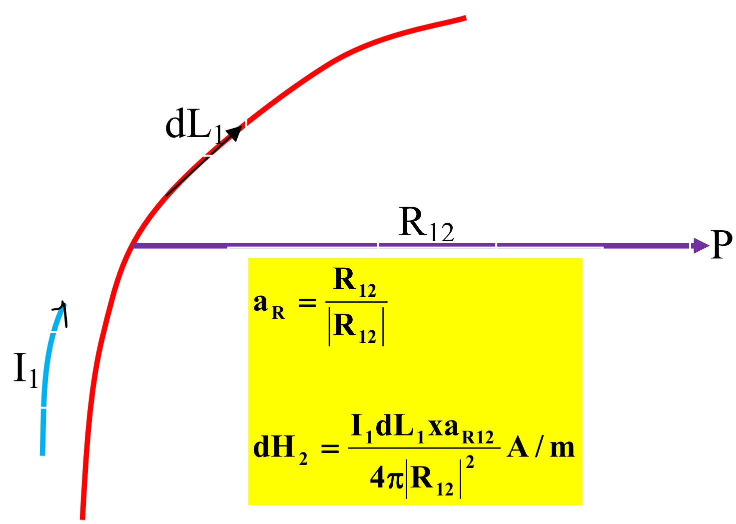
UNIT-II STATIC MAGNETIC FIELD

2.1 Biot - Savart Law

Assume a current ‘I’ flowing in differential vector length of the filament ‘dL’. The Biot-Savart law states that at any point ‘P’ the magnitude of the magnetic field intensity produced by the differential element is proportional to the product of current, magnitude of differential element and sine of the angle lying between the filament to point ‘P’ where the field is desired. The direction of magnetic field intensity is normal to the plane containing the differential filament and the line drawn from the filament to the point ‘P’.

The magnitude of magnetic field intensity is inversely proportional to the square of the distance from differential element to the point ‘P’.



$$dH \alpha \frac{IdL \sin\theta}{R^2} a_R$$

$$dH \alpha \frac{IdLxa_R}{R^2}$$

$$dH = \frac{1}{4\pi} \frac{IdLxa_R}{R^2}$$

$$dH = \frac{IdLxa_R}{4\pi |R|^2}$$

$$dH = \frac{IdLxR}{4\pi |R|^3}$$

The Biot-savart law may also be expressed in terms of distributed sources such as current density 'J' and surface current density 'K'.

Surface current density 'K'

Surface current density flows in a sheet of vanishingly small thickness and 'K' is measured in A/m width.

$$K = \frac{I}{b} \Rightarrow Kb = I$$

$$I = JdS$$

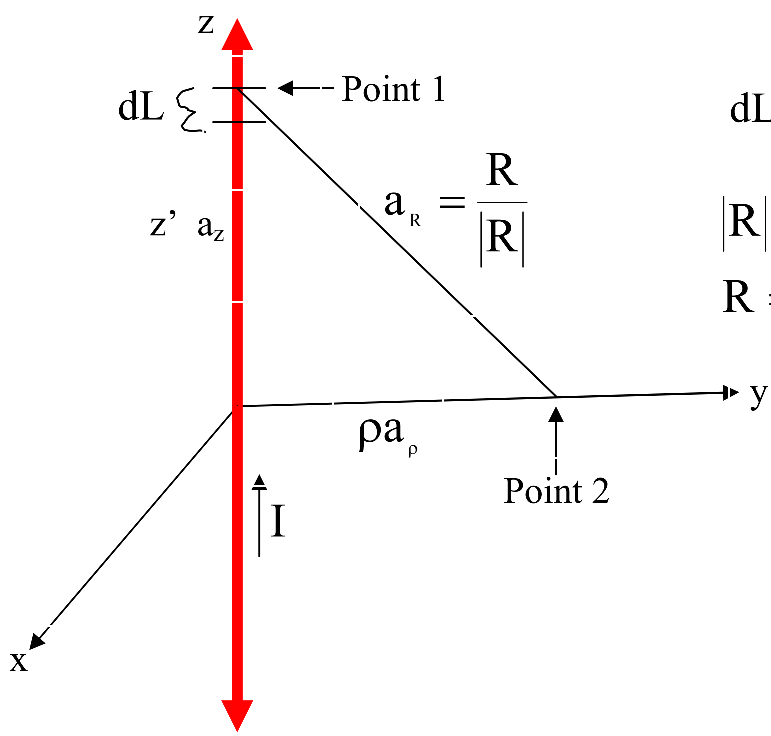
$$\therefore IdL = KdS = JdV$$

$$H = \int_L \frac{IdL x a_R}{4\pi R^2} \qquad \rightarrow (1)$$

$$H = \int_S \frac{KdS x a_R}{4\pi R^2} \qquad \rightarrow (2)$$

$$H = \int_V \frac{Jdv x a_R}{4\pi R^2} \qquad \rightarrow (3)$$

2.2 Magnetic field intensity due to infinite wire carrying current



$dL = dz' a_z$

$$|R| = \sqrt{\rho^2 + z'^2}$$

$$R = \rho a_\rho - z' a_z$$

$$dH = \frac{IdLxa_R}{4\pi R^2}$$

$$dH = \frac{Idz'a_z}{4\pi} x \frac{R}{|R|^3}$$

$$dH = \frac{Idz'a_z}{4\pi\left(\rho^2 + z'^2\right)^{\frac{3}{2}}} x\left(\rho a_\rho - z'a_z\right)$$

$$dH = \frac{Idz'\rho a_\phi}{4\pi\left(\rho^2 + z'^2\right)^{\frac{3}{2}}}$$

$$H = \int_{z'=-\infty}^{z'=+\infty} \frac{Idz'\rho a_\phi}{4\pi\left(\rho^2 + z'^2\right)^{\frac{3}{2}}}$$

$$z' = \rho \cot\theta; \quad dz' = -\rho \operatorname{cosec}^2\theta d\theta$$

$$z' = \infty; \quad z' = -\infty$$

upper limit   lower limit

$$\theta = 0; \qquad \theta = \pi$$

$$H = \int_{\theta=\pi}^{\theta=0} \frac{I\rho\left(-\rho \operatorname{cosec}^2\theta\right)d\theta}{4\pi\left(\rho^2 + \rho^2 \cot^2\theta\right)^{\frac{3}{2}}} a_\phi$$

$$H = \frac{I\rho^2}{4\pi\rho^3} \int_{\theta=\pi}^{\theta=0} \frac{-\operatorname{cosec}^2\theta d\theta}{\operatorname{cosec}^3\theta} a_\phi$$

$$H = \frac{I}{4\pi\rho} \int_{\pi}^{0} -\sin\theta d\theta\, a_\phi$$

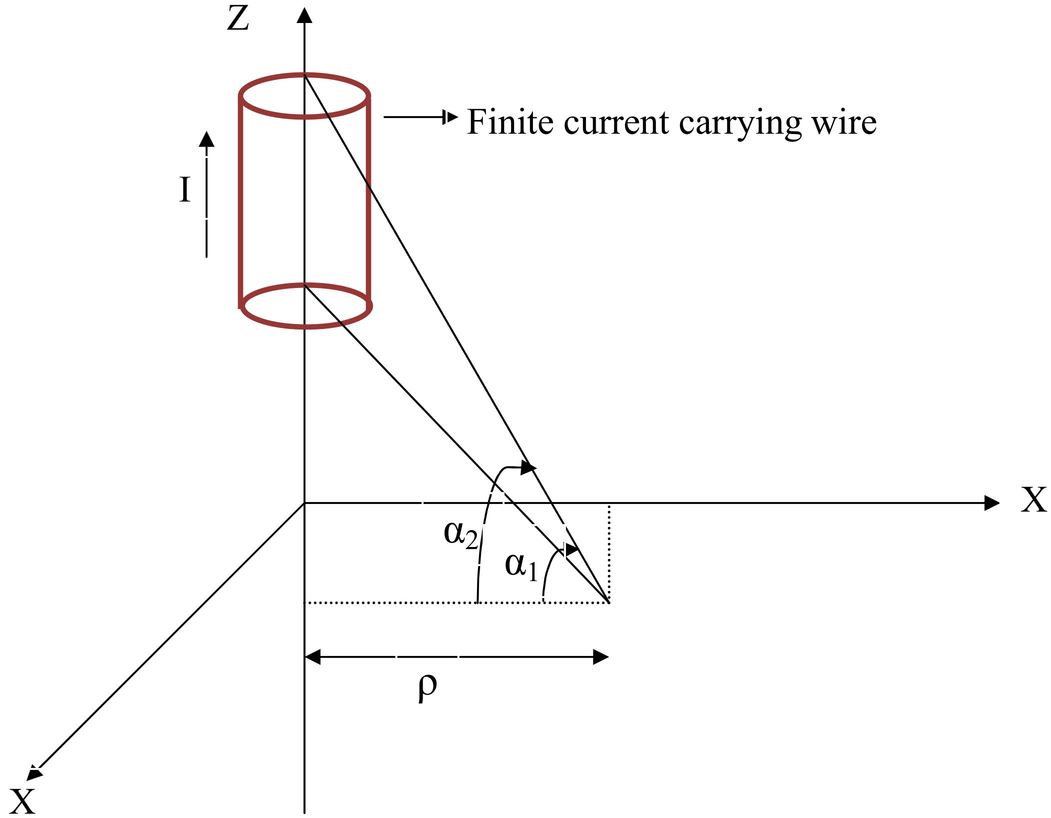
$$H = \frac{I}{4\pi\rho} (\cos\theta)_{\pi}^{0} a_\phi$$

$$H = \frac{I}{4\pi\rho} (2) a_\phi$$

$$H = \frac{I}{2\pi\rho} a_\phi$$

Note: The magnitude of field is not a function of Φ (or) z and it varies inversely as distance from the filament. Direction of the magnetic field intensity vector is circumferential.

2.3 Magnetic field intensity due to finite wire



$$H = H_2 = \frac{I\rho^2 a_\phi}{4\pi\rho^3}\int \frac{dz'}{\left(\rho^2 + z'^2\right)^{\frac{3}{2}}}$$

$$\text{put } z' = \rho\cot\theta \Rightarrow \quad dz' = -\rho\,\text{cosec}^2\,\theta d\theta$$

$$H_2 = \frac{I\rho^2 a_\phi}{4\pi\rho^3}\int_{\theta=\frac{\pi}{2}-\alpha_1}^{\theta=\frac{\pi}{2}-\alpha_2} -\sin\theta d\theta$$

$$H_2 = \frac{Ia_\phi}{4\pi\rho} (\cos\theta)_{\frac{\pi}{2}-\alpha_1}^{\frac{\pi}{2}-\alpha_2}$$

$$H_2 = \frac{Ia_\phi}{4\pi\rho} \left[\cos\left(\frac{\pi}{2} - \alpha_2\right) - \cos\left(\frac{\pi}{2} - \alpha_1\right) \right]$$

$$H_2 = \frac{Ia_\phi}{4\pi\rho} \left[\left(\cos\frac{\pi}{2}\cos\alpha_2 + \sin\frac{\pi}{2}\sin\alpha_2 \right) - \left(\cos\frac{\pi}{2}\cos\alpha_1 + \sin\frac{\pi}{2}\sin\alpha_1 \right) \right]$$

$$H_2 = \frac{Ia_\phi}{4\pi\rho} [\sin\alpha_2 - \sin\alpha_1] \quad A/m$$

2.4 Magnetic Flux Density 'B' (weber /m^2) or (Tesla)

Let us define magnetic flux density 'B' as $B = \mu_0 H$, where 'B' is measured in weber/m^2 (or) tesla and $\mu_0 = 4\Pi x 10^{-7}$ H/m (free space permeability).

2.5 Magnetic Flux 'Φ'(weber)

Let us represent magnetic flux by 'Φ' and define 'Φ' as the flux passing through any area. $\phi = \int_S B \bullet dS = BA$.

B – Magnetic flux density A - Area of the given surface

2.6 Gauss's Law for Magnetic Field

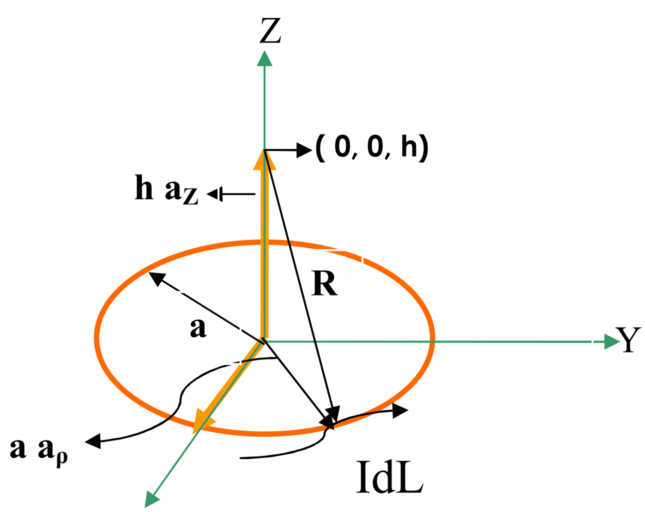
The Magnetic flux lines are closed and do not terminate on a magnetic charge.

For this reason, Gauss's law for magnetic field is $\oint_S B \bullet dS = 0$.

From divergence theorem $\oint_S B \bullet dS = \int_V (\nabla \bullet B) dV = 0$

$$\therefore \nabla \bullet B = 0$$

2.7 Magnetic Field Intensity on the axis of a circular loop



$$dH = \frac{IdLxa_R}{4\pi R^2}$$

$$dH = \frac{Iad\phi a_\phi}{4\pi\left(a^2 + h^2\right)} x \frac{\left(-aa_\rho + ha_z\right)}{\left(a^2 + h^2\right)^{\frac{1}{2}}}$$

$$dH = \frac{Iad\phi\left(aa_z + ha_\rho\right)}{4\pi\left(a^2 + h^2\right)^{\frac{3}{2}}}$$

The figure shows that diametrically opposite current elements produce 'ρ' components which cancel each other.

$$dH = \frac{Ia^2 d\phi}{4\pi\left(a^2 + h^2\right)^{\frac{3}{2}}} a_z$$

$$H = \int \frac{Ia^2 d\phi}{4\pi\left(a^2 + h^2\right)^{\frac{3}{2}}} a_z$$

$$H = \frac{Ia^2}{4\pi} a_z \int \frac{d\phi}{\left(a^2 + h^2\right)^{\frac{3}{2}}}$$

$$H = \frac{Ia^2}{4\pi\left(a^2 + h^2\right)^{\frac{3}{2}}} a_z \int_{\phi=0}^{\phi=2\pi} d\phi$$

$$H = \frac{Ia^2}{4\pi\left(a^2 + h^2\right)^{\frac{3}{2}}} a_z x 2\pi$$

$$H = \frac{Ia^2}{2\left(a^2 + h^2\right)^{\frac{3}{2}}} a_z \qquad A/m$$

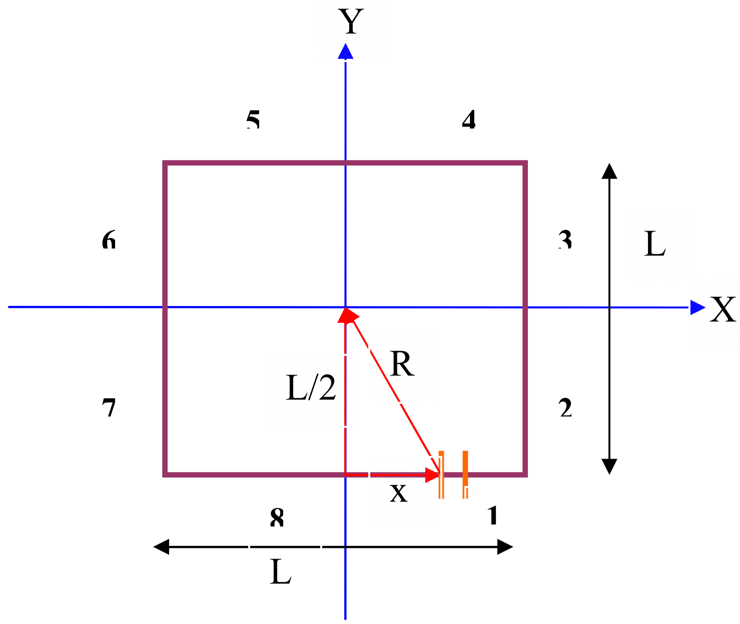
At center (i.e.) $h \to 0$

$$H = \frac{Ia^2}{2\left(a^2 + 0^2\right)^{\frac{3}{2}}} a_z$$

$$H = \frac{Ia^2}{2\left(a^2\right)^{\frac{3}{2}}} a_z$$

$$H = \frac{I}{2a} a_z$$

2.8 Magnetic Field intensity at the center of a Rectangular current carrying Loop



$$dH = \frac{IdLxa_R}{4\pi R^2}$$

$$dH = \frac{Idxa_x x\left(-xa_x + \left(\frac{L}{2}\right)a_y\right)}{4\pi\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$dH = \frac{Idx\left(\frac{L}{2}\right)a_z}{4\pi\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$H = 8\int \frac{Idx\left(\frac{L}{2}\right)a_Z}{4\pi\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$H = 8\int_{x=0}^{x=L/2} \frac{Idx\left(\frac{L}{2}\right)a_Z}{4\pi\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{3}{2}}}$$

$$H = \frac{8I}{4\pi}\left(\frac{L}{2}\right)x\left(\frac{2}{L}\right)^2 a_z \left[\frac{x}{\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}}\right]_0^{\frac{L}{2}}$$

$$H = \frac{8I}{4\pi}\left(\frac{2}{L}\right)a_z\left[\frac{\frac{L}{2}}{\left(\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}}\right]$$

$$H = \frac{8I}{4\pi}\left(\frac{2}{L}\right)a_z\left[\frac{\frac{L}{2}}{\left(2x\left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}}\right] = \frac{8I}{4\pi}\left(\frac{2}{L}\right)a_z\left[\frac{\frac{L}{2}}{\frac{L}{\sqrt{2}}}\right]$$

$$H = \frac{4I}{\pi L}a_z\left(\frac{1}{\sqrt{2}}\right)$$

$$H = \frac{2\sqrt{2}I}{\pi L}a_z \qquad A/m$$

Case (i): 'H' at center of the square loop = $\left(\frac{2\sqrt{2}I}{\pi}\right)\left(\frac{1}{L}\right)a_Z$

Case (ii): 'H' at the axis of a square loop at a distance 'h'

$$H = \left(\frac{2\sqrt{2}I}{\pi}\right)\frac{L^2}{\left(L^2+h^2\right)^{\frac{3}{2}}}a_Z$$

Case (iii): 'H' at infinite distance $h \to \infty \qquad H = 0$

2.9 Lorentz Force Equation for Moving Charge

The definition of electric field intensity shows that the force on a charged particle is $F = QE \qquad \to (1)$.

The force is in the same direction as electric field intensity and is directly proportional to both 'E', 'Q'.

A charged particle in motion in a magnetic field of flux density 'B' is found experimentally it experiences a force whose magnitude is proportional to the product of the magnitude of charge 'Q' its velocity 'V' and flux density 'B' and to the sine of the angle between the vectors 'V', 'B'. The direction of the force is perpendicular to both 'V', 'B' and is given by a vector in the direction of VxB.

$$F = QVB\sin\theta$$

$$F = Q(VxB) \qquad \to (2)$$

The force on a moving particle due to combined electrical and magnetic fields is given by the vector sum of individual force.

$$F = QE + Q(VxB)$$

$$F = Q[E + (VxB)] \qquad \rightarrow (3)$$

This equation is known as Lorentz Force Equation.

Applications:

The solution is required in determining the electron orbits in magnetrons, proton paths in cyclotron, plasma characteristics in magneto hydrodynamic generator (or) in general, charged particle motion in combined electric and magnetic fields.

2.10 Force on a wire carrying a current 'I' placed in the magnetic field

Force on a filamentary closed circuit is given by $F = \oint IdLxB$

$I \rightarrow$ Current through wire

$dL \rightarrow$ Length of the filament

$B \rightarrow$ Magnetic field acting on the filament

$$F = -I\int BxdL \qquad \rightarrow (1)$$

For a straight conductor in a uniform magnetic field

$$F = I\int dLxB \qquad \rightarrow (2)$$

$$F = ILxB$$

$$F = ILB\sin\theta \qquad \rightarrow (3)$$

where 'θ' is angle between vectors representing the direction of current and direction of magnetic flux density. For uniform magnetic flux density 'B' is removed from the integral

$$F = -IBx\oint dL \qquad \rightarrow (4)$$

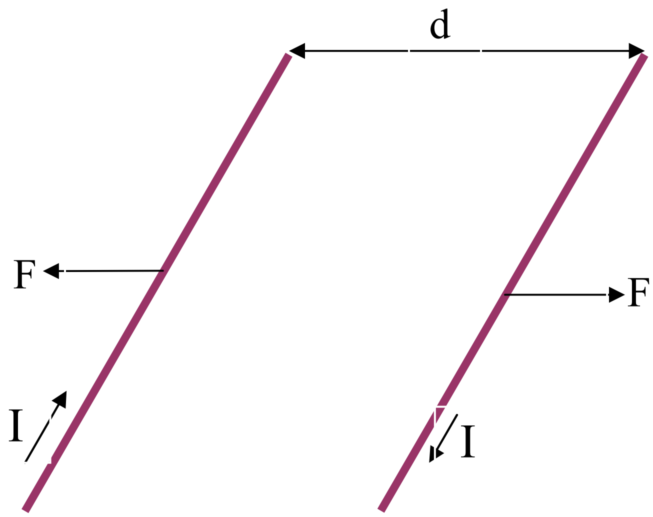
We know that closed line integrals in an electrostatics field $\oint B \bullet dL = 0$. Therefore force on the closed filamentary circuit in a uniform magnetic field is zero.

Force between Differential Current Elements

$$F_2 = \frac{\mu_0 I_1 I_2}{4\pi}\left[\oint(dL_2)\oint\frac{dL_1 x a_{R12}}{R_{12}{}^2}\right]$$

$$F_2 = \frac{\mu_0 I_1 I_2}{4\pi}\oint\left[\oint\frac{a_{R12} x dL_1}{R_{12}{}^2}\right]x dL_2$$

Force between two infinite parallel filaments with separation'd' and equal but opposite currents 'I' experience a repulsive force which is given by



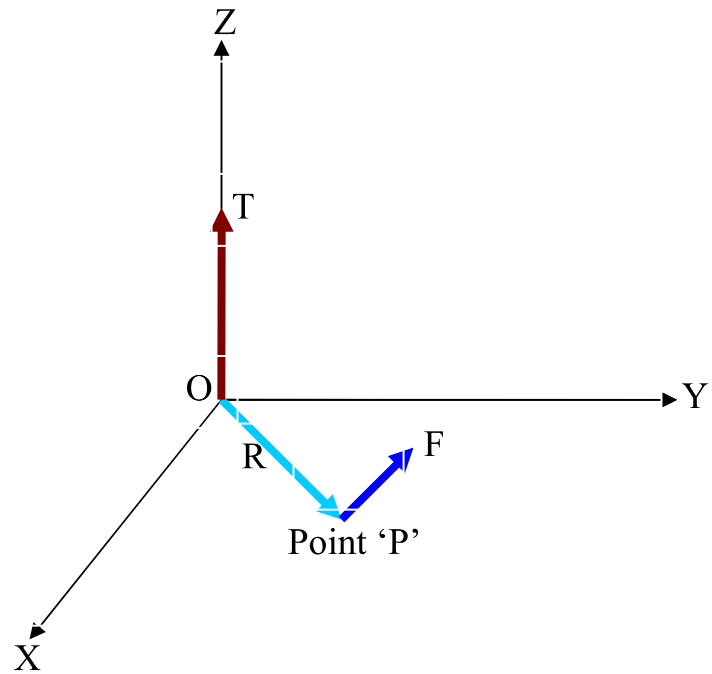
$$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi d}\left(\frac{\text{Newton}}{\text{meter}}\right)$$

If the currents are different

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}\left(\frac{\text{Newton}}{\text{meter}}\right)$$

2.11 Torque

Torque about point 'O' is the vector whose magnitude is product of magnitude of 'R', 'F' (R –lever arm; F – force) and of the sine angle between these two vectors.

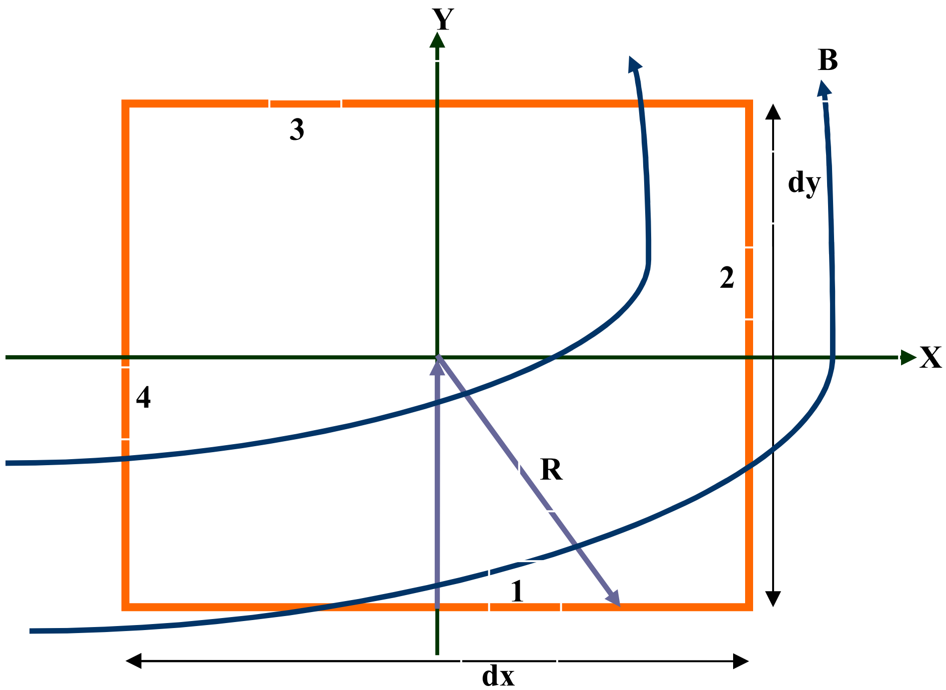


Direction of the vector torque 'T' is normal to both force 'F' and lever arm 'R'

$$T = |R||F|\sin\theta$$

$$T = RxF$$

2.12 Torque on a loop carrying current 'I'



Vector force on side 1 is $dF_1 = IdLxB_0$

$$dF_1 = Idxa_x x\left(B_{0x}a_x + B_{0y}a_y + B_{0z}a_z\right)$$

$$dF_1 = Idx\left(B_{0y}a_z - B_{0z}a_y\right) \qquad \rightarrow (1)$$

$$dF_2 = Idya_y x\left(B_{0x}a_x + B_{0y}a_y + B_{0z}a_z\right)$$

$$dF_2 = Idy\left(-B_{0x}a_x + B_{0z}a_z\right) \qquad \rightarrow (2)$$

$$dF_3 = Idx\left(-B_{0y}a_z + B_{0z}a_y\right) \qquad \rightarrow (3)$$

$$dF_4 = Idy\left(B_{0x}a_z - B_{0z}a_z\right) \qquad \rightarrow (4)$$

Torque acting on side 1 is $dT_1 = R_1 x dF_1$

$$dT_1 = -\frac{1}{2}dya_y xIdS\left(B_{0y}a_z - B_{0z}a_y\right)$$

$$dT_1 = -\frac{1}{2}IdxdyB_{0y}a_x \qquad \rightarrow (5)$$

$$dT_3 = -\frac{1}{2}IdxdyB_{0y}a_x \qquad \rightarrow (6)$$

$$dT_2 = \frac{1}{2}dxa_x xIdy\left(-B_{0x}a_z + B_{0z}a_x\right)$$

$$dT_2 = \frac{1}{2}IdxdyB_{0x}a_y \qquad \rightarrow (7)$$

$$dT_4 = \frac{1}{2}IdxdyB_{0x}a_y \qquad \rightarrow (8)$$

(5)+(6)+(7)+(8) =>

$$dT = -IdxdyB_{0y}a_x + IdxdyB_{0x}a_y$$

$$dT = Idxdy(B_{0x}a_y - B_{0y}a_x) \rightarrow (9)$$

$$dT = Idxdy(a_z xB) \rightarrow (10)$$

$$dT = IdSxB \rightarrow (11)$$

$dS \rightarrow$ vector area of differential loop $dxdya_z = dS$

2.13 Magnetic moment (or) Magnetic Dipole Moment

It is defined as product of the loop current and vector area of the loop gets differential magnetic dipole 'dm' with units **A.m^2**.

$$dm = IdS \rightarrow (12)$$

$$dT = dmxB$$

$$T = mxB \rightarrow (13)$$

$$T = ISxB$$

2.14 Magnetic Vector Potential (A): Weber/m (or) Tesla. Metre

Let 'A' be vector magnetic potential

$$\nabla \bullet B = 0 \qquad \rightarrow (1)$$

Divergence of curl of any vector field is zero

$$\nabla \bullet (\nabla x A) = 0 \qquad \rightarrow (2)$$

$$(1),(2) \Rightarrow B = \nabla x A \qquad \rightarrow (3)$$

$$H = \frac{B}{\mu_0} = \frac{1}{\mu_0}(\nabla x A) \qquad \rightarrow (4)$$

$$A = \oint_{line} \frac{\mu_0 IdL}{4\pi R}$$

$$A = \oint_{surface} \frac{\mu_0 KdS}{4\pi R}$$

$$A = \oint_{volume} \frac{\mu_0 JdV}{4\pi R}$$

2.15 Ampere's Circuital law

$$\oint H \bullet dL = I$$

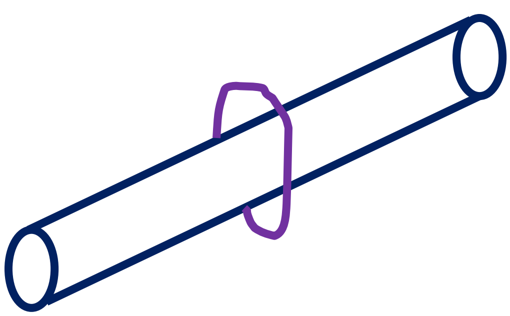
2.16 Applications of Ampere's Circuital law

$$1) \oint H \bullet dL = \int_{\phi=0}^{\phi=2\pi} H_\phi \bullet \rho d\phi = H_\phi \rho . 2\pi$$

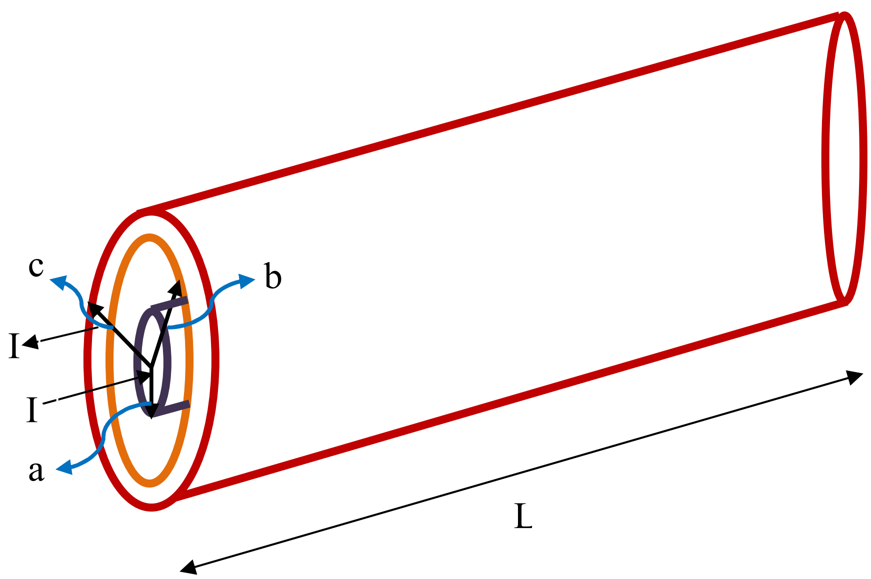
$$H_\phi \rho (2\pi) = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$H = \frac{I}{2\pi\rho} a_\phi$$



2) coaxialcable



Case (i): $\rho < a$

$$I_{enclosed} = I\frac{\pi\rho^2}{\pi a^2}$$

$$2\pi\rho H_\phi = \frac{I\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho^2}{a^2(2\pi\rho)}$$

$$H = \frac{I\rho}{2\pi a^2} a_\phi$$

Case (ii): $a < \rho < b$

$$H = \frac{I}{2\pi a} a_\phi$$

Case (iii): $b < \rho < c$

$$I_{enclosed} = I - I\left(\frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}\right)$$

$$I_{enclosed} = I - I\left(\frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

$$I_{enclosed} = I\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)$$

$$H_\phi = \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)$$

$$H = \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right) a_\phi$$

Case (iv): $\rho > c$

The current enclosed $I = 0 \qquad \Rightarrow H = 0$

Note: The internal field is zero, equal positive and negative currents enclosed by path each produces an external field of magnitude $\frac{I}{2\pi\rho}$ complete cancellation occurs. This is an example of shielding. Such a coaxial cable carrying large current would not produce noticable effect in an adjacent circuit.

Fig: Magnetic field intensity as a function of radius in an infinitely long co-axial transmission.

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