UNIT IV TIME VARYING ELECTRIC AND MAGNETIC FIELDS

4.1 Faradays Law of Electromagnetic Induction

$$\text{emf} = -\frac{d\phi}{dt}$$

The total induced emf in a circuit is equal to the time rate of decrease of the total magnetic flux linking with a circuit. The negative sign indicates that the emf is in such a direction as to produce a current whose flux if added to the original flux would reduce the magnitude of the emf.

4.2 Maxwell's Equation from Faraday's Law

I. Integral Form

From Faraday's Law

$$\text{Induced emf} = -\frac{d\phi}{dt} \quad \rightarrow (1)$$

$$\text{Emf} = V$$

$$\text{Induced emf} = -V = -\left[-\int E \bullet dL\right] \quad \rightarrow (2)$$

$$(1) = (2)$$

$$\int E \bullet dL = -\frac{d\phi}{dt}$$

$$\oint E \bullet dL = -\frac{d\phi}{dt} \qquad \rightarrow (3)$$

$$\phi = \int_S B \bullet dS \qquad \rightarrow (4)$$

sub (4) in (3)

$$\oint E \bullet dL = -\frac{d}{dt}\int_S B \bullet dS$$

$$\oint E \bullet dL = -\int_S \frac{\partial B}{\partial t} \bullet dS \qquad \rightarrow (5)$$

Equation (5) is the maxwell's equation from Faraday's law in integral form.

II. Point form (or) Differential form

From Stoke's theorem

$$\oint E \bullet dL = \int_S (\nabla x E) \bullet dS \qquad \rightarrow (6)$$

$$(5) = (6)$$

$$\oint E \bullet dL = \int_S (\nabla x E) \bullet dS = -\int_S \frac{\partial B}{\partial t} \bullet dS$$

$$\int_S (\nabla x E) \bullet dS = -\int_S \frac{\partial B}{\partial t} \bullet dS$$

Removing surface integral on both sides

$$\nabla x E = -\frac{\partial B}{\partial t} \qquad \rightarrow (7)$$

The relation in equation (7) is referred to as maxwell's equation derived from faraday's law in differential form (or) point form.

4.3 Derivation of Equation for Motional EMF (Moving conductor in a magnetic Field)

When a conductor moves in a magnetic field 'B', an electric field 'E' appears along the conductor. The force on a charge 'Q' moving at a velocity 'V' in a magnetic field 'B' is $F = q(vxB) \qquad \rightarrow (1)$

$$\frac{F}{q} = vxB \qquad \rightarrow (2)$$

Force per unit charge is called motional electric field intensity.

$$E_m = \frac{F}{q} = vxB$$

$$E_m = vxB \qquad \rightarrow (3)$$

The motional emf produced by the conductor is

$$\text{emf} = \oint E_m \bullet dL = \oint (vxB) \bullet dL \qquad \rightarrow (4)$$

According to faraday's law of electromagnetic induction,

$$\text{Induced emf} = \oint E \bullet dL = -\int_S \left(\frac{\partial B}{\partial t}\right) \bullet dS \qquad \rightarrow (5)$$

Note: Equation (5) gives the emf induced in a closed circuit due to the time rate of change of 'B' (transformer induction). Equation (4) gives the emf induced in a closed circuit due to its motion and it is said to be motional induction. When both kinds of changes are occurring simultaneously (i.e.) 'B' changing with time and circuit is in motion the total emf produced is equal to the sum of the emf given in equation (4) and (5).

$$\text{emf} = -\int_S \left(\frac{\partial B}{\partial t}\right) \bullet dS + \int_S (vxB) \bullet dS \qquad \rightarrow (6) \qquad \text{General case}$$

4.4 Displacement Current (I_D)

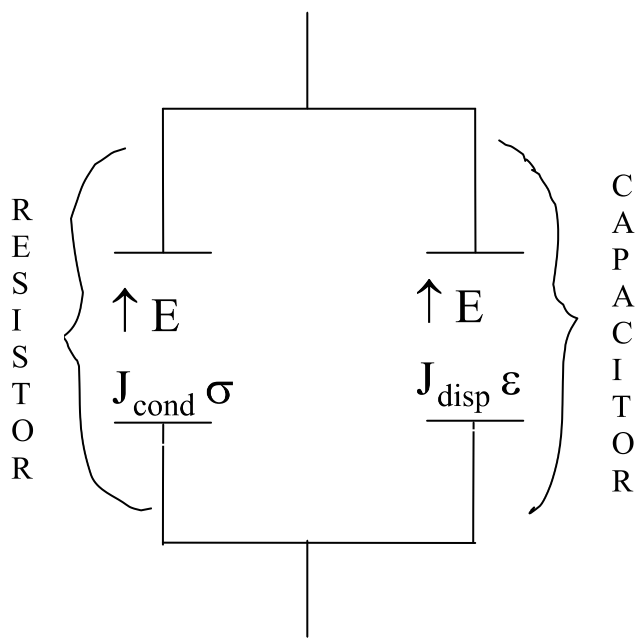


Fig. (a) Conduction current through resistor and displacement current through capacitor.

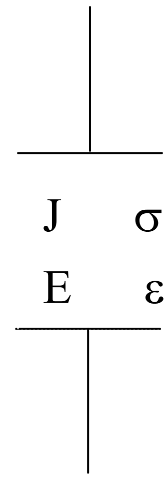


Fig. (b) Conduction current and displacement current through a conductive dielectric medium

A constant voltage across a resistor produces a continuous flow of current of constant value. On the other hand there will be a current through the capacitor only while the voltage is changing.

Current flow through the resistor $i_1 = \dfrac{V}{R} \qquad \rightarrow (1)$

Current flow through the capacitor $i_2 = C\frac{dV}{dt}$ $\rightarrow (2)$

i_1 is the conduction current through the resistor and i_2 is the displacement current through the capacitor.

Displacement Current :

Actually current does not flow through the capacitor as much current flows out of one plate as flows in to the opposite plate.

The current density J_1 inside the resistor $= \frac{i_1}{A}$ $\rightarrow (3)$

$$i_2 = C\frac{dV}{dt} \qquad \rightarrow (4)$$

$$C = \frac{\varepsilon A}{d} \qquad \rightarrow (5)$$

$$\text{sub(4)in(5)}$$

$$i_2 = \frac{\varepsilon A}{d}\frac{dV}{dt} \qquad \rightarrow (6)$$

$$V = Ed \qquad \text{sub in} \quad (6)$$

$$i_2 = \frac{\varepsilon A}{d}\frac{d(Ed)}{dt}$$

$$i_2 = \frac{\varepsilon A}{d} \frac{d \cdot dE}{dt}$$

$$i_2 = A \cdot \frac{d(\varepsilon E)}{dt} = A \cdot \frac{dD}{dt}$$

$$i_2 = I_D = A \cdot \frac{dD}{dt} \qquad \rightarrow (7)$$

J_D - displacement current density

$$J_D = \frac{I_D}{A} = \frac{dD}{dt} \qquad \rightarrow (8)$$

The total I_D crossing any given surface is expressed as the surface integral of J_D. Equation (7) & (8) are in point form (or) differential form and integral form is given by equation (9).

$$I_D = \int_S J_D \bullet dS = \int_S \frac{\partial D}{\partial t} \bullet dS \qquad \rightarrow (9)$$

4.5 Maxwell Equation From Ampere Circuital Law

I. Integral Form

Ampere's Circuital Law $\oint H \bullet dL = I_{enclosed}$

` When both conduction and displacement current are present, the total current is the sum of these two currents.

$$\oint H \bullet dL = I_{enclosed}$$

$$\oint H \bullet dL = I_c + I_d$$

$$\oint H \bullet dL = \int_S I_c ds + \int_S I_d ds$$

$$\oint H \bullet dL = \int_S (\sigma E) ds + \int_S \left(\frac{\partial D}{\partial t} \right) ds \qquad \rightarrow (1)$$

$$\oint H \bullet dL = \int_S (\sigma E) ds + \int_S \left(\frac{\varepsilon \partial E}{\partial t} \right) ds \qquad \rightarrow (2)$$

$$\oint H \bullet dL = \int_S \left(\sigma E + \frac{\varepsilon \partial E}{\partial t} \right) ds \qquad \rightarrow (3)$$

Equation (1), (2) and (3) called as Maxwell's equation from ampere circuital law in integral law.

II. Point Form (or) Differential Form

From Stoke's Theorem $\oint H \bullet dL = \int_S (\nabla x H) ds \qquad \rightarrow (4)$

From (3) and (4)

$$\oint H \bullet dL = \int_S (\nabla x H) ds = \int_S \left(\sigma E + \frac{\varepsilon \partial E}{\partial t} \right) ds$$

$$\nabla x H = \sigma E + \frac{\varepsilon \partial E}{\partial t} \qquad \rightarrow (5)$$

$$\nabla x H = J + \frac{\partial D}{\partial t} \qquad \rightarrow (6)$$

Equation (4) and (5) are called as Maxwell's equation from ampere's circuital law in point form (or) differential form.

4.6 Maxwell's Four Equation in Integral form and Differential Form for Time Varying Field

Integral Form	Differential Form (or) Point Form
(1) Maxwell's First Equation By Applying Ampere's Circuital Law $\oint H \bullet dL = \int_S \left(J + \frac{\partial D}{\partial t} \right) ds$	 By Applying Stoke's Theorem $\nabla x H = J + \frac{\partial D}{\partial t}$
(2) Maxwell's Second Equation From Faraday's Law $\oint E \bullet dL = -\int_S \left(\frac{\partial B}{\partial t} \right) \bullet ds$	 By Applying Stoke's Theorem $\nabla x E = -\frac{\partial B}{\partial t}$
(3) Maxwell's Third Equation From Gauss's Law for Electric Field $\oint D \bullet ds = q = \int_v \rho_v dv$	 $\oint_S D \bullet ds = \int_V (\nabla \bullet D) dV = \int_V \rho_v dV$ $\nabla \bullet D = \rho_V$
(4) Maxwell's Fourth Equation $\oint_S B \bullet ds = 0$	Divergence Theorem $\oint B \bullet ds = \int_V (\nabla \bullet B) dV = 0$ $\nabla \bullet B = 0$

4.7 Maxwell's Four Equation for Non-Time Varying Field

Integral Form	Differential Form (or) Point Form
(1) Maxwell's First Equation By Applying Ampere's Circuital Law $\oint H \bullet dL = I = \int_S J \bullet ds$	 By Applying Stoke's Theorem $\nabla x H = J$
(2) Maxwell's Second Equation From Faraday's Law $\oint E \bullet dL = 0$	 By Applying Stoke's Theorem $\nabla x E = 0$
(3) Maxwell's Third Equation From Gauss's Law for Electric Field $\oint_S D \bullet dS = \int_V \rho_V dV = Q$	$\nabla \bullet D = \rho_V$
(4) Maxwell's Fourth Equation $\oint_S B \bullet dS = 0$	Divergence Theorem $\nabla \bullet B = 0$

4.8 Maxwell's Four Equation for Free Space

Integral Form	Differential Form (or) Point Form
(1) Maxwell's First Equation By Applying Ampere's Circuital Law $\oint H \bullet dL = \int_S \frac{\partial D}{\partial t} \bullet ds$	 By Applying Stoke's Theorem $\nabla x H = \frac{\partial D}{\partial t}$
(2) Maxwell's Second Equation From Faraday's Law $\oint E \bullet dL = -\int_S \left(\frac{\partial B}{\partial t}\right) ds$	 By Applying Stoke's Theorem $\nabla x E = 0$
(3) Maxwell's Third Equation From Gauss's Law for Electric Field $\oint_S D \bullet dS = 0$	 $\nabla \bullet D = 0$
(4) Maxwell's Fourth Equation $\oint_S B \bullet dS = 0$	Divergence Theorem $\nabla \bullet B = 0$

4.9 Poynting Vector and Power Flow

Poynting Theorem

As electromagnetic waves propagates through space from their source to distant receiving points there is a transfer of energy from the source to the receivers. There exist a simple and direct relation between the rate of this energy transfer and the amplitudes of electric and magnetic field strength of the electromagnetic wave.

Poynting theorem is a power theorem for the electromagnetic field used to find the power in an uniform plane wave.

Derivation of Poynting Theorem

Maxwell's curl equation for time varying field in point form is given by

$$\nabla x H = J + \frac{\partial D}{\partial t}$$

$$\nabla x H = J + \frac{\varepsilon \partial E}{\partial t} \qquad \rightarrow (1)$$

Dot product on each side of the equation with 'E'

$$E \bullet (\nabla x H) = E \bullet J + E \bullet \frac{\partial D}{\partial t} \qquad \rightarrow (2)$$

By using vector identity

$$\nabla \bullet (E x H) = -E \bullet \nabla x H + H \bullet \nabla x E$$

$$-E \bullet \nabla xH = \nabla \bullet (ExH) - H \bullet \nabla xE$$

$$E \bullet \nabla xH = -\nabla \bullet (ExH) + H \bullet \nabla xE \qquad \rightarrow (3)$$

sub (3) in equation (2)

$$-\nabla \bullet (ExH) + H \bullet (\nabla xE) = E \bullet J + E \bullet \frac{\partial D}{\partial t} \qquad \rightarrow (4)$$

From Maxwell's second equation for time varying field

$$\nabla xE = -\frac{\partial B}{\partial t}$$

Sub. this value in (4)

$$-\nabla \bullet (ExH) + H\left(-\frac{\partial B}{\partial t}\right) = E \bullet J + E \bullet \frac{\partial D}{\partial t}$$

$$-\nabla \bullet (ExH) = E \bullet J + E \bullet \frac{\partial \varepsilon E}{\partial t} + H\left(\frac{\partial B}{\partial t}\right)$$

$$-\nabla \bullet (ExH) = E \bullet J + E \bullet \varepsilon \frac{\partial E}{\partial t} + H\left(\mu \frac{\partial H}{\partial t}\right) \qquad \rightarrow (5)$$

$$\varepsilon E \bullet \frac{\partial E}{\partial t} = \frac{\varepsilon}{2} \bullet \frac{\partial E^2}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2}\right)$$

$$\mu H \bullet \frac{\partial H}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}\right)$$

Sub. the values in (5)

$$-\nabla \bullet (ExH) = E \bullet J + \frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2}\right) + \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}\right) \qquad \to (6)$$

Integrate equation (6) throughout the volume on both sides

$$-\int_{vol} \nabla \bullet (ExH) dv = \int_{vol} E \bullet J dv + \frac{\partial}{\partial t}\int_{vol}\left(\frac{\varepsilon E^2}{2}\right) dv + \frac{\partial}{\partial t}\int_{vol}\left(\frac{\mu H^2}{2}\right) dv \to (7)$$

$$-\int_{vol} \nabla \bullet (ExH) dv = \int_{vol} E \bullet J dv + \frac{\partial}{\partial t}\int_{vol}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv \qquad \to (8)$$

By using divergence theorem

$$\int_{vol} \nabla \bullet (ExH) dv = \oint_S (ExH) \bullet ds$$

$$(8) \Rightarrow -\oint_S (ExH) ds = \int_{vol} E \bullet J dv + \frac{\partial}{\partial t}\int_{vol}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv \qquad \to (9)$$

Physical Interpretation of equation (9)

1. The integral on the right is the total ohmic power dissipated with the volume (i.e.)

$\int_{vol} JE \; dv$. If 'E' is electric field strength due to source of power, 'E' & 'J' are in the same direction, then the power represented by the integral expression

$\int_{vol}(JE)dv$ would be used in driving the current against the battery voltage and hence charging the battery. If the direction of 'E' is opposite to that of 'J', the dissipated power represented by the $\int_{vol}(JE)dv$ would be negative. In this case, battery would be generating electric power.

2. The integral in the second term on the right $\frac{\partial}{\partial t}\int_{vol}\left(\frac{\varepsilon E^2}{2}+\frac{\mu H^2}{2}\right)dv$ is the total energy stored in the electric and magnetic fields and the partial derivatives with respect to time cause this term to be the time rate of increase of energy stored within this volume.

3. The sum of the expressions on the right side be the total power flowing in to this volume (i.e.)

$$P = -\oint_S (ExH)ds \qquad \rightarrow (10)$$

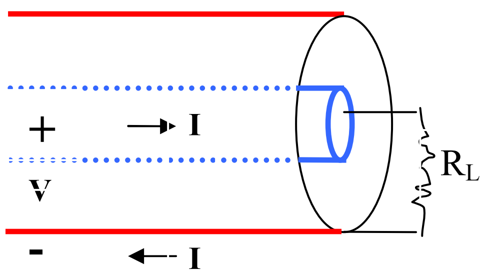
The total power flowing out of the volume is

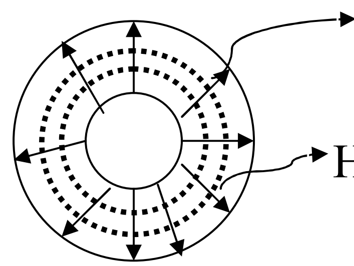
$$P = \oint_S (ExH)ds \qquad \rightarrow (11)$$

The integral of ExH over any closed surface gives the rate of energy flow through that surface.

4. Vector $P = ExH$ (watts/m^2). The poynting theorem states that the vector product $(ExH) = P$ at any point is a measure of the rate of energy flow per unit area at that point. The direction of flow is perpendicular to 'E' and 'H' in the direction of the vector ExH.

4.10 Power Flow in a Coaxial Cable





$$E_\rho = \frac{\rho_L}{2\pi\varepsilon\rho}$$

$$H_\phi = \frac{I}{2\pi\rho}$$

Consider the transfer of power to a load resistance are along a concentric cable which has DC voltage 'V' within conductors and a steady current 'I' flowing inner and outer conductors, the radius of the inner conductor is 'a', the radius of the outer conductor is 'b'. The magnetic field strength 'H' will be directed in circles about the axis. By ampere's, the MMF around any of the circles will be equal to the current enclosed.

$$\oint H \bullet dL = I$$

$$\oint H_\phi a_\phi \bullet \rho d\phi a_\phi = I$$

$$\int_{\phi=0}^{\phi=2\pi} H_\phi \rho d\phi = I$$

$$H_\phi \rho 2\pi = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$H = \frac{I}{2\pi\rho} a_\phi$$

$$V = \frac{\rho_L}{2\pi\varepsilon} \ln(b/a) \qquad \rightarrow (1)$$

$$E = \frac{\rho_L}{2\pi\varepsilon\rho} a_\rho \qquad \rightarrow (2)$$

$\text{from}(1) \Rightarrow$

$$\frac{\rho_L}{2\pi\varepsilon} = \frac{V}{\ln(b/a)} \quad \text{sub} \quad \text{this value} \quad \text{in}(2)$$

$$E = \frac{V}{\rho(\ln(b/a))} a_\rho \qquad \rightarrow (3)$$

P = Poynting Vector = E x H watts/m2

$$P = \frac{V}{\rho \ln(b/a)} a_\rho \, x \, \frac{I}{2\pi\rho} a_\phi$$

$$P = \frac{VI}{2\pi\rho^2 \ln(b/a)} a_z \text{ watts}/m^2$$

$$P = \text{powerflow} = w = \int P \bullet dS$$

$$P = w = \int_{\phi=0}^{2\pi} \int_{\rho=a}^{\rho=b} \frac{VI}{2\pi\rho^2 \ln(b/a)} a_z \bullet \rho d\rho d\phi a_z$$

$$P = \frac{VI}{\ln(b/a)2\pi} \int_{\phi=0}^{\phi=2\pi} \int_{\rho=a}^{\rho=b} \frac{d\rho d\phi}{\rho}$$

$$P = \frac{VI}{\ln(b/a)2\pi} \int_{\phi=0}^{2\pi} [\ln\rho]_a^b \, d\phi$$

$$P = \frac{VI}{\ln(b/a)2\pi} x \ln(b/a) \int_{\phi=0}^{2\pi} d\phi$$

$$P = w = \frac{VI}{2\pi} x \int_{\phi=0}^{2\pi} d\phi$$

$$P = w = VI \qquad \text{watts}$$

Note: When a transmission line is used to deliver power from a generator to a load, power transmission takes place through all the non conducting region of space and none of the power flow through the conductor that make up the transmission.

4.11 Instantaneous, Average and Complex Poynting Vector

(1) Instantaneous Power $\tilde{\omega}$

It is always given by the product of instantaneous voltage $\tilde{V}$ and instantaneous current $\tilde{I}$ (i.e.) $\tilde{\omega} = \tilde{V}\tilde{I}$. The quantities $\tilde{V}$ and $\tilde{I}$ may be expressed in terms of phasors $\tilde{V}$ and $\tilde{I}$ as follows.

$$\tilde{V} = \operatorname{Re}\left\{V.e^{j\omega t}\right\} = \operatorname{Re}\left\{|V| e^{j\theta_V} .e^{j\omega t}\right\}$$

$$\tilde{V} = |V| \cos(\theta_V + \omega t) \qquad \rightarrow (1)$$

$$\tilde{I} = \operatorname{Re}\left\{I.e^{j\omega t}\right\} = \operatorname{Re}\left\{|I|.e^{j\theta i} .e^{j\omega t}\right\}$$

$$\tilde{I} = |I|\cos(\theta_i + \omega t) \qquad \rightarrow (2)$$

Instantaneous Power $\tilde{w} = \tilde{V}\tilde{I}$

$$\tilde{w} = |V|\cos(\omega t + \theta_V) x |I|\cos(\omega t + \theta_i)$$

$$\tilde{w} = \frac{|V||I|\cos(\theta_V - \theta_i)\cos(2\omega t + \theta_V + \theta_i)}{2} \qquad \rightarrow (3)$$

Equation (3) gives the instantaneous power

(II) Average Power: (Wav)

Equation (3) consists of an average part and oscillating part.

Average part = Average Power = $W_{av} = \frac{|V||I|}{2}\cos\theta, \quad \theta = \theta_v - \theta_i$

(III) Complex Power:

It is defined as one half the product of **'V'** & complex conjugate of **'I'**.

$$w = \frac{1}{2}VI^* = \frac{1}{2}|V|e^{j\theta_V}.|I|e^{-j\theta_i}$$

$$w = \frac{|V||I|}{2}e^{j(\theta_V - \theta_i)}$$

$$w = \frac{|V||I|}{2}e^{j\theta}, \qquad \theta = \theta_v - \theta_i$$

W = Waverage + Wreactive

This shows that average and reactive power may be recovered from the complex power by taking the real and imaginary part.

$$W_{average} = \mathrm{Re}\{w\} = \mathrm{Re}\left\{\frac{|V||I|}{2} e^{j\theta}\right\}$$

$$W_{reactive} = \mathrm{Im}\{w\} = \mathrm{Im}\left\{\frac{|V||I|}{2} e^{j\theta}\right\}$$

(IV) Complex Poynting Vector 'P'

Define the complex pointing vector 'P' as $P = \frac{1}{2} EH^{*}$ where **H*** is the complex conjugate of **'H'**. The average and reactive part of the power flow per square meter.

$$P_{average} = \frac{1}{2} \mathrm{Re}\{ExH^{*}\}$$

$$P_{reactive} = \frac{1}{2} \mathrm{Im}\{ExH^{*}\}$$

The product of 'E' and 'H' in above equations is a vector product, only mutually perpendicular components of 'E' and 'H' contribute anything to power flow and the direction of flow is normal to the plane containing 'E' & 'H'.