)(p-z)}$); $a = l_x \cdot R$, $b = l_y \cdot R$, $c = l_z \cdot R$; a, b, c – magnitude of legs of a spherical triangle ABC, expressed in radians; x, y, z – magnitude of legs of a flat triangle ABC.

In connection with that at reflection of a sum of two variables x and y $(x+y)$ from adjacent space $\omega_0{}'$ in ω_0 we shall receive $z = \sqrt[p]{x^p + y^p}$ $\left(x + y \setminus \omega_0{}' \to \omega_0 \setminus \sqrt[p]{x^p + y^p}; x, y, z, p \in R_+\right)$, we shall try to find any regularities ω-reflection $z = x + y$ at transformation of this sum in a triangle, to which there corresponds a spherical rectangular triangle. Here we suppose a hypothesis about a reflection of a triangle with legs x, y, z, where $z = \sqrt[p]{x^p + y^p}$, $p \geq z$, $p \in Z$, on an orb or hyperspherical. This hypothesis is based on a special case: at $p = 2$ the ω-reflection of a sum of two variables x and y is transformed in $z = \sqrt{x^2 + y^2}$, i.e., if x and y – legs of a rectangular triangle, z – are it a hypotenuse: $x + y \setminus \omega_0{}' \to \omega_0 \setminus \sqrt{x^2 + y^2}$.

Proceeding from this hypothesis, in the given example the image of flat triangles with legs x, y, z ($z = \sqrt[p]{x^p + y^p}$, $p > 2$, $p \in \mathbf{Z}$) on an orb provided that one angle of an appropriate spherical triangle always direct is carried out. The research of this situation is carried out *numerically* at various x, y, p.

Let's reduce some *elementary* conclusions, which can be made, analyzing obtained outcomes: [48]

a). At magnification $p\,(p \to \infty)$ the triangle ABC *tends to isosceles*, as at $p \to \infty$ the set of adjacent spaces $\{\omega_0{}'\}$ turns in ω-space of a higher rank ω_1.

b). In case of equality x and y irrespective of their magnitude a lot of performances of a spherical triangle ABC becomes to *constants*. For example, *spherical excess* (for $p = 3$ $\varepsilon \approx 0{,}508584$), ratio $k = \frac{s_1}{s_2}$ (for $p = 3$ $k \approx 1{,}25975$), $l_x + l_y$ (for $p = 3$, $l_x + l_y = 0{,}9430$ radian), $l_x + l_y + l_z$ (for $p = 3$, $l_x + l_y + l_z = 3{,}1045$ radian) etc.

c). The replacement x both y on mx and my $(m = const, m \in \mathbf{R}_+, m \in \mathbf{Z})$ reduces in respective alteration of a radius of an orb, i.e. the radius becomes equal mR (for example, for $p = 3$ at $x = 10$, $y = 11$, $R = 11{,}5698$; at $x = 100$, $y = 110$, $R = 115{,}698$; at $x = 1000$, $y = 1100$, $R = 1156{,}98$).

d). There are precise assotiations of all performances of a spherical triangle ABC from magnitude p…

In summary, we shall remark, that both *norm* of a vector x $\left(\|\bar{x}\| = \sqrt{\sum_{i=1}^{n} x_i{}^2}\right)$, and *spectral radius* $|x| = \lim_{p \to \infty} \|x^p\|^{\frac{1}{p}}$ In $\mathbf{X}$-banach algebra above $\mathbf{C}$, being a *seminorm*, if it is *uniform is continuous* on $\mathbf{X}$ (this condition is equivalent commutabilities of a quotient algebra on a radical), are connected with ω-reflection in adjacent spaces and passage to the limit at $p \to \infty$ set of adjacent spaces $\{\omega_0{}'\}$ in space of a higher rank ω_1 (and, this passage is realized is *uniform continuously*).

Let's mark also some facts, which can be used at a solution of the practical tasks:

1. *All physical laws have infinite a set ω-images, which are the independent laws.*

[48] All conclusions, reduced in the given example, (outcomes) can be obtained by analytical methods.

For example, the fields of temperatures, strength and potential of an electrical field, velocity of current of a liquid (gas) etc. between two parallel slices ($f_1 = a_1 \cdot x + b_1 \cdot y$, $f_2 = a_2 \cdot x + b_2 \cdot y$) are known, if the appropriate boundary conditions are known. Slanting, that the slices are located in ω_0', we shall reflection them in space ω_0. Let's receive cones $\left(f_1^* = \sqrt{a_1 \cdot x^2 + b_1 \cdot y^2},\ f_1^* = \sqrt{a_1 \cdot x^2 + b_1 \cdot y^2}\right)$. It is uneasy to note ω-images of fields of physical magnitudes known in space between parallel slices. Obtained the ω-images will describe quasifield between two conic surfaces.

2. *For each process there is such ω-space, after reflection which from in ω_0 the mathematical model of the process is rather close to theoretical exposition.*

It is connected that the goal function (response function) is influenced by a set of the factors. At ω-reflections it is possible to reach a situation, when the major factors will be “defining”, and others – infinitesimal, i.e. to strengthen an approximation of the theory to practice.

3. *Exists infinite a set of mathematical methods identified well-known. This set defines infinite of mathematical expositions of any object.*

For example, it is possible to select such ω-image of a method Euler, which considerably improves primary.

§ 5.10. A problematics of application ω-images

In connection with heterogeneity of a material of chapter 5 in whole has a problematic character. Nevertheless, we shall focus attention on the most important moment.

1. It is necessary to collect, to systematize and to explain meeting in the theory ω-reflections, paradoxes. For example, paradox $(a \cdot b)^{\Delta c} \neq a^{\Delta c} \cdot b^{\Delta c}$, at $a > 0$, $b > 0$. $\left((2 \cdot 3)^{\Delta 2} = 6^{\Delta 2} = -6^2 = -36,\quad 2^{\Delta 2} \cdot 3^{\Delta 2} = \left(-2^2\right) \cdot \left(-3^2\right) = 36\right)$ it is uneasy to explain from a paradox reduced in §2.3 (“The note 10”).

$$\lg(a \cdot b)^{\Delta c} \neq (\lg a + \lg b) \cdot (\Delta c) = \lg a \cdot (\Delta c) + \lg b \cdot (\Delta c),$$

$$\left(a^* + b^*\right) \cdot \left(\Delta c^*\right) \neq a^* \cdot \left(\Delta c^*\right) + b^* \cdot \left(\Delta c^*\right).$$

2. To classify and to describe the special operations and numbers of a new nature, similarly to represented in the given chapter.

3. To generate the theory of functions $j-$variable.

4. To apply the theory ω-reflections at a solution of the various practical tasks.

5. To solve the *direct* task in item 5.5., i.e. to develop algorithm of a determination ω-space, where any differential equation is solved already in known receptions.

6. To investigate application ω-reflections not only in a spherical trigonometry, but also in a trigonometry based on a hyperboloid of one sheet or other surface. To try to prove the great theorem the Fermat.

7. To study restrictions superimposed on the differential equations and their system, which can be solved by the upgraded numerical methods with the help of ω-reflections.

8. To find ω-images of various methods of searching of an optimum (extremums) of functions of a several variable.

9. Because of analysis of the theory ω-reflections to try deeper to understand a real, enclosing us.

Inference

The known mathematical tools of *exposition* of physical objects complicated, are not rational and, as a rule, do not ensure analyticities of a solution of the practical tasks. The spreaded methods of approximation at a numerical solution differential and integral equations allow with a defined approximation to judge a final outcome, but do not satisfy desires to learn to the full physical object.

The *material*, *explained* in the given book, is only *example* or special case in a developed methods ω-reflection and **common theory of objects**. Nevertheless, the author hopes, that offered in the book the material will allow not only to attract the inquisitive reader in creativity and innovation, but also will give him primary skills for understanding the common theory of objects.

$\mathbf{N}_1$, $\mathbf{N}_2$ $-$ accordingly, set of odd and even numbers;

$\mathbf{R}$, $\mathbf{R}_+$, $\mathbf{R}_f$, $\mathbf{R}_{ir}$, $\mathbf{R}_-$ $-$ accordingly set real positive, fractional, irrational and negative numbers;

Δ_0 or $\mathbf{R}_\Delta\left(\mathbf{R}_\Delta \equiv \Delta_0\right)$ $-$ set $\Delta -$numbers being solutions of an equation $x \circ a = (-\infty)$, $a \in \mathbf{R}$; $\theta \equiv (-\infty)$;

"$\circ$" – the operation of an order zero $(n = 0)$ is easier than addition, and "Δ"–operation inverse "$\circ$" (thus the addition has an order $n = 1$); $\mathbf{R}_0 = \mathbf{R} \cup \Delta_0 \equiv \mathbf{R} \cup \mathbf{R}_\Delta$;

Δa $-$ label $\Delta -$number $(a \in \mathbf{R})$;

$\mathbf{R}_{\triangle}$ $-$ the set $\triangle -$numbers, where $\triangle -$operation $n = (-1)$, i.e. *second* operation is easier than addition and first operation easier than operation "$\circ$", $\triangle a$ $-$ label $\triangle -$number;

$\mathbf{R}_{\Delta i}$ $-$ sets $\Delta i -$numbers, where $\Delta i -$the operation $(-i)-$of the order is easier than addition (exist also other label: Δ_n $-$ it is possible to connect a set of numbers, which origin with operation Δ_n $(-n)-$of the order easier than addition, and $\Delta_n \equiv \mathbf{R}_{\Delta n}$);

ω_0 – well-known mathematical space with dominating operation of addition;

ω_i, ω_j $-$ i and j – spaces absolutely similar ω_0 on an interior structure, and the difference from ω_0 is exhibited only at reflection (passage) of objects from ω_i (or ω_j) in ω_0 (thus the transformation of objects) happens;

$\backslash \omega_i \to \omega_j \backslash$ $-$ Operation ω- Image of object from space ω_i in space ω_j;

f_c $-$ function of connection between ω-spaces (for example, in the present book $f_c = k^x$, $k > 1$, k $-$ factor of reflection);

$\odot$ $-$ reflexive multiplication (or reflexive multiplication of the *first* order), $a \odot b = a^{\log b / \log k} = b \odot a = b^{\log a / \log k}$, $\odot$ $-$ image in ω_0 operation of *multiplication* in ω_1;

$\triangle$ $-$ reflexive division (or reflexive division of the *first* order),

$a \triangle b = a^{\log_b(k)}$, $(a,b) \in \mathbf{R}$, $k \in \mathbf{R}$, $k \neq 1$, $\triangle -$image in ω_0 operation of *division* in ω_1 (reflexive division);

$\triangle a -$number derivated by operation "$\triangle$" $(\triangle a = k \triangle a = = k^{1/\log_k a}, \quad k \in \mathbf{R}, \quad k \neq 1)$;

$a^{\rightarrow b} -$reflexive exponentation b number a $\left(a^{\rightarrow b} = \underbrace{a \odot a \odot a \odot \ldots \odot a \odot a}_{(\log_k b) \in Z}, (a,b,k) > 0, k \neq 1\right)$, $a^{\rightarrow b} - \omega$-reflection in ω_0 operation of *exponentation* in ω_1;

$:a -$label of fractional number $\frac{1}{a}$, where $a \in \mathbf{R}$, i.e. $:a \equiv \frac{1}{a}$ (by analogy to a label of a negative number $-a = 0 - a$);

$\overset{i}{\underline{a}} - \omega -$image in ω_0 number a, displayed from ω_i space, i.e. $a \setminus \omega_1 \rightarrow \omega_0 \setminus \overset{i}{\underline{a}}$;

$\mathrm{ilog}_a b -$image in ω_0 $\log_a b$ from ω_1, i.e. $\log_a b \setminus \omega_1 \rightarrow \rightarrow \omega_0 \setminus \mathrm{ilog}_a b$;

$\odot_i -$reflexive multiplication i of the order, i.e. operation of multiplication for numbers concerning to i to a class (on to the ω-factor), $\bullet \setminus \omega_i \rightarrow \omega_0 \setminus \odot_i$ by analogy to reflection $\bullet \setminus \omega_1 \rightarrow \omega_0 \setminus \odot_i$;

$y = {}^x k \quad (k \in \mathbf{R}, k \neq 1) -$superexponential degree, if ${}^x k = \left.k^{k^{\cdot^{\cdot^{\cdot k}}}}\right\} x$ $(x = \mathrm{var}, k = const)$;

$y = \mathrm{slog}_k x \ (k \in \mathbf{R}, k \neq 1) -$superlogarithmic (or superlogarithmic) the function being inverse superexponential, i.e. from $y = \mathrm{slog}_k x$ follows $x = {}^k y$;

$y = {}^{\underline{x}} k \left(y = \underbrace{k^{\cdot^{\cdot^{\cdot k}}} k}_{x}, k \in \mathbf{Z}, k > 1\right) -$ exponential function of the second order;

$y = \mathrm{sslog}_k x -$ the superlogarithmic (superlogarithmic) function of the *second* order, i.e. from $y = \mathrm{sslog}_k x$ follows $x = \underbrace{k^{\cdot^{\cdot^{k^k}}}}_{y}$, where $k \in \mathbf{R}, k \neq 1$;

$y = {}^n\!\!\int\!\!\overline{x}$ $(n \in \mathbf{Z}) -$ superradical n of a degree (this function inverse superdegree, i.e. from $y = {}^n\!\!\int\!\!\overline{x}$ follows $x = {}^n y$;

${}^n_i\mathfrak{R}^b_a$ - outcome of algebraic operation between operands a and b, where n and $i -$ accordingly orders of direct and inverse operations (for example,

$${}^1_1\mathfrak{R}^b_a = b + a, \quad {}^1_2\mathfrak{R}^b_a = b - a, \quad {}^2_1\mathfrak{R}^b_a = b \cdot a, \quad {}^2_2\mathfrak{R}^b_a = b/a \qquad \text{etc.}),$$

and, ${}^n_i\mathfrak{R}^b_a = {}^n_i\mathfrak{R}_a(b)$, and in ${}^{\;\;k}_{n}{}_1\mathfrak{R}^b_a$ $k -$ number of repeated operations $k \in \mathbf{Z}$;

$v_n -$ unit element of operation $(-n)$ of the order;

$a//b -$ division a on b second inversion $(a//b = c \Rightarrow a = c \cdot b$, i.e. multiplication on $b -$ on the right as against division by first inversion $\frac{a}{b} = c \;\Rightarrow\; a = b \cdot c$);

$\underline{\overset{i}{f}} -$ image of function f, obtained at reflection in ω_0 from ω_i, i.e. $f \setminus \omega_1 \to \omega_0 \setminus \underline{\overset{i}{f}}$ (by analogy, with $a \setminus \omega_1 \to \omega_0 \setminus \underline{\overset{i}{a}}$);

$\boxtimes$, $\blacklozenge -$ of accordingly reflexive multiplication *second* and *third* is ordinal;

$\oplus$, $\ominus$, $\boxtimes$, $\oslash$ – accordingly ω-images in ω_0 operations $+, -, \times, \div$ from ω_{-1};

$⌑a = {}^2k ⌑a = {}_k k^{\left(1/\log_k \log_k a\right)}$

$✧a = {}_k k^{k^{\left(1/\log_k \log_k \log_k a\right)}}$ etc.;

$\mathbf{R}_{◬}$, $\mathbf{R}_{⌑}$, $\mathbf{R}_{✧}$, ... – accordingly set of numbers ◬, ⌑, ✧ etc.;

$'f, \underline{'}f, \overset{o}{f}$ – images in ω_0 derivative from ω_1, not reduced to "scale" ω_0 $\left({}'f\right)$, reduced to "scale" ω_0 $\left(\underline{'}f\right)$ and derivative from ω_{-1} $\left(\overset{o}{f}\right)$;

${}^{p}f$ – image in ω_0 derivative from ω_2, not reduced to "scale" ω_0;

f'_{k2} – image in ω_i derivative from $\omega_i{}'$ $\left(\omega_i = \omega_{i,k1};\ \omega_i{}' = \omega_{i,k2}\right)$

$\underline{(n)}f$, $\overset{o}{\underline{(n)}}f$, $f_{k2}^{(n)}$ – images in ω_0 derivative n-го of the order accordingly from ω_1, ω_{-1}, $\omega_0{}'$.

δx – generalized label ω-image (reduced and not reduced to a scale ω_0) differential at image it in ω_0 from ω_1 and ω_{-1};

$\delta_0 x$ – label ω-image of a differential (image in ω_0 differential from ω_2, not reduced to "to a scale" ω_0).

$\int_{k2}$ – ω-image of an integral obtained by reflection $\backslash \omega_0 \to \omega_0{}' \backslash$;

$p_1 = \log_{k_1} k_2$ – the space parameter, k_1, k_2 – factors of connection of spaces ω_0 and $\omega_0{}'$ with space ω_1 (meets also label k instead of p_1, i.e. $k \equiv p_1$);

$\int(\delta x)^{f(x)}$ and $\int(\delta x)^{\log_k f(x)}$ – superintegrals of the first sort (accordingly not reduced and reduced to "to a scale" to space ω_0);

$\int f(x) + \nu x$ – image of an integral obtained by reflection $\backslash \omega_{-1} \to \omega_0 \backslash$, and νx – image of a differential obtained of themes by reflection;

$\overline{i_*}, \overline{j_*}, \overline{k_*}$ – basis quasivector $\left(\overline{i_*} = \log_k \bar{i}, \quad \overline{j_*} = \log_k \bar{j}, \right.$ $\left. \overline{k_*} = \log_k \bar{k}\right)$, $\overline{e_{i_*}}$ – generalized basis quasivector;

$\overline{a_*}$ – quasivector $\left(\log_k \overline{a_*} = (\log_k a_x) \cdot \overline{i_*} + (\log_k a_y) \cdot \overline{j_*} + \right.$ $\left. + (\log_k a_z) \cdot \overline{k_*}\right)$;

$\odot$ – ω-image of a scalar product quasivectorial $\left(\bar{a} \cdot \bar{b} \setminus \omega_1 \to \omega_0 \setminus A_* \odot B_*\right)$;

$\otimes$ – ω-image of a vector product quasivectorial $\left(\bar{a} \times \bar{b} \setminus \omega_1 \to \omega_0 \setminus A_* \otimes B_*\right)$;

$\underline{f}, \underline{a}$ – label of function f and number a in space ω_1;

$\blacklozenge$ – reflexive division second inversion $(// \setminus \omega_1 \to \omega_0 \setminus \blacklozenge)$;

y_{i+1}, y_i – $i+1$ and i of a value of a primitive of function in a method of the Euler of a numerical solution of the differential equations;

$f(x,y)$ – right member of the equation $y' = f(x,y)$;

ε_0, β_0 – accordingly error and summarized error in a usual method of the Euler $(\beta_0 = \varepsilon_0)$;

ε_1, β_1 – images in ω_0 summarized error in a method of the Euler not reduced (ε_1) and reduced (β_1) to a scale ω_0;

S-Euler, P-Euler – upgraded methods of the Eulers obtained by reflections in ω_0 from ω_1 and ω_0' – of spaces;

h, H, H_s – pitch in a usual method of the Euler (h), in ω – image of this method (H) at reflection $\setminus \omega_1 \to \omega_0 \setminus$ and in ω-image of this method at reflection $\setminus \omega_2 \to \omega_0 \setminus$;

Δ_s, Δ_{2s} – ω-images of the absolute error of a method of the Euler accordingly at reflections $\setminus \omega_1 \to \omega_0 \setminus$ and $\setminus \omega_2 \to \omega_0 \setminus$.

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