UNIT V
ELECTROMAGNETIC WAVES

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Reference:

E.C.Jordan & K.G.Balmain "Electromagnetic Waves and Radiating Systems"

Derivation of Wave Equation

$$\nabla x \mathbf{H} = \dot{\mathbf{D}} = \frac{\partial \mathbf{D}}{\partial t} \qquad \rightarrow 1$$

$$\nabla x \dot{\mathbf{H}} = \ddot{\mathbf{D}} = \varepsilon \ddot{\mathbf{E}} \qquad \rightarrow 2$$

$$\nabla x \mathbf{E} = -\dot{\mathbf{B}} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \rightarrow 3$$

$$\nabla x \mathbf{E} = -\mu \dot{\mathbf{H}}$$

$$3 \Rightarrow \nabla x \nabla x \mathbf{E} = -\mu \nabla x \dot{\mathbf{H}} \qquad \rightarrow 4$$

Sub 2 in 4

$$\nabla x \nabla x \mathbf{E} = -\mu\varepsilon \ddot{\mathbf{E}} \qquad \rightarrow 5$$

According to vector identity

$$\nabla x \nabla x \mathbf{E} = \nabla\nabla \bullet \mathbf{E} - \nabla^2 \mathbf{E}$$

$$\nabla\nabla \bullet \mathbf{E} - \nabla^2 \mathbf{E} = -\mu\varepsilon \ddot{\mathbf{E}} \qquad \rightarrow 6$$

$$\rho_v = 0 \text{ for charge free region}$$

$$\nabla \bullet \mathbf{E} = \frac{1}{\varepsilon} \nabla \bullet \mathbf{D} = 0$$

$$6 \Rightarrow \nabla^2 \mathbf{E} = \mu\varepsilon \ddot{\mathbf{E}} \quad \rightarrow 7$$

$$\text{Similarly} \quad \nabla^2 \mathbf{H} = \mu\varepsilon \ddot{\mathbf{H}} \quad \rightarrow 8$$

The Wave equation reduces to a simple form in the special case where 'E' and 'H' are considered to be independent of two dimensions say 'y' and 'z' then

$$\nabla^2 \mathbf{E} = \frac{\partial^2 \mathbf{E}}{\partial \mathbf{x}^2}$$

$$\frac{\partial^2 \mathbf{E}}{\partial \mathbf{x}^2} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial \mathbf{t}^2}$$

Equations 7 and 8 are known as the wave equations

Uniform Plane Waves

$$\mathbf{E} = \mathbf{f}_1(\mathbf{x} - \mathbf{v}_0\mathbf{t}) \quad \to 1$$

The above equation is a solution of the wave equation for the particular case where the electric field is independent of ‘y’ and ‘z’ and is a function of ‘x’ and ‘t’ only. Such a wave is called a uniform plane wave.

The plane wave equation

$$\frac{\partial^2 \mathbf{E}}{\partial \mathbf{x}^2} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial \mathbf{t}^2}$$

In terms of the components Of ‘E’

$$\frac{\partial^2 \mathbf{E}_x}{\partial \mathbf{x}^2} = \mu\varepsilon \frac{\partial^2 \mathbf{E}_x}{\partial \mathbf{t}^2}$$

$$\frac{\partial^2 \mathbf{E}_y}{\partial \mathbf{x}^2} = \mu\varepsilon \frac{\partial^2 \mathbf{E}_y}{\partial \mathbf{t}^2}$$

$$\frac{\partial^2 \mathbf{E}_z}{\partial \mathbf{x}^2} = \mu\varepsilon \frac{\partial^2 \mathbf{E}_z}{\partial \mathbf{t}^2}$$

Relation between ‘E’ and ‘H’ in a Uniform Plane Wave

For a uniform plane wave traveling in the ‘x’ direction, ‘E’ and ‘H’ are both independent of ‘y’ and ‘z’ and ‘E’ and ‘H’ have no ‘x’ component

$$\nabla \mathrm{x} \mathbf{E} = -\frac{\partial E_z}{\partial x}\hat{y} + \frac{\partial E_y}{\partial x}\hat{z}$$

$$\nabla \mathrm{x} \mathbf{H} = -\frac{\partial H_z}{\partial x}\hat{y} + \frac{\partial H_y}{\partial x}\hat{z}$$

From Maxwell Equations

$$-\frac{\partial H_z}{\partial x}\hat{y} + \frac{\partial H_y}{\partial x}\hat{z} = \varepsilon\left(\frac{\partial E_y}{\partial t}\hat{y} + \frac{\partial E_Z}{\partial t}\hat{z}\right) \quad \rightarrow 2$$

$$-\frac{\partial E_z}{\partial x}\hat{y} + \frac{\partial E_y}{\partial x}\hat{z} = -\mu\left(\frac{\partial H_y}{\partial t}\hat{y} + \frac{\partial H_Z}{\partial t}\hat{z}\right) \quad \rightarrow 3$$

Equating terms on both sides of equations 2 and 3 yields following four relations

$$-\frac{\partial H_z}{\partial x} = \varepsilon \frac{\partial E_y}{\partial t} \rightarrow 4$$

$$\frac{\partial H_y}{\partial x} = \varepsilon \frac{\partial E_z}{\partial t} \rightarrow 5$$

$$\frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t} \rightarrow 6$$

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \rightarrow 7$$

if $E_y = f_1(x - v_0 t),$ where $v_0 = \frac{1}{\sqrt{\mu\varepsilon}}$, then

$$\frac{\partial E_y}{\partial t} = \frac{\partial f_1}{\partial t} = \frac{\partial f_1}{\partial (x - v_0 t)} \frac{\partial (x - v_0 t)}{\partial t} = -v_0 \frac{\partial f_1}{\partial (x - v_0 t)}$$

from equation 1 $\frac{\partial H_z}{\partial x} = v_0 \varepsilon \frac{\partial f_1}{\partial (x - v_0 t)} = v_0 \varepsilon f_1'$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} \int f_1' dx + c$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} \int \frac{\partial f_1}{\partial x} dx + c$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} f_1 + c$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} E_y + c$$

$$H_z = \sqrt{\frac{\varepsilon}{\mu}} E_y$$

$$\frac{E_y}{H_z} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} \quad \rightarrow 8$$

For free space

$$\mu = \mu_0 = 4\pi \times 10^{-7} \quad H/m$$

$$\varepsilon = \varepsilon_0 = 8.854 \times 10^{-12} \quad F/m$$

$$\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \text{ohms}$$

Equation ‘8’ states that in a travelling plane electomagnetic wave there is a definite ratio between the amplitude of ‘E’ and ‘H’ and that this ratio is equal to the square root of the ratio of permeability to the dielectric constant of the medium. Since the units of ‘E’ are volts per meter and the units of ‘H’ are amperes pre meter, the ratio given in equation ‘8’ will have the dimensions of impedance (or) ohms. For this reason the ratio in equation ‘8’ is referred to as the characteristic impedance (or) intrinsic impedance of the medium.

Wave Equation for a Conducting Medium

For regions in which the conductivity is not zero and conduction Current may exist the more general solution obtained from Maxwell's equation as follows

$$\nabla \times \mathbf{H} = \varepsilon \dot{\mathbf{E}} + \mathbf{J} \qquad \rightarrow 1$$

$$\nabla \times \mathbf{E} = -\mu \dot{\mathbf{H}} \qquad \rightarrow 2$$

If the medium has a conductivity 'σ', the conduction current Density given by

$$\mathbf{J} = \sigma \mathbf{E}$$

from equation '1'

$$\nabla \times \mathbf{H} = \varepsilon \dot{\mathbf{E}} + \sigma \mathbf{E} \qquad \rightarrow 3$$

From '2'

$$\nabla x \nabla x E = -\mu \nabla x \dot{H}$$

From '3'

$$\nabla x \nabla x E = -\mu\varepsilon \ddot{E} - \mu\sigma \dot{E}$$

From Vector Identity

$$\nabla x \nabla x E = \nabla\nabla \bullet E - \nabla^2 E$$

$$\nabla^2 E - \mu\varepsilon \ddot{E} - \mu\sigma \dot{E} = \nabla\nabla \bullet E \qquad \to 4$$

Charge density $\rho_v = 0$ within conductor

$$\therefore \nabla\nabla \bullet E = 0$$

Equation $4 \to \quad \nabla^2 E - \mu\varepsilon \ddot{E} - \mu\sigma \dot{E} = 0 \qquad \to 5$

From equation '3'

$$\nabla \times \nabla \times H = \varepsilon \nabla \times \dot{E} + \sigma \nabla \times E \rightarrow 6$$

where $\nabla \times E = -\mu \dot{H}$

From equation '6'

$$\nabla \times \nabla \times H = -\mu\varepsilon \ddot{H} - \mu\sigma \dot{H} \rightarrow 7$$

According to vector identity equation

$$\nabla \times \nabla \times H = \nabla\nabla \bullet H - \nabla^2 H$$

Equation $7 \Rightarrow$

$$\nabla\nabla \bullet H - \nabla^2 H = -\mu\varepsilon \ddot{H} - \mu\sigma \dot{H} \rightarrow 8$$

From Gauss 's Law for Magneticfi eld

$$\nabla \bullet H = \frac{1}{\mu} \nabla \bullet B = 0$$

$$\nabla^2 H - \mu\varepsilon \ddot{H} - \mu\sigma \dot{H} = 0 \rightarrow 9$$

Equation ‘5’ is the wave equation for ‘E’ in a conducting medium and Equation ‘9’ is the wave equation for ‘H’ in a conducting medium

Plane Waves in Lossy Dielectrics- Wave Propagation in Good Dielectrics

$\sigma / \omega\varepsilon << 1$ for good dielectrics

$$\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} \cong \left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right) \rightarrow 1$$

Attenuation Constant 'α' is given as

$$\alpha = \text{Real Part of } \sqrt{(j\omega\mu)(\sigma + j\omega\varepsilon)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}$$

From Equation '1'

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}-1\right)} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Phase Constant 'β'

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}+1\right)} = \omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

Velocity of Wave in Dielectric

$$\upsilon = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)} \cong \upsilon_o\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

where $\upsilon_o = \frac{1}{\sqrt{\mu\varepsilon}}$ **is the velocity of the in the dielctric when conductivity is zero**

The general expression for the intrinsic or characteristic impedance of a medium which has a finite conductivity

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

For a Good Dielectric

$$\eta = \sqrt{\frac{\mu}{\varepsilon}\left(\frac{1}{1+\frac{\sigma}{j\omega\varepsilon}}\right)}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{j\sigma}{2\omega\varepsilon}\right)$$

$\sqrt{\frac{\mu}{\varepsilon}}$ → is the intrinsic impedance of the dielectric when σ = 0, the effect of a small amount of loss is to add a small reactive component to the intrinsic impedance

Wave Propagation in Good Conductors

$$\sigma / \omega\varepsilon >> 1 \quad \text{for good Conductors}$$

$$\gamma = \sqrt{(j\omega\mu\sigma)\left(1 + j\frac{\omega\varepsilon}{\sigma}\right)} \cong \sqrt{j\omega\mu\sigma}$$

$$\gamma = \sqrt{\omega\mu\sigma}\angle 45^\circ$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

The Velocity of the wave in the condcutor

$$\upsilon = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

The intrinsic impedance of the condcutor is

$$\eta \cong \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$$

In good conductors σ is very large, both α and β are also large. This means that the wave is attenuated greatly as it progresses through the conductor and the phase shift per unit length is also great.

The Velocity of the wave, being inversely proportional to β, is very small in a good conductor.

The Characteristic impedance is also very small and has a reactive component. The angle of this impedance is always 45 degrees for good conductors.

Depth of Penetration ‘δ’ (or) Skin Depth (or) Skin Effect

In a medium which has conductivity the wave is attenuated as it progress. In a good conductor at radio frequencies the rate of attenuation is very great and the wave may penetrate only a very short distance. A term that has Significance under such circumstances is the depth of penetration.

The depth of penetration ‘δ’, *is defined as that depth in which the wave has been attenuated to 1/e or approximately 37 percent of its original value.*

The depth of penetration is $\alpha\delta=1$ or $\delta=1/\alpha$

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}}$$

For a good conductor the depth of penetration is

$$\delta = \frac{1}{\alpha} \cong \sqrt{\frac{2}{\omega\mu\sigma}} \qquad \sigma/\omega\varepsilon >> 1$$

Skin Effect – All fields in a good conductor such as copper are Essentially zero at distances greater than few skin depths from the Surface. Any current density or electric field intensity established at the surface of a good conductor decays rapidly as it progress into the conductor. Electromagnetic energy is not transmitted in the interior of a conductor; it travels in the region surrounding the conductor, while the conductor merely guides the waves.

Maxwell's Equation in Phasor form

Point form or Differential form

From Amperes circuital law

$$\nabla \mathbf{x} \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \qquad \longrightarrow \quad \boxed{1}$$

For sinusoidal steady state substitute the phasor relations

$$\nabla \mathbf{x}(\mathbf{H}e^{j\omega t}) = \frac{\partial(\mathbf{D}e^{j\omega t})}{\partial t} + \mathbf{J}e^{j\omega t}$$

$$(\nabla \mathbf{x} \mathbf{H})e^{j\omega t} = (j\omega \mathbf{D} + \mathbf{J})e^{j\omega t}$$

$$\nabla \mathbf{x} \mathbf{H} = j\omega \mathbf{D} + \mathbf{J} \qquad \longrightarrow \quad \boxed{2}$$

From Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \longrightarrow \boxed{3}$$

$$\nabla \times (\mathbf{E}e^{j\omega t}) = -\frac{\partial (\mathbf{B}e^{j\omega t})}{\partial t}$$

$$(\nabla \times \mathbf{E})e^{j\omega t} = -j\omega \mathbf{B}e^{j\omega t}$$

$$(\nabla \times \mathbf{E})e^{j\omega t} = -j\omega \mathbf{B}e^{j\omega t}$$

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \longrightarrow \boxed{4}$$

From Gauss's law for Electric Field

$$\nabla \bullet \mathbf{D} = \rho_v \longrightarrow \boxed{5}$$

From Gauss's law for Magnetic Field

$$\nabla \bullet \mathbf{B} = 0 \quad \longrightarrow \quad \boxed{6}$$

Maxwell's Equation in Point form (Phasor form)	Maxwell's Equation in Integral form (Phasor form)
$\nabla x \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}$	$\oint \mathbf{H} \bullet d\mathbf{L} = \int_S (j\omega \mathbf{D} + \mathbf{J}) \bullet d\mathbf{S}$
$\nabla x \mathbf{E} = -j\omega \mathbf{B}$	$\oint \mathbf{E} \bullet d\mathbf{L} = -\int_S j\omega \mathbf{B} \bullet d\mathbf{S}$
$\nabla \bullet \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho_V dV$
$\nabla \bullet \mathbf{B} = 0$	$\oint_S \mathbf{B} \bullet d\mathbf{S} = 0$

Wave Equation in Phasor Form

Wave Equations for loss less medium (i.e.) for dielectric medium

$$\nabla^2 \mathbf{E} = \mu\varepsilon \ddot{\mathbf{E}}$$

$$\nabla^2 \mathbf{H} = \mu\varepsilon \ddot{\mathbf{H}}$$

'E' and 'H' phasor form

$$\mathbf{E} = |\mathbf{E}|e^{j\omega t} \text{ and } \mathbf{H} = |\mathbf{H}|e^{j\omega t}$$

$$\ddot{\mathbf{E}} = (j^2)\omega^2 \mathbf{E}e^{j\omega t} = -\omega^2 \mathbf{E}e^{j\omega t}$$

$$\ddot{\mathbf{H}} = (j^2)\omega^2 \mathbf{H} e^{j\omega t} = -\omega^2 \mathbf{H} e^{j\omega t}$$

$$\nabla^2 \mathbf{E} e^{j\omega t} = \mu\varepsilon \ddot{\mathbf{E}} = -\omega^2 \mu\varepsilon \mathbf{E} e^{j\omega t}$$

$$\nabla^2 \mathbf{E} = -\omega^2 \mu\varepsilon \mathbf{E} \qquad \rightarrow (1)$$

similarly for 'H'

$$\nabla^2 \mathbf{H} = -\omega^2 \mu\varepsilon \mathbf{H} \qquad \rightarrow (2)$$

Equations (1) and (2) are wave equations in phasor form for lossless medium (i.e.) σ = 0. They are called as *Helmholtz equations.*

Wave Equations for lossy medium (i.e.) for conducting Medium $\sigma \neq 0$

$$\nabla^2 \mathbf{E} - \mu\varepsilon \ddot{\mathbf{E}} - \mu\sigma \dot{\mathbf{E}} = 0 \qquad \rightarrow (3)$$

$$\nabla^2 \mathbf{H} - \mu\varepsilon \ddot{\mathbf{H}} - \mu\sigma \dot{\mathbf{H}} = 0 \qquad \rightarrow (4)$$

$$\ddot{\mathbf{E}} = -\omega^2 \mathbf{E} e^{j\omega t} \quad \text{and} \quad \dot{\mathbf{E}} = j\omega \mathbf{E} e^{j\omega t} \qquad \rightarrow (5)$$

sub these values in equ(3)

$$\nabla^2 \mathbf{E} e^{j\omega t} + (\omega^2 \mu\varepsilon - j\omega\mu\sigma) \mathbf{E} e^{j\omega t} = 0$$

$$\nabla^2 \mathbf{E} + (\omega^2 \mu\varepsilon - j\omega\mu\sigma)\mathbf{E} = 0 \qquad \rightarrow (6)$$

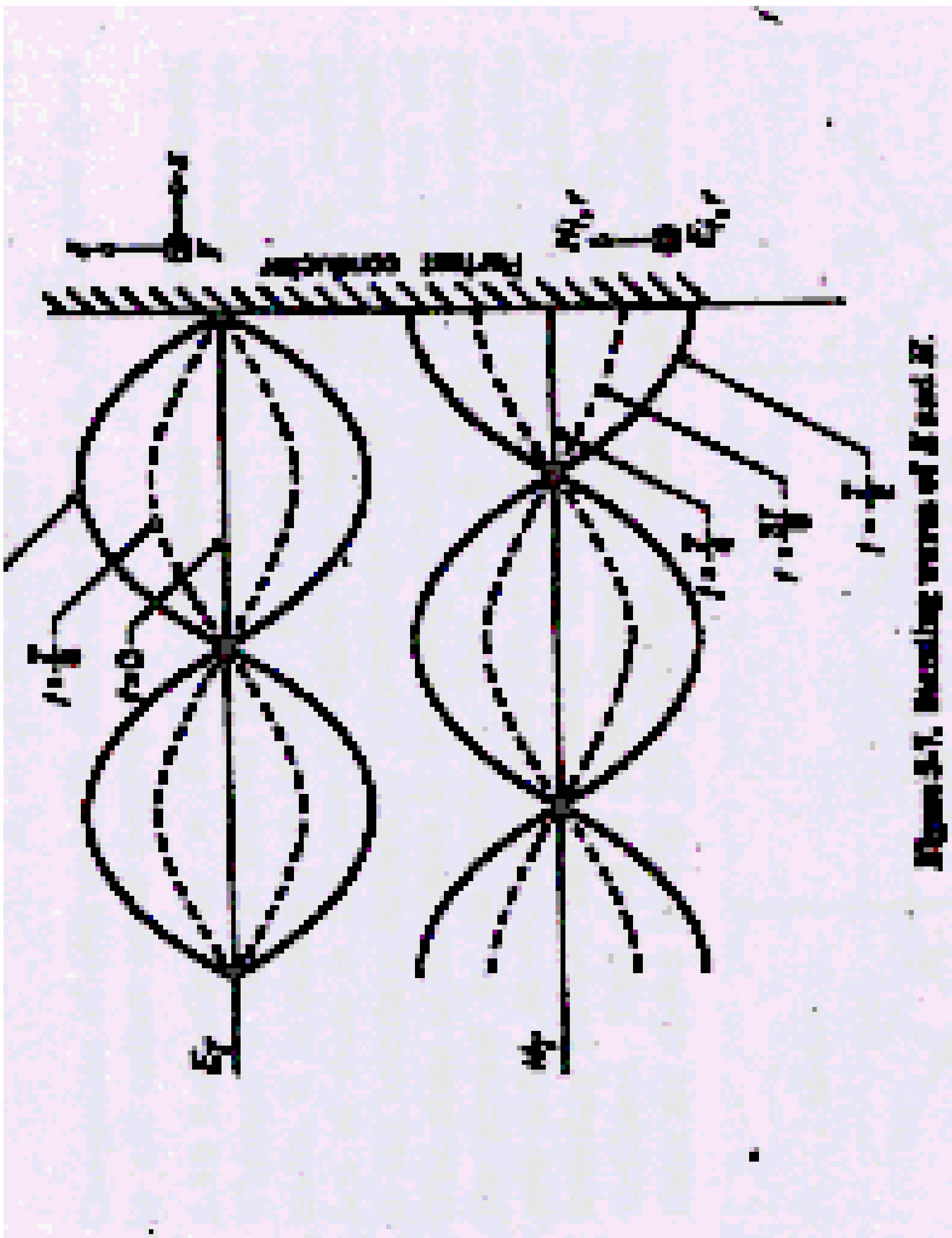
similarly for 'H'

$$\nabla^2 \mathbf{H} + (\omega^2 \mu\varepsilon - j\omega\mu\sigma)\mathbf{H} = 0 \qquad \rightarrow (7)$$

Equations (6) and (7) are Wave Equations in phasor form for lossy medium (i.e.) for conducting Medium $\sigma \neq 0$

Reflection by a Perfect Conductor – Normal Incidence

- When an electromagnetic wave traveling in one medium impinges upon a second medium having a different dielectric constant, permeability or Conductivity, the wave in general will be partially transmitted and Partially reflected.
- In the case of a plane wave in air incident normally upon the surface of a perfect conductor, the wave is entirely reflected.
- The expression for the electric field of the incident wave is $E_i e^{-j\beta x}$
- The expression for the electric field of the reflected wave is $E_r e^{-j\beta x}$
- The tangential component of 'E' must be continuous across the boundary and 'E' is zero with in the conductor, the tangential component of 'E' just outside the conductor must also be zero.



- From the above statement, the tangential component of electric field within the conductor and outside the conductor, both are zero. This requires that the sum of the electric field strengths in the initial and reflected waves add to give zero resultant field strength in the plane x=0.

$$E_r = -E_i$$

- The resultant electric field strength at any point a distance –x from the x=0 plane will be the sum of the field strengths of the incident and reflected waves at that point and it is given by

$$E_T(x) = E_i e^{-j\beta x} + E_r e^{j\beta x}$$

$$E_r = -E_i$$

$$E_T(x) = E_i(e^{-j\beta x} - e^{j\beta x})$$

$$E_T(x) = -2jE_i \sin\beta x$$

$$E_T(x,t) = \mathrm{Re}\left\{-2jE_i \sin\beta x \quad e^{j\omega t}\right\}$$

$$E_T(x,t) = 2E_i \sin\beta x \quad \sin\omega t$$

The above equation shows that the incident and reflected waves combine to produce a standing wave which does not progress.

The expression for the resultant magnetic field will be

$$\mathbf{H_T(x) = H_i e^{-j\beta x} + H_r e^{j\beta x}}$$

$$\mathbf{H_r = H_i}$$

$$\mathbf{H_T(x) = 2H_i (e^{-j\beta x} + e^{j\beta x})}$$

$$\mathbf{H_T(x) = 2H_i \cos\beta x}$$

$$\mathbf{H_T(x,t) = Re\left\{H_T(x) \quad e^{j\omega t}\right\}}$$

$$\mathbf{H_T(x,t) = 2H_i \cos\beta x \quad \cos\omega t}$$

Reflection by a Perfect Dielectric- Normal Incidence

- When a plane electromagnetic wave is incident normally on the surface of a perfect dielectric, part of the energy is transmitted and part of it is reflected.

- A perfect dielectric is one with zero conductivity, so that there is no loss or absorption of power in propagation through the dielectric.

$$E_i = \eta_1 H_i$$

$$E_r = -\eta_1 H_r$$

$$E_t = \eta_2 H_t$$

$\eta_1 \longrightarrow$ Intrinsic impedance of medium '1'

$\eta_2 \longrightarrow$ Intrinsic impedance of medium '2'

The continuity of the tangential components of ‘E’ and ‘H’ require that

$$\mathbf{H_i} + \mathbf{H_r} = \mathbf{H_t}$$

$$\mathbf{E_i} + \mathbf{E_r} = \mathbf{E_t}$$

$$\mathbf{H_i} + \mathbf{H_r} = \frac{\mathbf{1}}{\eta_1}\left(\mathbf{E_i} - \mathbf{E_r}\right) = \mathbf{H_t} = \frac{\mathbf{1}}{\eta_2}\left(\mathbf{E_i} + \mathbf{E_r}\right)$$

$$\eta_2\left(\mathbf{E_i} - \mathbf{E_r}\right) = \eta_1\left(\mathbf{E_i} + \mathbf{E_r}\right)$$

$$\mathbf{E_i}\left(\eta_2 - \eta_1\right) = \mathbf{E_r}\left(\eta_2 + \eta_1\right)$$

$$\frac{\mathbf{E_r}}{\mathbf{E_i}} = \frac{\left(\eta_2 - \eta_1\right)}{\left(\eta_2 + \eta_1\right)}$$

$$\frac{\mathbf{E_t}}{\mathbf{E_i}} = \frac{\mathbf{E_i} + \mathbf{E_r}}{\mathbf{E_i}} = 1 + \frac{\mathbf{E_r}}{\mathbf{E_i}}$$

$$\frac{\mathbf{E_t}}{\mathbf{E_i}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\frac{\mathbf{H_r}}{\mathbf{H_i}} = -\frac{\mathbf{E_r}}{\mathbf{E_i}} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$\frac{\mathbf{H_t}}{\mathbf{H_i}} = \frac{\eta_1 \mathbf{E_t}}{\eta_2 \mathbf{E_i}} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

$$\mu_1 = \mu_2 = \mu_v$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\mu_v}{\varepsilon_2}} - \sqrt{\frac{\mu_v}{\varepsilon_1}}}{\sqrt{\frac{\mu_v}{\varepsilon_2}} + \sqrt{\frac{\mu_v}{\varepsilon_1}}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

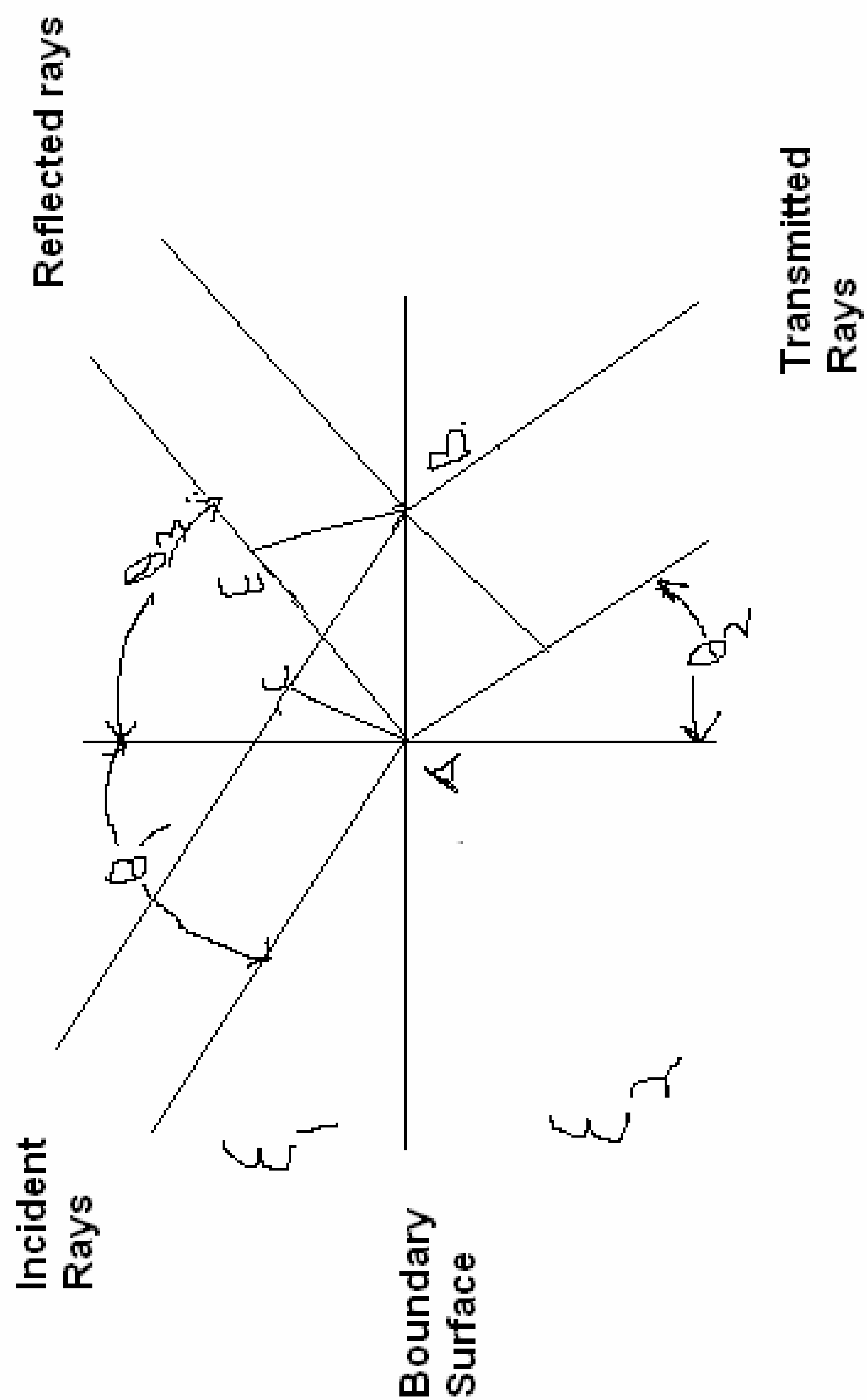
$$\frac{E_t}{E_i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\frac{H_r}{H_i} = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$\frac{H_t}{H_i} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

Reflection by a Perfect Insulator- Oblique Incidence

If a Plane wave is incident upon a boundary surface that is not parallel to the plane containing ‘E’ and ‘H’, the boundary conditions are more complex. Again part of the wave will be transmitted and part of it reflected, but in this case the transmitted wave will be refracted (i.e.) the direction of propagation will be altered.


Reflected rays
Transmitted Rays
Incident Rays
Boundary Surface

In the diagram incident ray travels the distance CB, whereas transmitted ray travels the distance AD and reflected ray travels from A to E. If v_1 is the velocity of the wave in medium(1) and v_2 is the velocity in medium(2).

$$\frac{CB}{AD} = \frac{v_1}{v_2} \qquad \rightarrow (1)$$

$$CB = AB \sin\theta_1 \quad \text{and} \quad AD = AB \sin\theta_2$$

sub in equation(1)

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \qquad \rightarrow (2)$$

$$v_1 = \frac{1}{\sqrt{\mu_1 \varepsilon_1}} = \frac{1}{\sqrt{\mu_v \varepsilon_1}}$$

$$v_2 = \frac{1}{\sqrt{\mu_2 \varepsilon_2}} = \frac{1}{\sqrt{\mu_v \varepsilon_2}}$$

$$\mathbf{sub\ the\ value\ of\ v_1\ and\ v_2\ in (2)}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \rightarrow (3)$$

$$\mathbf{AE = CB} \Rightarrow \therefore \sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2 \rightarrow (4)$$

<u>Snell’s Law (or) Law of sines</u>

From equation (4) -The angle of incidence is equal to the angle of refraction. The angle of incidence is related to the angle of refraction by equation (3), which in optics is known as the *law of sines* (or) *Snell’s law*

The power transmitted per square meter in a wave is the vector product of ‘E’ and ‘H’.

(i.e.) $P = E \times H$ or $P = EH$

where $H = E/\eta$

therefore $P = E^2/\eta$

The power in the incident wave $\longrightarrow (1/\eta_1)E_i^2 \cos\theta_1$

Power in the refracted wave $\longrightarrow (1/\eta_1)E_r^2 \cos\theta_1$

Power in the transmitted wave $\longrightarrow (1/\eta_2)E_t^2 \cos\theta_2$

By Conservation of Energy

$$(1/\eta_1)E_i^{\ 2}\cos\theta_1 = (1/\eta_1)E_r^{\ 2}\cos\theta_1 + (1/\eta_2)E_t^{\ 2}\cos\theta_2$$

$$1 = \frac{(1/\eta_1)E_r^{\ 2}\cos\theta_1}{(1/\eta_1)E_i^{\ 2}\cos\theta_1} + \frac{(1/\eta_2)E_t^{\ 2}\cos\theta_2}{(1/\eta_1)E_i^{\ 2}\cos\theta_1}$$

$$1 = \frac{E_r^{\ 2}}{E_i^{\ 2}} + \frac{\eta_1 E_t^{\ 2}\cos\theta_2}{\eta_2 E_i^{\ 2}\cos\theta_1}$$

$$\frac{E_r^{\ 2}}{E_i^{\ 2}} = 1 - \frac{\eta_1 E_t^{\ 2}\cos\theta_2}{\eta_2 E_i^{\ 2}\cos\theta_1}, \text{where } \eta_1 = \sqrt{\frac{\mu_v}{\varepsilon_1}}, \quad \eta_2 = \sqrt{\frac{\mu_v}{\varepsilon_2}}$$

$$\frac{E_r^{\ 2}}{E_i^{\ 2}} = 1 - \frac{\sqrt{\varepsilon_2}E_t^{\ 2}\cos\theta_2}{\sqrt{\varepsilon_1}E_i^{\ 2}\cos\theta_1} \quad \rightarrow (5)$$

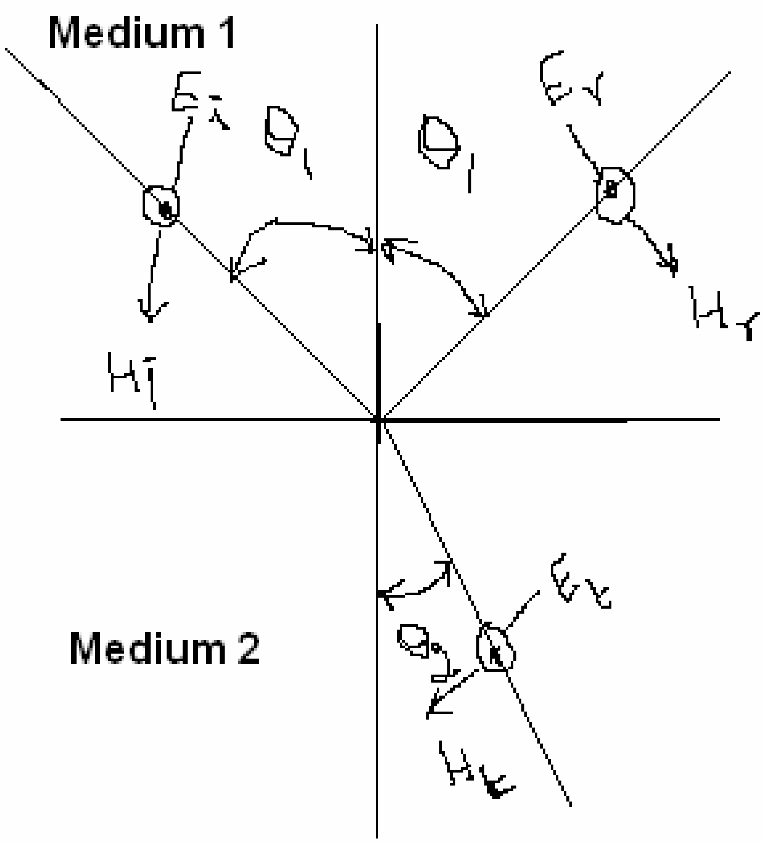


Fig (a)

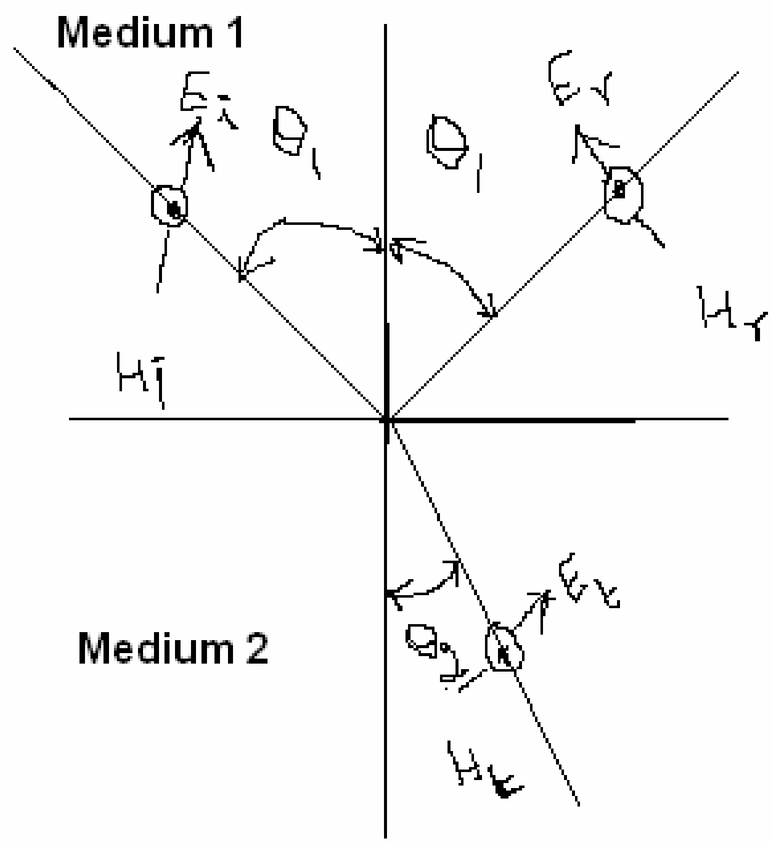


Fig (b)

Fig shows Reflection and Refraction of waves having (a) Perpendicular (horizontal) polarization and (b) parallel (vertical) polarization

Dependence of Polarization

CASE I : *Perpendicular (Horizontal) Polarization.*

In this case the electric vector E is perpendicular to the plane of incidence and Parallel to the reflecting surface.

Let the electric field strength E_i of the incident wave be in the positive *x* direction and E_r and E_t of the reflected and transmitted waves also be in the positive *x* direction. Then applying the boundary conditions that the tangential component of E is continuous across the boundary.

$$E_i + E_r = E_t \qquad (6)$$

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} \qquad \rightarrow (7)$$

$$(5) \Rightarrow \frac{E_r^{\,2}}{E_i^{\,2}} = 1 - \frac{\sqrt{\varepsilon_2}\,E_t^{\,2}\cos\theta_2}{\sqrt{\varepsilon_1}\,E_i^{\,2}\cos\theta_1}$$

$$\frac{E_r^{\,2}}{E_i^{\,2}} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \quad \frac{E_t^{\,2}}{E_i^{\,2}} \quad \frac{\cos\theta_2}{\cos\theta_1}$$

using (7) in (5)

$$\frac{E_r^{\,2}}{E_i^{\,2}} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$1 - \frac{E_r^{\,2}}{E_i^{\,2}} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$\left(1 + \frac{E_r}{E_i}\right)\left(1 - \frac{E_r}{E_i}\right) = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 + \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$\left(1-\frac{E_r}{E_i}\right)=\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1+\frac{E_r}{E_i}\right)\frac{\cos\theta_2}{\cos\theta_1}$$

$$\sqrt{\varepsilon_1}\cos\theta_1-\frac{E_r}{E_i}\sqrt{\varepsilon_1}\cos\theta_1=\left(\sqrt{\varepsilon_2}\cos\theta_2+\frac{E_r}{E_i}\sqrt{\varepsilon_2}\cos\theta_2\right)$$

$$\sqrt{\varepsilon_1}\cos\theta_1-\frac{E_r}{E_i}\sqrt{\varepsilon_1}\cos\theta_1=\left(\sqrt{\varepsilon_2}\cos\theta_2+\frac{E_r}{E_i}\sqrt{\varepsilon_2}\cos\theta_2\right)$$

$$\sqrt{\varepsilon_1}\cos\theta_1-\sqrt{\varepsilon_2}\cos\theta_2=\frac{E_r}{E_i}\sqrt{\varepsilon_2}\cos\theta_2+\frac{E_r}{E_i}\sqrt{\varepsilon_1}\cos\theta_1$$

$$\frac{\sqrt{\varepsilon_1}\cos\theta_1-\sqrt{\varepsilon_2}\cos\theta_2}{\sqrt{\varepsilon_2}\cos\theta_2+\sqrt{\varepsilon_1}\cos\theta_1}=\frac{E_r}{E_i}$$

$$\frac{E_r}{E_i}=\frac{\sqrt{\varepsilon_1}\cos\theta_1-\sqrt{\varepsilon_2}\cos\theta_2}{\sqrt{\varepsilon_2}\cos\theta_2+\sqrt{\varepsilon_1}\cos\theta_1} \longrightarrow (8)$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2}\sqrt{1-\sin^2\theta_2}}{\sqrt{\varepsilon_2}\sqrt{1-\sin^2\theta_2} + \sqrt{\varepsilon_1}\cos\theta_1}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2 - \varepsilon_2\sin^2\theta_2}}{\sqrt{\varepsilon_2 - \varepsilon_2\sin^2\theta_2} + \sqrt{\varepsilon_1}\cos\theta_1}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2 - \varepsilon_2\sin^2\theta_2}}{\sqrt{\varepsilon_2 - \varepsilon_2\sin^2\theta_2} + \sqrt{\varepsilon_1}\cos\theta_1} \quad \rightarrow (9)$$

$$\text{from}(3) \Rightarrow \quad \sin\theta_2 = \sin\theta_1\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

sub in equation (9)

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_1}}{\sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_1} + \sqrt{\varepsilon_1}\cos\theta_1}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_1}}{\sqrt{\varepsilon_1}\cos\theta_1 + \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_1}} \rightarrow (10)$$

$$\frac{E_r}{E_i} = \frac{\cos\theta_1 - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1}} \rightarrow (11)$$

Equation (8),(9),(10) and (11) give the ratio of reflected to Incident electric field strength for the case of a perpendicularly polarized wave

CASE II : *Parallel (Vertical) Polarization.*

In this case E is parallel to the plane of incidence and H is parallel to the reflecting surface. Again applying the boundary condition that the tangential component of E is continuous across the boundary and in this case

$$E_i \cos\theta_1 - E_r \cos\theta_1 = E_t \cos\theta_2$$

$$\frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_1}{\cos\theta_2}$$

sub in equation(5)

$$\frac{E_r^{\,2}}{E_i^{\,2}} = 1 - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(\left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_2}{\cos\theta_1}\right)^2 \frac{\cos\theta_1}{\cos\theta_2}$$

$$\frac{E_r^{\ 2}}{E_i^{\ 2}} = 1 - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_1}{\cos\theta_2}\right)$$

$$1 - \frac{E_r^{\ 2}}{E_i^{\ 2}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_1}{\cos\theta_2}\right)$$

$$\left(1 - \frac{E_r}{E_i}\right)\left(1 + \frac{E_r}{E_i}\right) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos\theta_1}{\cos\theta_2}$$

$$\left(1 + \frac{E_r}{E_i}\right) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_1}{\cos\theta_2}$$

$$\sqrt{\varepsilon_1}\cos\theta_2 + \sqrt{\varepsilon_1}\cos\theta_2 \frac{E_r}{E_i} = \sqrt{\varepsilon_2}\cos\theta_1 - \frac{E_r}{E_i}\sqrt{\varepsilon_2}\cos\theta_1$$

$$\frac{E_r}{E_i}\left(\sqrt{\varepsilon_1}\cos\theta_2 + \sqrt{\varepsilon_2}\cos\theta_1\right) = \sqrt{\varepsilon_2}\cos\theta_1 - \sqrt{\varepsilon_1}\cos\theta_2$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_1 - \sqrt{\varepsilon_1}\cos\theta_2}{\sqrt{\varepsilon_2}\cos\theta_1 + \sqrt{\varepsilon_1}\cos\theta_2} \rightarrow (12)$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_1 - \sqrt{\varepsilon_1\left(1-\sin^2\theta_2\right)}}{\sqrt{\varepsilon_2}\cos\theta_1 + \sqrt{\varepsilon_1\left(1-\sin^2\theta_2\right)}} \rightarrow (13)$$

$$\frac{E_r}{E_i} = \frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_1 - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_1 + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1}} \rightarrow (14)$$

The equations (12),(13) and (14) give the reflection coefficient for parallel or vertical polarization (i.e.) the ratio of reflected to incident electric field strength when E is parallel to the plane of incidence

Brewster Angle

There is no reflection at a particular angle. This occurs when the numerator in the following equation is zero.

$$\frac{\mathbf{E_r}}{\mathbf{E_i}} = \frac{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\cos\theta_1 - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1}}{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\cos\theta_1 + \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1}}$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_1 - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1} = 0$$

$$\sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_1$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1 = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \cos^2\theta_1$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_1 = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 (1 - \sin^2\theta_1)$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 = \sin^2\theta_1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_1$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\left(1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\right) = \sin^2\theta_1\left(1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2\right)$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\left(1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\right) = \sin^2\theta_1\left(1 - \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\right)\left(1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\right)$$

$$\left(\frac{\varepsilon_2}{\varepsilon_1}\right) = \sin^2\theta_1\left(1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\right)$$

$$\frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}{\left(1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\right)} = \sin^2\theta_1$$

$$\sin^2\theta_1 = \frac{\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)}$$

$$\cos^2\theta_1 = \frac{\varepsilon_1}{(\varepsilon_1 + \varepsilon_2)}$$

$$\tan\theta = \sqrt{\frac{\sin^2\theta_1}{\cos^2\theta_1}} = \sqrt{\frac{\frac{\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)}}{\frac{\varepsilon_1}{(\varepsilon_1 + \varepsilon_2)}}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\tan\theta_1 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \rightarrow (15)$$

At this angle, which is called the *Brewster angle,* there is no reflected wave when the incident wave is parallel (or vertically)polarized.

For perpendicular polarization there is no corresponding Brewster angle.

Polarization- Linear, Circular and Elliptical polarization

Definition – The Polarization of a uniform plane wave refers to the time-varying behaviour of the electric field strength vector at some fixed point in space.

If a uniform plane wave traveling in the z direction with the E and H vectors lying *x*-*y* plane , if E_y =0 and only E_x is present, the wave is said to be polarized in the *X* direction.

1. Linear Polarization

If both E_x and E_y are present and are in phase, the resultant electric field has a direction dependent on the relative magnitude of E_x and E_y. The angle which this direction makes with the x- axis is $\tan^{-1}(Ey/Ex)$ and this angle will be constant with time.

For all the cases the direction of the resultant vector is constant with time, the wave is said to be *linearly polarized.*

2. Elliptical Polarization

If Ex and Ey are not in phase and they reach their maximum values at different instants of time, then the direction of the resultant electric vector will vary with time. In this case the locus of the end point of the *resultant E will be an ellipse* and the wave is said to be *elliptically polarized*.

3. Circular Polarization

If Ex and Ey have equal magnitudes and a 90 degree phase difference, the locus of the *resultant E is a circle* and the wave is said to be *circularly polarized.*

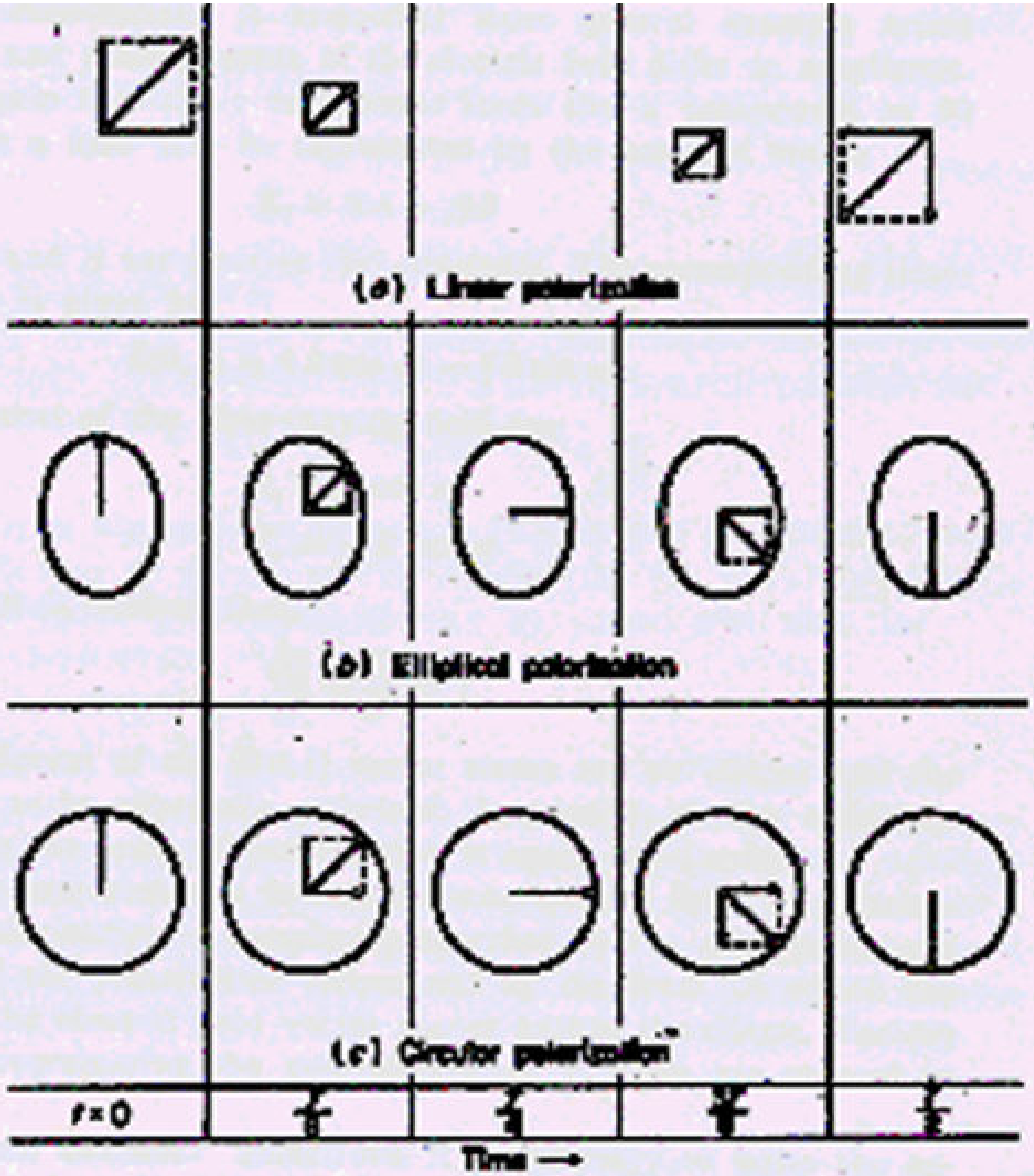


Figure 6-4. Linear, elliptical and circular polarization.

The Electric field of a uniform plane wave traveling in the z direction may be expressed in phasor form as

$$E(z) = E_0 \, e^{-j\beta z}$$

in time-varying form as

$$E(z) = \text{Re}\,\{E_0 \, e^{-j\beta z} \, e^{j\omega t}\} \longrightarrow (1)$$

Since the wave travels in z- direction, E lies in the x-y plane. E_0 is a complex vector and it is written in the form of

$$E_0 = E_r + j\,E_i \longrightarrow (2)$$

At z =0 the resultant time-varying electric field is

$$E(0,t) = \text{Re}\,\{(E_r + j\,E_i)\, e^{j\omega t}\} \longrightarrow (3)$$

$$E(0,t) = E_r \cos \omega t - E_i \sin \omega t \longrightarrow (4)$$

It is clear that E not only changes its magnitude but also changes its direction as time varies.

***Circular Polarization* -**

x and y components of the electric field are equal in magnitude. If the y component leads the x component by 90 degrees and if both components have amplitude Ea, then the electric field at z =0 is given by the complex vector

$$\mathbf{E}(0,t) = (\hat{x} + j\hat{y})E_a$$

$$\mathbf{E}(0,t) = (\hat{x}\cos \omega t - \hat{y}\sin \omega t)E_a \longrightarrow$$

$$E_x = E_a \cos \omega t$$

$$E_y = -E_a \sin \omega t$$

$$E_x^2 + E_y^2 = E_a^2$$

The equation indicates that the endpoint of E(0,t) traces out a circle of radius Ea as time progress. The direction of rotation is that of a left handed screw advancing in the z direction. Thus the wave is said to be ***left circularly polarized***. The right circular polarization represented by the complex vector

$$\mathbf{E}(\mathbf{0},\mathbf{t}) = (\hat{\mathbf{x}} - \mathbf{j}\hat{\mathbf{y}})\mathbf{E}_{\mathbf{a}}$$

The reversal of rotation obtained by a 180 degree phase shift applied either to the ‘x’ component or to the ‘y’ component of the electric field.

Elliptical Polarization -

x and y components of the electric field are unequal in magnitude. If the y component leads the x component by 90 degrees and if both components have different amplitude ‘A’ and ‘B’, then the electric field at z =0 is given by the complex vector

$$\mathbf{E}(0,t) = (\hat{x} - j\hat{y})E_a$$

$$\mathbf{E}(0,t) = \hat{x}A \quad + j\hat{y}B$$

$$\mathbf{E}(0,t) = \hat{x}A\cos\omega t - \hat{y}B\sin\omega t$$

→

$$E_x = A\cos\omega t$$

$$E_y = -B\sin\omega t$$

$$\frac{E_x^{\,2}}{A^2} + \frac{E_y^{\,2}}{B^2} = 1$$

Thus the endpoint of the E(0,t) vector traces out an ellipse and the wave is Said to be ***elliptically polarized***. From the above equations the **sense** or rotation Of polarization is **left handed**.