

Prove that an irrational raised to an irrational can be rational.

**Proof:** Consider  $\sqrt{2}$ .  $\sqrt{2}^{\sqrt{2}}$  can either be rational or irrational.

Case 1:  $\sqrt{2}^{\sqrt{2}}$  is rational. Then we are done.

Case 2:  $\sqrt{2}^{\sqrt{2}}$  is irrational. Then  $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^{\sqrt{2} \cdot \sqrt{2}} = \sqrt{2}^2 = 2$ . So an irrational raised to an irrational is rational.

Therefore an irrational raised to an irrational can be rational.