Prove that for all $x \in \mathbb{N}$ where x is not a perfect square, that $\sqrt{x}$ is irrational.

Proof: This proof is by contraction. Let $x \in \mathbb{N}$ and not be a perfect square.
Also, assume that $\sqrt{x}$ is rational.

Then $\frac{m}{n} = \sqrt{x}$, where $m, n \in \mathbb{Z}$ and $n \neq 0$. Thus, $m^2 = xn^2$ so $n^2 | m^2$
Taking the square root of both sides leaves $m = \sqrt{x} n$

Since x is not a perfect square, $\sqrt{x}$ is not an integer. This means that $n \nmid m$
But, because $n \nmid m$, $n^2 \nmid m^2$

Therefore, contradiction, so $\sqrt{x}$ is irrational. QED.