Prove that for any real number x, $x^0 = 1$

Proof:

Let x and a be real numbers.

Case 1: $x \neq 1$

Let $x^0 = a$. So $\log_x a = 0$. This means that for any real base c:

$$\frac{\log_c a}{\log_c x} = 0$$

Since $\log_c x \neq 0$ because $x \neq 1$. By multiplying by $\log_c x$ on both sides, we get:

$$\log_c a = 0$$

Raising each side to the base of c it becomes:

$$c^0 = a$$

For all real numbers c.

This means that if we can show $x^0 = a$ for some real number x, it will be true for all real numbers.

Now consider the equation $\int_1^e \frac{dy}{y} = n$ where n is a real number. This integrates to:

$$n = \ln|y| \text{ from 1 to } e.$$

By the fundamental theorm of calculus this becomes:

$$n = \ln|e| - \ln|1|$$

This means that $n = \ln\left|\frac{e}{1}\right|$, so, $n = \ln(e)$ and $n = 1$. Now because $n = 1$, using our original equation, $1 = \ln(e) - \ln(1)$ so, $1 = 1 - \ln(1)$ which means that $\ln(1) = 0$. Now when we raise both sides to the base of e, we get, $e^0 = 1$

Therefore, since we showed that $e^0 = 1$ for base e, it must be true for all real bases except 1.

Case 2: $x = 1$

Now we must show that $1^0 = 1$.

Let m be a real number.

If $m = 0$ then $\log_1(1^m) = 0$ which can be written as $\log_1(\frac{1^m}{1}) = 0$. By the laws of logarithms, $\log_1(1^m) - \log_1(1) = 0$. Thus, $\log_1(1^m) = \log_1(1)$. Raising each to the base of 1, we get $1^m = 1$. Thus $1^0 = 1$

Therefore, $x^0 = 1$ for any real number x. QED.

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