CHAPTER 1 - STATIC ELECTRIC FIELDS

1.1 Different Co-ordinate Systems

1. Rectangular (or) Cartesian Coordinate system

$$dL = (dx)a_x + (dy)a_y + (dz)a_z$$

Elemental length = $\sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

Elemental area = $dxdy,\ dydz, dzdx$

Volume = $dxdydz$

Right handed Cartesian coordinate system

$$a_x x a_y = a_z$$
$$a_y x a_z = a_x$$
$$a_z x a_x = a_y$$

and

$$a_y x a_x = -a_z$$
$$a_z x a_y = -a_x$$
$$a_x x a_z = -a_y$$

2. Circular Cylindrical Coordinate System

Consider any point in this coordinate system as the intersection of three mutually perpendicular surfaces. The surfaces are:

1) Circular Cylinder (ρ = constant)

2) a plane (Φ = constant)

3) another plane (Z = constant).

Any point 'P' in this coordinate system is represented by (ρ, Φ, z). Units of 'ρ' and 'z' are in meters and unit of 'Φ' is radians. The unit vectors of this coordinate system are a_ρ, a_Φ and a_z.

$$dL = (d\rho)a_\rho + (\rho d\phi)a_\phi + (dz)a_z$$

Elemental length = $\sqrt{(d\rho)^2 + (\rho d\phi)^2 + (dz)^2}$

Elemental area = $\rho d\rho d\phi, \rho d\phi dz, dz d\rho$

Volume = $\rho d\rho d\phi dz$

Right handed Cylindrical coordinate system

$$a_\rho x a_\phi = a_z$$

$$a_\phi x a_z = a_\rho$$

$$a_z x a_\rho = a_\phi$$

Rectangular to Cylindrical coordinate sytem

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$

Cylindrical to Rectangular coordinate system

$$x = \rho \cos\phi$$

$$y = \rho \sin\phi$$

$$z = z$$

Dot product of unit vectors in cylindrical and Cartesian coordinate system

.	a_ρ	a_Φ	a_z
a_x	$\cos\Phi$	$-\sin\Phi$	0
a_y	$\sin\Phi$	$\cos\Phi$	0
a_z	0	0	1

3. Spherical coordinate system

$$dL = (dr)a_r + (rd\theta)a_\theta + (r\sin\theta d\phi)a_\phi$$

Elemental length = $\sqrt{(dr)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2}$

Elemental area ds = $rdrd\theta, r\sin\theta drd\phi, r^2\sin\theta d\theta d\phi$

Volume $dv = r^2\sin\theta d\theta drd\phi$

Right handed spherical coordinate system $a_R x a_\theta = a_\phi$

Spherical to Cartesian coordinate system

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

Cartesian to Spherical coordinate system

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \tan^{-1}(y/x)$$

Dot products of unit vectors in spherical and Cartesian systems

.	$\mathbf{a}_R$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x$	Sinθ CosΦ	Cosθ CosΦ	-sin Φ
$\mathbf{a}_y$	Sinθ SinΦ	Cosθ SinΦ	CosΦ
$\mathbf{a}_z$	Cosθ	-sinθ	0

1.2 Coulomb's Law

Coulomb stated that force between any two objects (two charges) in free space (or) vacuum separated by a distance is

- Directly proportional to the product of the two charges
- Inversely proportional to the square of the distance between them
- Directed along the line joining the charges.

$$F_2 \alpha \frac{Q_1 Q_2}{{R_{12}}^2} a_{12}$$

$$F_2 = K \frac{Q_1 Q_2}{{R_{12}}^2} a_{12}$$

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_o {R_{12}}^2} a_{12} \qquad \text{Newton}$$

$$a_{12} = \frac{R_{12}}{|R_{12}|} = \frac{r_2 - r_1}{|r_2 - r_1|} \quad \text{and constant } K = \frac{1}{4\pi\varepsilon_o}$$

$$\varepsilon_o = 8.854 x 10^{-12} F/m$$

$$F_1 = -F_2 = -\frac{Q_1 Q_2}{4\pi\varepsilon_o {R_{12}}^2} a_{12}$$

$$F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_o {R_{21}}^2} a_{21}$$

F_1 is the force exerted by q_2 on q_1

1.3 Electric Field Intensity (E)

The Electric Field Intensity ‘E’ is defined as force per unit charge.

$$F_{1t} = \frac{Q_1 Q_t}{4\pi\varepsilon_o {R_{t1}}^2} a_{t1}$$

Q_t – is the test charge and F_{1t} – force acting on Q_2 by Q_1

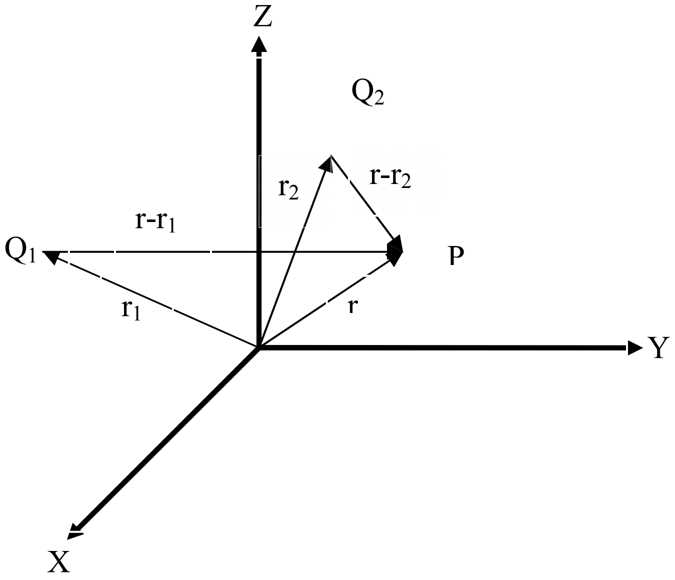
$$E = \frac{F_{1t}}{Q_t}$$

$$E = \frac{Q_1}{4\pi\varepsilon_0 {R_{t1}}^2} a_{t1}$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} a_R \qquad \text{Volt/meter}$$

$$\text{where} \qquad a_R = \frac{R}{|R|}$$

1.4 Electric Field due to Discrete Charges



From figure E = Resultant electric field Intensity due to two discrete charges = $E_1 + E_2$

$$E = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2} \frac{r - r_1}{|r - r_1|} + \frac{Q_2}{4\pi\varepsilon_0 |r - r_2|^2} \frac{r - r_2}{|r - r_2|}$$

The above equation gives the electric field intensity due to two discrete charges. For ‘n’ number of discrete charges, the electric field intensity is given by,

$E = E_1 + E_2 + E_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \, En.$

1.5 Principle of Superposition of Fields

Principle of Superposition of Electric Field Intensity:

The total or Resultant electric fields intensity at a point is the vector sum of the individual component fields at the point.

Principle of Superposition of Potential Fields:

The total potential acting at a point is the algebraic sum of individual potentials at the point.

1.6 Electric Field Intensity due to Continuous Volume Charge Distribution

$$\Delta Q = \rho_V \Delta V \qquad \rightarrow (1)$$

$$\rho_V = \lim_{\Delta V \rightarrow 0} \frac{\Delta Q}{\Delta V}$$

$$\Delta E = \frac{\Delta Q}{4\pi\varepsilon_0 R^2} a_R \qquad \rightarrow (2)$$

$$\text{from(1)}\, \Delta E = \frac{\rho_V \Delta V}{4\pi\varepsilon_0 R^2} a_R$$

$$= \frac{\rho_V \Delta V}{4\pi\varepsilon_0 |r - r'|^2} \frac{r - r'}{|r - r'|}$$

$$E = \iiint \frac{\rho_V \Delta V}{4\pi\varepsilon_0 |r - r'|^2} \frac{r - r'}{|r - r'|} \rightarrow (3)$$

Equation (3) is the electric field intensity due to continuous volume charge distribution.

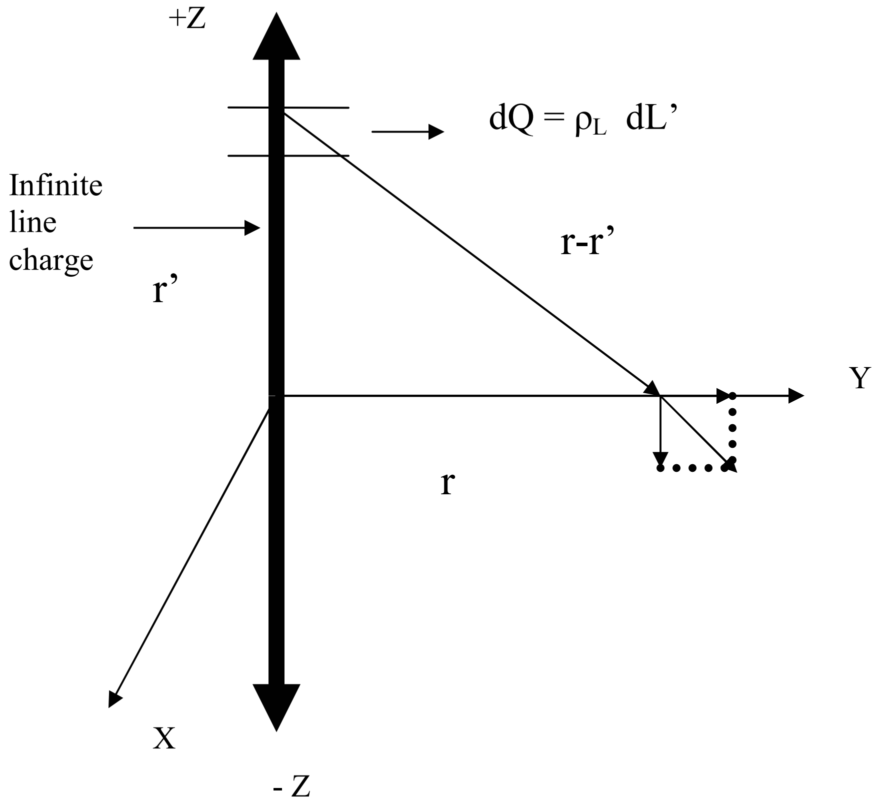
Then the electric field intensity due to continuous line charge density

$$E = \int_L \frac{\rho_L dL}{4\pi\varepsilon_0 |r - r'|^3}(r - r')$$

Electric field intensity due to continuous Surface charge density

$$E = \int_S \frac{\rho_S dS}{4\pi\varepsilon_0 |r - r'|^3}(r - r')$$

1.7 Electric Field due to charges distributed uniformly on an infinite line



$$r = \rho a_\rho \qquad \text{and} \quad r' = z' a_z$$

$$R = r - r' = \rho a_\rho - z' a_z$$

$$|R| = \sqrt{\rho^2 + z'^2}$$

$$E = E_\rho a_\rho + E_\phi a_\phi + E_z a_z$$

No element of charge produces a 'Φ' component of electric field intensity, therefore $E_\Phi = 0$.

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} a_R$$

$$\text{from fig} \quad dQ = \rho_L dL' \qquad \text{and} \quad dL' = dz'$$

$$dE = \frac{\rho_L dz'}{4\pi\varepsilon_0 |R|^2} \frac{R}{|R|} = \frac{\rho_L dz'}{4\pi\varepsilon_0 |r - r'|^2} \frac{r - r'}{|r - r'|}$$

$$dE = \frac{\rho_L dz'}{4\pi\varepsilon_0 |r - r'|^3} r - r' = \frac{\rho_L dz'}{4\pi\varepsilon_0 \left(\sqrt{\rho^2 + z'^2}\right)^3} \left(\rho a_\rho - z' a_z\right)$$

$$dE = \frac{\rho_L dz'}{4\pi\varepsilon_0 \left(\rho^2 + z'^2\right)^{\frac{3}{2}}} \left(\rho a_\rho - z' a_z\right) \rightarrow (1)$$

Equation (1) is in the form of $E = E_\rho\, a_\rho + E_z\, a_z$. The contribution to '$E_z$' by elements of charge which are equal distances above and below the point at which the field is going to be determined will cancel each other. Therefore only 'E_ρ' will exist and it varies with respect to 'ρ'.

$$(1) \rightarrow dE = \frac{\rho_L dz'}{4\pi\varepsilon_0\left(\rho^2 + z'^2\right)^{\frac{3}{2}}}\rho a_\rho$$

$$dE = \int \frac{\rho_L dz' \rho a_\rho}{4\pi\varepsilon_0\left(\rho^2 + z'^2\right)^{\frac{3}{2}}}$$

$$E = \int_{z=-\infty}^{z=+\infty} \frac{\rho_L dz' \rho a_\rho}{4\pi\varepsilon_0\left(\rho^2 + z'^2\right)^{\frac{3}{2}}} \qquad \rightarrow (2)$$

$$\text{put} \qquad z' = \rho\cot\theta \qquad ; dz' = -\rho\,\mathrm{cosec}^2\,\theta\, d\theta$$

$$\text{upper limit}\; z' = \infty \quad ; z' = -\infty$$

$$\rho\cot\theta = \infty \qquad ; \rho\cot\theta = -\infty$$

$$\theta = \cot^{-1}(\infty) \qquad ; \theta = \cot^{-1}(-\infty)$$

$$\theta = \cot^{-1}(\cot(0)) \qquad ; \theta = \pi + \cot^{-1}(\cot(0))$$
$$\theta = \text{upper limit} = 0 \qquad ; \theta = \text{lower limit} = \pi$$

$$E = \int_{\theta=\pi}^{\theta=0} \frac{\rho_L \rho\left(-\rho \operatorname{cosec}^2\theta\right)d\theta}{4\pi\varepsilon_0\left(\rho^2 + \rho^2\cot^2\theta\right)^{\frac{3}{2}}} a_\rho$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0} \int_{\theta=\pi}^{\theta=0} \frac{\rho^2\left(-\operatorname{cosec}^2\theta\right)d\theta}{\rho^3\left(1 + \cot^2\theta\right)^{\frac{3}{2}}}$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0} \int_{\theta=\pi}^{\theta=0} \frac{\left(-\operatorname{cosec}^2\theta\right)d\theta}{\rho\left(\operatorname{cosec}^2\theta\right)^{\frac{3}{2}}}$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0} \int_{\theta=\pi}^{\theta=0} \frac{\left(-\operatorname{cosec}^2\theta\right)d\theta}{\rho\left(\operatorname{cosec}^3\theta\right)}$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0} \int_{\theta=\pi}^{\theta=0} \frac{-d\theta}{\rho(\operatorname{cosec}\theta)}$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0} \int_{\theta=\pi}^{\theta=0} \frac{-\sin\theta d\theta}{\rho}$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0\rho} \int_{\theta=\pi}^{\theta=0} -\sin\theta d\theta$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0 \rho}\left(-\cos\theta\right)_\pi^0$$

$$E = \frac{\rho_L a_\rho}{4\pi\varepsilon_0 \rho} x 2$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho \qquad \rightarrow (3)$$

Equation number (3) gives the electric field intensity due to infinite line charge.

1.8 Electric Field due to Finite Line Charge

$$E = \frac{\rho_L \rho}{4\pi\varepsilon_0}\left[\frac{1}{\rho^2}\frac{z'}{\left(\rho^2 + z'^2\right)^{\frac{1}{2}}}\right]_{z'=-L}^{z'=L} a_\rho$$

$$E = \frac{\rho_L}{4\pi\varepsilon_0 \rho}\left[\frac{z'}{\left(\rho^2 + z'^2\right)^{\frac{1}{2}}}\right]_{z'=-L}^{z'=L} a_\rho$$

$$E = \frac{\rho_L}{4\pi\varepsilon_0 \rho}\left[\frac{L'}{\left(\rho^2 + L^2\right)^{\frac{1}{2}}} - \frac{-L'}{\left(\rho^2 + L^2\right)^{\frac{1}{2}}}\right] a_\rho$$

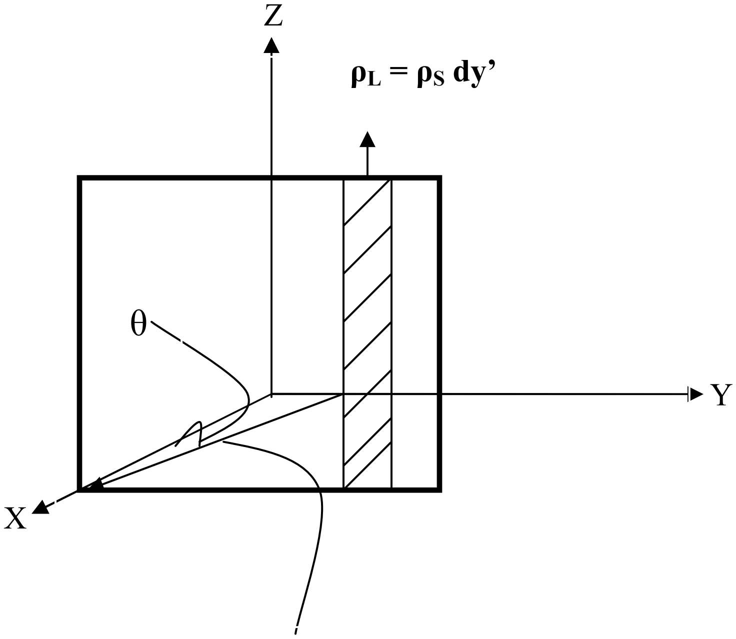
$$E = \frac{\rho_L}{4\pi\varepsilon_0\rho}\left[\frac{2L}{\left(\rho^2 + L^2\right)^{\frac{1}{2}}}\right]a_\rho$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0\rho}\left[\frac{L}{\left(\rho^2 + L^2\right)^{\frac{1}{2}}}\right]a_\rho \qquad \rightarrow (4)$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0\rho}\cos\theta\, a_\rho \qquad \rightarrow (5)$$

Equation (4) and (5) give the electric field intensity due to finite line charge.

1.9 Electric Field Intensity due to Infinite Sheet of Charge



$$R = \sqrt{x^2 + y'^2}$$

$$dE = \frac{\rho_L}{2\pi\varepsilon_0 R}\cos\theta \qquad a_x$$

$$dE = \frac{\rho_S dy'}{2\pi\varepsilon_0 R}\cos\theta \qquad a_x$$

$$R = \sqrt{x^2 + y'^2} \qquad \text{and} \cos\theta = \frac{x}{R} = \frac{x}{\sqrt{x^2 + y'^2}}$$

$$dE = \frac{\rho_S dy'}{2\pi\varepsilon_0 \sqrt{x^2 + y'^2}} \frac{x}{\sqrt{x^2 + y'^2}} a_x$$

$$dE = \frac{\rho_S dy'}{2\pi\varepsilon_0} \frac{x}{\left(x^2 + y'^2\right)} a_x$$

$$E = \int \frac{\rho_S dy'}{2\pi\varepsilon_0} \frac{x}{\left(x^2 + y'^2\right)} a_x$$

$$E = \int_{y'=-\infty}^{y'=\infty} \frac{\rho_S dy'}{2\pi\varepsilon_0} \frac{x}{\left(x^2 + y'^2\right)} a_x$$

$$E = \frac{\rho_S}{2\pi\varepsilon_0} a_x \int_{y'=-\infty}^{y'=\infty} \frac{dy' x}{\left(x^2 + y'^2\right)}$$

$$E = \frac{\rho_S}{2\pi\varepsilon_0} a_x \left[\tan^{-1}\left(\frac{y'}{x} \right) \right]_{-\infty}^{\infty}$$

$$E = \frac{\rho_S}{2\pi\varepsilon_0} a_x \left[\tan^{-1}(\infty) - \tan^{-1}(-\infty) \right]$$

$$E = \frac{\rho_S}{2\pi\varepsilon_0} a_x \left[\tan^{-1}\left(\tan\frac{\pi}{2} \right) - (-) \tan^{-1}\left(\tan\frac{\pi}{2} \right) \right]$$

$$E = \frac{\rho_S}{2\pi\varepsilon_0} a_x \left[\frac{\pi}{2} - (-)\frac{\pi}{2} \right] = \frac{\rho_S}{2\pi\varepsilon_0} a_x x\pi$$

$$E = \frac{\rho_S}{2\varepsilon_0} a_x$$

The above equation gives the electric field intensity due to infinite sheet of charge lies in the 'yz' plane.

$$E = \frac{\rho_S}{2\varepsilon_0} a_y$$ Infinite sheet of charge lies in 'xz' plane

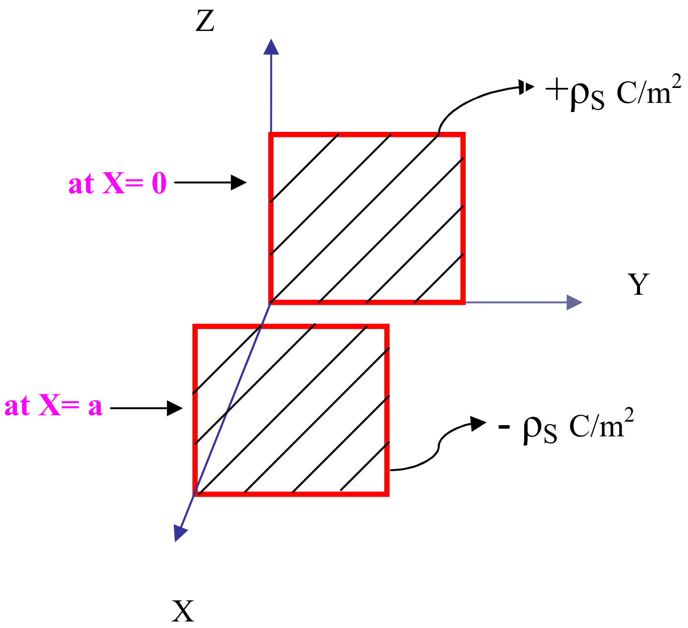
$$E = \frac{\rho_S}{2\varepsilon_0} a_z$$ Infinite sheet of charge lies in 'xy' plane

In general electric field intensity due to infinite sheet of charge lies in any plane is given by

$$E = \frac{\rho_S}{2\varepsilon_0} a_N$$

where a_N is the unit normal vector perpendicular to the plane containing sheet of charge.

Two Sheets of Charge



Case (i) **When $x > a$**

$$E = E_+ + E_-$$

$$E_+ = \frac{\rho_S}{2\varepsilon_0} a_x \qquad E_- = \frac{-\rho_S}{2\varepsilon_0} a_x$$

$$\therefore E = E_+ + E_- = 0$$

Case (ii) **When $x < 0$**

$$E_+ = \frac{\rho_S}{2\varepsilon_0}(-a_x) \qquad E_- = \frac{-\rho_S}{2\varepsilon_0}(-a_x)$$

$$\therefore E = E_+ + E_- = 0$$

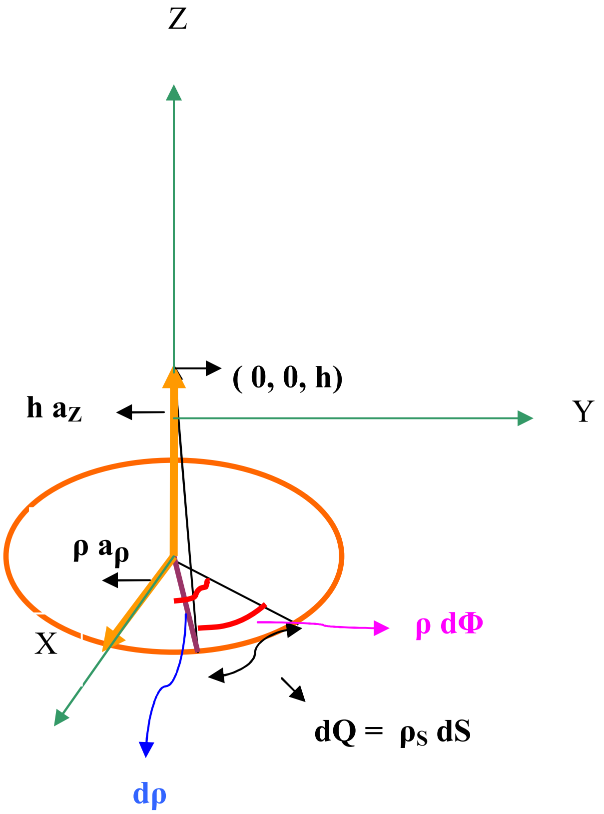
Case (iii) **when 0 < x <a**

$$E_+ = \frac{\rho_S}{2\varepsilon_0} a_x \qquad E_- = \frac{-\rho_S}{2\varepsilon_0}(-a_x) = \frac{\rho_S}{2\varepsilon_0} a_x$$

$$\therefore E = E_+ + E_- = 2x\frac{\rho_S}{2\varepsilon_0} a_x$$

$$E = \frac{\rho_S}{\varepsilon_0} a_x$$

1.10 Electric Field on the axis of a uniformly charged circular disc



$$dE = \frac{dQ}{4\pi\varepsilon_0 |R|^2} a_R$$

$$dE = \frac{\rho_S dS}{4\pi\varepsilon_0 |R|^2} a_R \qquad \rightarrow (1)$$

$$a_R = \frac{R}{|R|}$$

$$R = -\rho a_\rho + h a_z$$

$$|R| = \sqrt{\rho^2 + h^2}$$

$$dE = \frac{\rho_S dS}{4\pi\varepsilon_0 |R|^2} \frac{R}{|R|} = \frac{\rho_S dS}{4\pi\varepsilon_0 |R|^3} R$$

$$dE = \frac{\rho_S dS}{4\pi\varepsilon_0 \left(\sqrt{\rho^2 + h^2}\right)^3} \left(-\rho a_\rho + h a_z\right) = dE_\rho a_\rho + dE_z a_z$$

Due to opposite 'ρ' components dE_ρ gets cancelled each other and we are having only dE_z component only.

$$dE = \frac{\rho_S dS}{4\pi\varepsilon_0 \left(\rho^2 + h^2\right)^{\frac{3}{2}}} \left(h a_z\right)$$

where

$$dS = \rho d\rho d\phi$$

$$dE = \frac{\rho_S \rho d\rho d\phi}{4\pi\varepsilon_0 \left(\rho^2 + h^2\right)^{\frac{3}{2}}} \left(h a_z\right)$$

$$E = \int_S \frac{\rho_S \rho d\rho d\phi}{4\pi\varepsilon_0 \left(\rho^2 + h^2\right)^{\frac{3}{2}}} \left(h a_z\right)$$

$$E = \frac{\rho_S h}{4\pi\varepsilon_0} a_z \int_S \frac{\rho d\rho d\phi}{\left(\rho^2 + h^2\right)^{\frac{3}{2}}}$$

$$E = \frac{\rho_S h}{4\pi\varepsilon_0} a_z \int_{\phi=0}^{\phi=2\pi} \int_{\rho=0}^{\rho=a} \frac{\rho d\rho d\phi}{\left(\rho^2 + h^2\right)^{\frac{3}{2}}}$$

$$E = \frac{\rho_S h}{4\pi\varepsilon_0} a_z \int_{\phi=0}^{\phi=2\pi} \left[\frac{-1}{\left(\rho^2 + h^2\right)^{\frac{1}{2}}} \right]_{\rho=0}^{\rho=a} d\phi$$

$$E = \frac{\rho_S h}{4\pi\varepsilon_0} a_z \int_{\phi=0}^{\phi=2\pi} \left[\frac{1}{h} - \frac{1}{\left(h^2 + a^2\right)^{\frac{1}{2}}} \right] d\phi$$

$$E = \frac{\rho_S h}{4\pi\varepsilon_0} a_z \left[\frac{1}{h} - \frac{1}{\left(h^2 + a^2\right)^{\frac{1}{2}}} \right] x 2\pi$$

$$E = \frac{\rho_S h}{2\varepsilon_0} a_z \left[\frac{1}{h} - \frac{1}{\left(h^2 + a^2\right)^{\frac{1}{2}}} \right] \qquad \rightarrow (2)$$

Equation (2) gives the electric field intensity 'E' at a point

$$E = \frac{\rho_S}{2\varepsilon_0} a_z \left[1 - \frac{1}{\left(1 + \left(\frac{a}{\infty}\right)^2\right)^{\frac{1}{2}}} \right]$$

$$E = \frac{\rho_S}{2\varepsilon_0} a_z \left[1 - \frac{1}{(1+0)^{\frac{1}{2}}} \right] = \frac{\rho_S}{2\varepsilon_0} a_z [1 - 1]$$

$$E = 0$$

Case (iii) Electric field Intensity exactly at the center of the axis of a circular loop that is $h = 0$

$$E = \frac{\rho_S}{2\varepsilon_0} a_z \left[1 - \frac{h}{(h^2 + a^2)^{\frac{1}{2}}} \right]$$

$$h \rightarrow 0$$

$$E = \frac{\rho_S}{2\varepsilon_0} a_z \left[1 - \frac{0}{(0^2 + a^2)^{\frac{1}{2}}} \right]$$

$$E = \frac{\rho_S}{2\varepsilon_0} a_z [1 - 0]$$

$$E = \frac{\rho_S}{2\varepsilon_0} a_z$$

1.11 Potential (or) Potential Difference

It is defined as work done in moving a point charge from one point to another point

$$W = -Q \int_{\text{initial}}^{\text{final}} E \bullet dL \quad \text{joules} \quad \rightarrow (1)$$

potential difference $V = \frac{W}{Q} = \frac{\text{Workdone}}{\text{unitch arg e}}$

$$V = -\int_{\text{initial}}^{\text{final}} E \bullet dL \qquad \rightarrow (2)$$

If 'B' is the initial point and 'A' is the final point

$$V_{AB} = -\int_B^A E \bullet dL \qquad \rightarrow (3)$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} a_r; \qquad dL = dra_r$$

$$V_{AB} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2} a_r \bullet dr a_r \qquad \rightarrow (3)$$

$$V_{AB} = -\int_{r_B}^{r_A} \frac{Q dr}{4\pi\varepsilon_0 r^2}$$

$$V_{AB} = -\frac{Q}{4\pi\varepsilon_0} \int_{r_B}^{r_A} \frac{dr}{r^2}$$

$$V_{AB} = -\frac{Q}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_{r_B}^{r_A}$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r_A} - \frac{Q}{4\pi\varepsilon_0 r_B} = V_A - V_B$$

In general
$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 |R|}$$

Conservative Field (or) Lamellar Field

Any vector field satisfies the following equation $\oint E \bullet dL = 0$ is said to be conservative field; the name arises from the fact that no work is done around a closed path.

Equipotential Surface

$$V_{AB} = -\int E \bullet dL = 0$$

$$\text{(i.e.)}V_{AB} = V_A - V_B = 0$$

In equipotential surface, potential difference between any two points is zero. That is all points are at equal potential. An euipotential surface or line is a contour along which a charge moves with zero. A maximum amount of work per unit distance is performed moving normal or perpendicular to the equipotential surface in the direction of electric field.

1.12 Relationship between potential (V) and Electric field Intensity (E)

$$V = -\int E \bullet dL$$

(or)

$$E = -\text{grad}V$$

$$E = -\nabla V$$

1.13 Gradient V (or) ∇V

Cartesian $$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

Cylindrical $$\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z$$

Spherical $$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi$$

Note: Gradient of any scalar is a vector

1.14 Potential due to Electric Dipole

An Electric Dipole or simply a dipole is the name given to two point charges of equal magnitude and opposite sign separated by a distance which is small compared to the point 'P' at which we want to know the electric 'E' and potential 'V' fields.

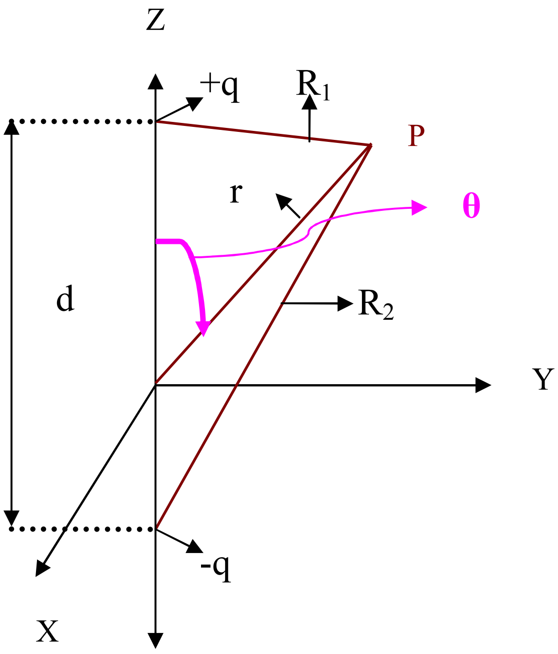


Fig (a) Point 'P' at a small distance

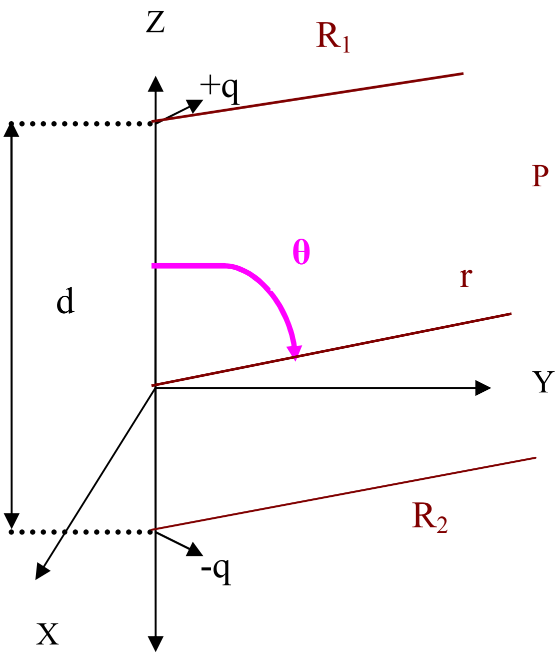


Fig (b) Point 'P' at a distant point

The positive charge $+q$ is at $\left(0,0,+\frac{d}{2}\right)$ and negative charge $-q$ is at $\left(0,0,-\frac{d}{2}\right)$.

The total potential acting at the point 'P' is the summation of individual potentials due to positive and negative charges.

$$V = \frac{q}{4\pi\varepsilon_0 |R_1|} + \frac{-q}{4\pi\varepsilon_0 |R_2|}$$

$$V = \frac{q}{4\pi\varepsilon_0 |R_1|} - \frac{q}{4\pi\varepsilon_0 |R_2|}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

from fig(b) $R_1 R_2 = r^2$

$$R_2 - R_1 = d\cos\theta$$

$$\therefore V = \frac{q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{r^2}\right) \quad \rightarrow (1)$$

$$d \bullet a_r = |d||a_r|\cos\theta = d\cos\theta \rightarrow (2)$$

using (2) equation (1) is

$$V = \frac{qd \bullet a_r}{4\pi\varepsilon_0 r^2} \quad \rightarrow (3)$$

$$(3) \Rightarrow V = \frac{p \bullet a_r}{4\pi\varepsilon_0 r^2} \qquad \rightarrow (4)$$

where $p = qd$ => dipole moment (Coloumb.meter)

Electric Field (E) due to Electric Dipole

$$E = -\nabla V$$

$$E = -\left[\frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} a_\phi \right]$$

$$E_\phi = 0$$

$$\therefore E = -\left[\frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta \right]$$

$$E = \left[\frac{-qd\cos\theta}{8\pi\varepsilon_0 r^3} a_r - \frac{qd\sin\theta}{4\pi\varepsilon_0 r^3} a_\theta \right]$$

$$E = \frac{qd}{4\pi\varepsilon_0 r^3} \left[2\cos\theta a_r + \sin\theta a_\theta \right] \qquad \frac{\text{Volt}}{\text{meter}}$$

1.15 Potential due to Infinite line Charge

Electric Field Intensity due to infinite line charge is $E = \frac{\rho_L}{2\pi\varepsilon_0\rho} a_\rho$

$$V = -\int E \bullet dL$$

$$V = -\int \frac{\rho_L}{2\pi\varepsilon_0\rho} a_\rho \bullet dL$$

$$\text{where}\, dL = d\rho a_\rho + \rho d\phi a_\phi + dz a_z$$

$$V = -\int \frac{\rho_L}{2\pi\varepsilon_0\rho} a_\rho \bullet \left(d\rho a_\rho + \rho d\phi a_\phi + dz a_z\right)$$

$$V = -\int \frac{\rho_L d\rho}{2\pi\varepsilon_0\rho} = -\frac{\rho_L}{2\pi\varepsilon_0}\int \frac{d\rho}{\rho}$$

$$V = -\frac{\rho_L}{2\pi\varepsilon_0}[\ln\rho]$$

$$V = \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln\left(\frac{1}{\rho}\right)\right]$$

1.16 Electric Flux Density (D) coulombs/ meter^2

Let ψ be coulombs of electric flux produced at the inner surface of sphere by the charge Q coulombs distributed uniformly over a surface having an area of $4\pi a^2\,m^2$ then the electric flux density (or) displacement density D is given by

$$D = \frac{\psi}{4\pi a^2} \text{(or)} \frac{Q}{4\pi a^2} \quad \frac{\text{coulombs}}{\text{meter}^2} \quad \rightarrow (1)$$

$$D = \frac{Q}{4\pi r^2} a_r \quad \rightarrow (2)$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2} a_r \quad \rightarrow (3)$$

$$(3) \Rightarrow \varepsilon_0 E = \frac{Q}{4\pi r^2} a_r \quad \rightarrow (4)$$

from (2) & (4)

$$D = \varepsilon_0 E$$

1.17 Gauss's Law for Electric Field

The Gauss's for electric field state that *"The electric flux passed through any closed surface is equal to the charge enclosed by that surface".*

$$\psi = \int d\psi = \oint_S D_S \bullet dS = Q$$

Proof:

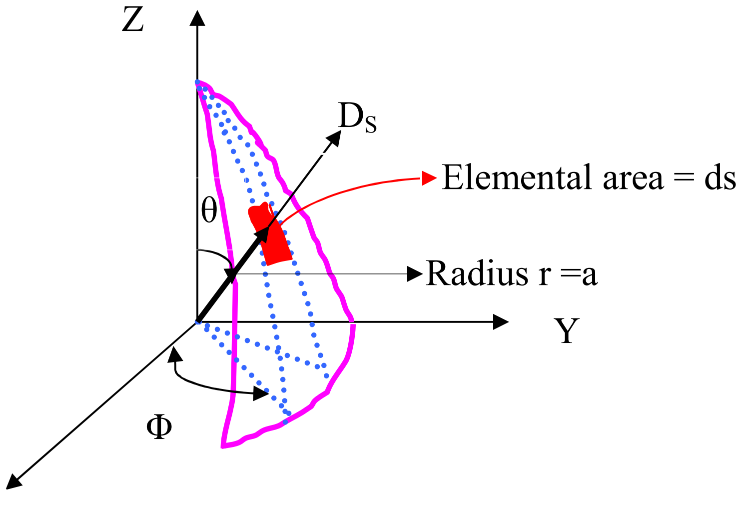


Fig . Gaussian Surface- A portion of Spherical surface

$$\oint D_S \bullet dS = \iint \frac{Q}{4\pi a^2} a_r \bullet dS$$

Where

$$dS = r^2 \sin\theta d\theta d\phi . a_r \qquad \text{and} \qquad r = a$$

$$\oint D_S \bullet dS = \iint \frac{Q}{4\pi a^2} a_r \bullet r^2 \sin\theta d\theta d\phi . a_r$$

$$\oint D_S \bullet dS = \int\int \frac{Q}{4\pi a^2} a^2 \sin\theta d\theta d\phi$$

$$\oint D_S \bullet dS = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{Q}{4\pi} \sin\theta d\theta d\phi$$

$$\oint D_S \bullet dS = \frac{Q}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi$$

$$\oint D_S \bullet dS = \frac{Q}{4\pi} \int_{\phi=0}^{2\pi} [-\cos\theta]_{\theta=0}^{\theta=\pi} d\phi$$

$$\oint D_S \bullet dS = \frac{Q}{4\pi} x 2 x \int_{\phi=0}^{\phi=2\pi} d\phi$$

$$\oint D_S \bullet dS = \frac{Q}{4\pi} x 2 x 2\pi = Q$$

Thus Gauss's law for electric field for the given surface is verified.

1.18 Applications of Gauss's Law

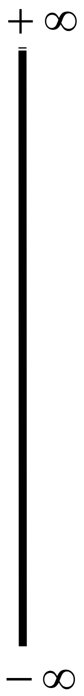
In order to apply Gauss's law for any closed surface it should satisfies the following two conditions:-

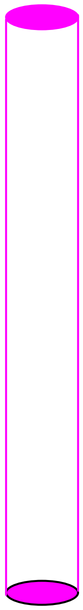
(1) D_S is everywhere either normal (or) tangential to the closed surface so that $D_S \bullet dS$ becomes either $D_S dS$ (or) zero respectively.

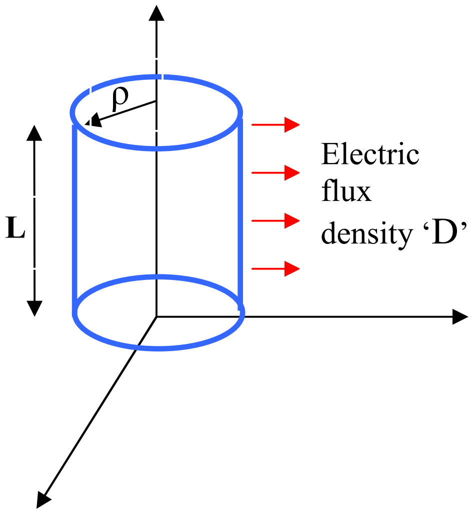
(2) On that portion of the closed surface for which $D_S \bullet dS$ is not zero, $D_S = \text{cons tan t}$.

Application (1): Application of Gauss law to an infinite line charge

The Gaussian surface for an infinite uniform line charge is a circular cylinder of length L, radius ρ, electric flux density D is constant in magnitude and everywhere perpendicular to the cylindrical surface and electric flux density D is parallel to end surfaces.







Fig(C) Gaussian surface for infinite line charge

Fig (a) Infinite Line Charge

Fig(b) Line charge looks like cylindrical surface

$$D = D_\rho a_\rho + D_\phi a_\phi + D_z a_z$$

only D_ρ component exist

$$\therefore D = D_\rho a_\rho$$

$$dS = \rho d\phi dz a_\rho$$

$$Q = \oint D_S \bullet dS = \oint D_\rho a_\rho \bullet \rho d\phi dz a_\rho$$

$$Q = \oint D_\rho \rho d\phi dz$$

$$Q = \int_{z=0}^{z=L} \int_{\phi=0}^{\phi=2\pi} D_\rho \rho d\phi dz$$

$$Q = D_\rho \rho x L x 2\pi$$

$$D_\rho = \frac{Q}{\rho L 2\pi}$$

$$D_\rho = \frac{(Q/L)}{2\pi\rho}$$

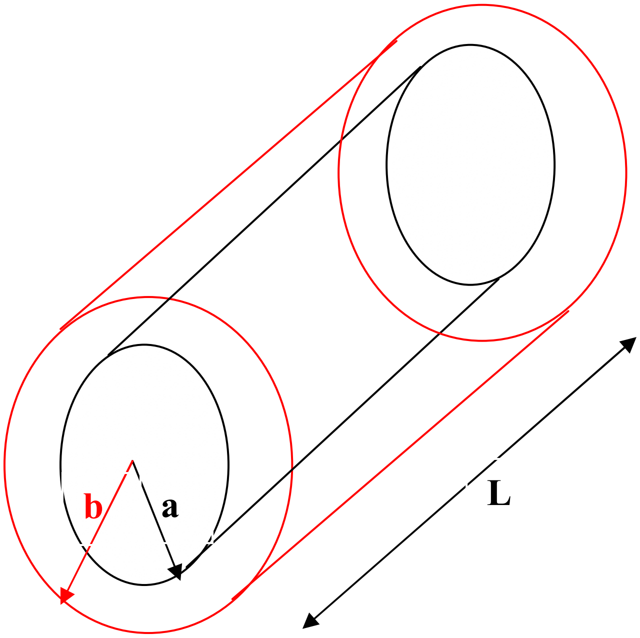
$$D_\rho = \frac{\rho_L}{2\pi\rho} \text{ where } \quad \rho_L = (Q/L)$$

$$D = D_\rho a_\rho$$

$$D = \frac{\rho_L}{2\pi\rho} a_\rho$$

and $$E = \frac{D}{\varepsilon_0} = \frac{\rho_L}{2\pi\varepsilon_0\rho} a_\rho$$

Application (2): Application of Gauss law to Co-axial Cylindrical Conductors



a – radius of inner conductor

b – radius of outer conductor

L– length of coaxial conductors

$$Q = D_\rho 2\pi\rho L \qquad \rightarrow (1)$$

$$Q = \iint \rho_S dS$$

$$Q = \int_{z=0}^{z=L} \int_{\phi=0}^{\phi=2\pi} \rho_S a d\phi dz \qquad \because \rho = a$$

$$Q = \rho_S a x 2\pi x L \qquad \rightarrow (2)$$

from(1) & (2)

$$D_\rho 2\pi\rho L = \rho_S a x 2\pi x L$$

$$D_\rho = \frac{\rho_S a}{\rho}$$

$$D = \frac{\rho_S a}{\rho} a_\rho \qquad \rightarrow (3)$$

case (i) : $a < \rho < b$

$$Q_{inner} = \rho_{Sinner} \quad 2\pi aL$$

$$\text{from Gauss Law}$$

$$Q_{Outer} = -Q_{inner} = -\rho_{Sinner} \quad 2\pi aL \qquad \rightarrow (4)$$

$$\text{but} \quad Q_{Outer} = \rho_{Souter} \quad 2\pi bL \qquad \rightarrow (5)$$

$$\text{using}(5) \,\&\, (4)$$

$$\rho_{Souter} \quad 2\pi bL = -\rho_{Sinner} \quad 2\pi aL$$

$$\rho_{Souter} = \frac{-\rho_{Sinner} \quad a}{b}$$

case (ii) : $\rho > b$

Total charge enclosed = 0. Equal and opposite charges on each conducting cylinder; therefore there is no external electric field.

case (iii) : $\rho < a$

There is no charge enclosed within the inner conductor, therefore no electric field within the inner conductor.

Note: In a co-axial cylinder there is no electric field within the inner conductor and outside the outer conductor and electric field exists between inner conductor and outer conductor.

1.19 Divergence

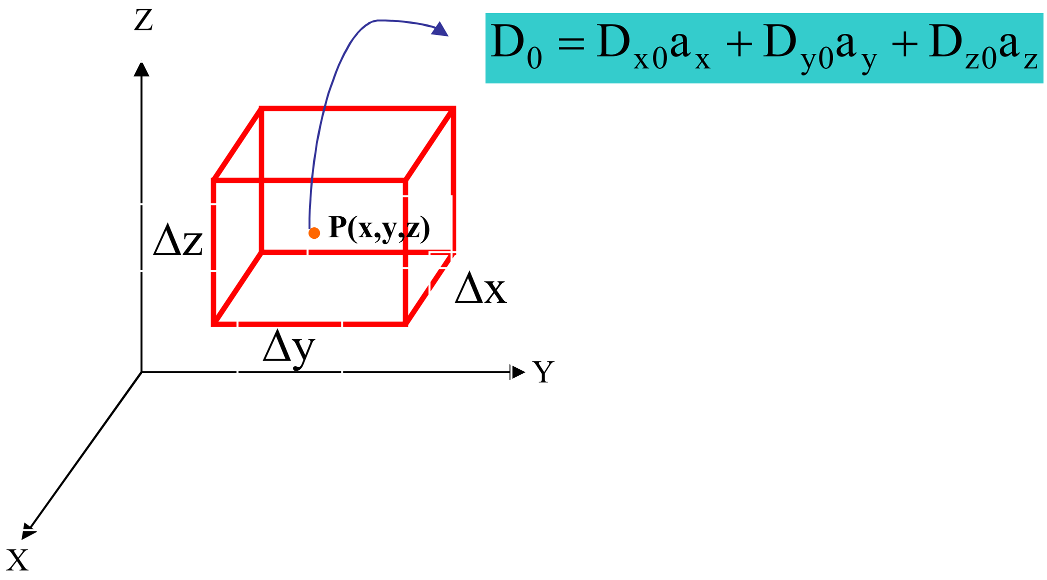
"The divergence of vector flux density D *is the outflow of flux from its small closed surface per unit volume as the volume shrinks to zero"*

$$\text{Divergence of } \mathrm{D} = \operatorname{div} \mathrm{D} = \lim_{\Delta V \to 0} \frac{\oint \mathrm{D} \bullet d\mathrm{S}}{\Delta V}$$

Where $\mathrm{D} \rightarrow$ Electric flux density C/m^2

$d\mathrm{S} \rightarrow$ Surface area m^2

$\Delta V \rightarrow$ Volume in m^3



$$\mathrm{D}_0 = \mathrm{D}_{x0}\mathrm{a}_x + \mathrm{D}_{y0}\mathrm{a}_y + \mathrm{D}_{z0}\mathrm{a}_z$$

$$\oint \mathrm{D} \bullet d\mathrm{S} = \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{front}} + \int_{\text{back}}$$

$$\int_{\text{front}} = D_{\text{front}} \bullet \Delta S_{\text{front}} = D_{\text{front}} \bullet \left(\Delta y \Delta z a_x\right)$$

$$\int_{\text{front}} = D_{x0,\text{front}} (\Delta y \Delta z) \qquad \rightarrow (1)$$

$$D_{x0,\text{front}} = D_{x0} + \frac{\Delta x}{2} \text{x rate of} \quad \text{change of} \quad D_x \text{ w.r.t.x}$$

$$D_{x0,\text{front}} = D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x} \qquad \rightarrow (2)$$

sub(2)in(1)

$$\int_{\text{front}} = \left[D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x} \right] \Delta y \Delta z \qquad \rightarrow (3)$$

$$\int_{\text{back}} = -D_{x0,\text{back}} (\Delta y \Delta z) \qquad \rightarrow (4)$$

$$D_{x0,\text{back}} = D_{x0} - \frac{\Delta x}{2} \frac{\partial D_x}{\partial x} \qquad \rightarrow (5)$$

$$\int_{\text{back}} = \left(- D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x} \right) \Delta y \Delta z \qquad \rightarrow (6)$$

$$\int_{\text{front}} + \int_{\text{back}} = \frac{\partial D_x}{\partial x}\Delta x \Delta y \Delta z \qquad \rightarrow (7)$$

$$\int_{\text{top}} + \int_{\text{bottom}} = \frac{\partial D_z}{\partial z}\Delta x \Delta y \Delta z \qquad \rightarrow (8)$$

$$\int_{\text{left}} + \int_{\text{right}} = \frac{\partial D_y}{\partial y}\Delta x \Delta y \Delta z \qquad \rightarrow (9)$$

$$\oint D \bullet dS = (7) + (8) + (9)$$

$$\oint D \bullet dS = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x \Delta y \Delta z$$

$$\oint D \bullet dS = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta V$$

$$\frac{\oint D \bullet dS}{\Delta V} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\lim_{\Delta V \to 0} \frac{\oint D \bullet dS}{\Delta V} = \lim_{\Delta V \to 0} \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\lim_{\Delta V \to 0} \frac{\oint D \bullet dS}{\Delta V} = \lim_{\Delta V \to 0} \frac{Q}{\Delta V} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\lim_{\Delta V \to 0} \frac{\oint D \bullet dS}{\Delta V} = \rho_V = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\lim_{\Delta V \to 0} \frac{\oint D \bullet dS}{\Delta V} = \operatorname{div} D = \nabla \bullet D = \rho_V = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\operatorname{div} D = \nabla \bullet D = \rho_V = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \bullet D = \rho_V \qquad \rightarrow (10)$$

$$\operatorname{div} D = \nabla \bullet D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} (\text{cartesian})$$

$$\operatorname{div} D = \nabla \bullet D = \frac{1}{\rho} \frac{\partial (\rho D_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} (\text{Cylindrical})$$

$$\mathrm{div}D = \nabla \bullet D = \frac{1}{r^2}\frac{\partial\left(r^2 D_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta D_\theta\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

[Spherical]

General Formula to calculate the divergence of any vector in different coordinate systems

$$\nabla \bullet D = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial\left(h_3 h_2 D_u\right)}{\partial u} + \frac{\partial\left(h_3 h_1 D_v\right)}{\partial v} + \frac{\partial\left(h_1 h_2 D_w\right)}{\partial w}\right]$$

cartesian

$u = x; \quad v = y; \quad w = z$

$h_1 = 1; \quad h_2 = 1; \quad h_3 = 1;$

cylindrical

$u = \rho; \quad v = \phi; \quad w = z$

$h_1 = 1; \quad h_2 = \rho; \quad h_3 = 1;$

Spherical

$u = r; \quad v = \theta; \quad w = \phi$

$h_1 = 1; \quad h_2 = r; \quad h_3 = r\sin\theta;$

1.20 Divergence Theorem

"The integral of normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface".

From Gauss's Law for electric field $\oint \mathrm{D} \bullet \mathrm{dS} = \mathrm{Q} \qquad \rightarrow (1)$

$$\mathrm{Q} = \int_{\mathrm{vol}} \rho_{\mathrm{V}} \mathrm{dV} \qquad \rightarrow (2)$$

$$\mathrm{divD} = \nabla \bullet \mathrm{D} = \rho_{\mathrm{V}} \qquad \rightarrow (3)$$

from(1) & (2)

$$\oint \mathrm{D} \bullet \mathrm{dS} = \int_{\mathrm{vol}} \rho_{\mathrm{V}} \mathrm{dV}$$

from(3)

$$\oint \mathrm{D} \bullet \mathrm{dS} = \int_{\mathrm{vol}} (\nabla \bullet \mathrm{D}) \mathrm{dV}$$

1.21 Ampere's Circuital Law

"It states that the line integral of magnetic field intensity H *about any closed path is exactly equal to the direct current enclosed by that path"*

$$\oint \mathrm{H} \bullet d\mathrm{L} = \mathrm{I}$$

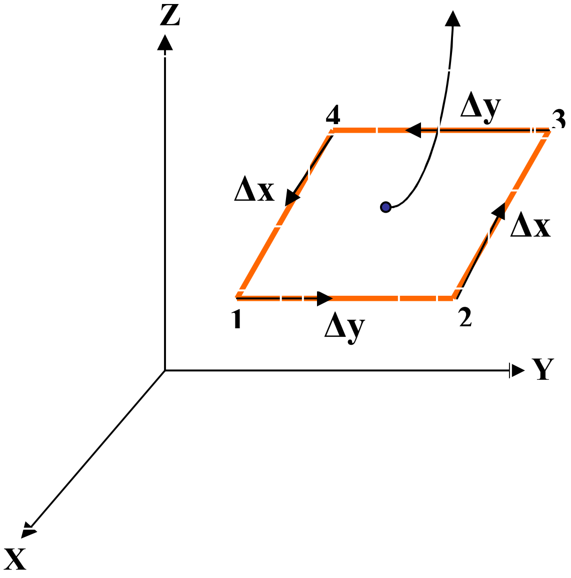
Where H → is the magnetic field intensity in Ampere/meter^2

I → is the direct current in Amperes

Note: Gauss's Law involves finding the total charge enclosed by a closed surface. The application of Ampere's Circuital law involves finding the total current enclosed by a closed path.

1.22 CURL

$$\mathrm{H}_0 = \mathrm{H}_{x0}\mathrm{a}_x + \mathrm{H}_{y0}\mathrm{a}_y + \mathrm{H}_{z0}\mathrm{a}_z$$



$$\oint H \bullet dL = [H \bullet \Delta L]_{1-2} + [H \bullet \Delta L]_{2-3} + [H \bullet \Delta L]_{3-4} + [H \bullet \Delta L]_{4-1}$$

$$[H \bullet \Delta L]_{1-2} = H_{y1-2}\Delta y \qquad \rightarrow (1)$$

$$H_{y1-2} = H_{y0} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x} \qquad \rightarrow (2)$$

$$(1) \Rightarrow [H \bullet \Delta L]_{1-2} = \left[H_{y0} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y \qquad \rightarrow (3)$$

$$[H \bullet \Delta L]_{2-3} = -[H_{x2-3}]\Delta x \qquad \rightarrow (4)$$

$$[H \bullet \Delta L]_{2-3} = -\left[H_{x0} + \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x \qquad \rightarrow (5)$$

$$[H \bullet \Delta L]_{3-4} = -\left[H_{y0} - \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y \qquad \rightarrow (6)$$

$$[H \bullet \Delta L]_{4-1} = \left[H_{x0} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x \qquad \rightarrow (7)$$

(3)+(5)+(6)+(7) =>

$$\oint H \bullet dL = \frac{\partial H_y}{\partial x} \Delta x \Delta y - \frac{\partial H_x}{\partial y} \Delta x \Delta y$$

$$\oint H \bullet dL = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \Delta x \Delta y$$

$$\oint H \bullet dL = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \Delta x \Delta y = \Delta I$$

where $\Delta I = J_z \, \Delta x \Delta y$

$$\therefore \oint H \bullet dL = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \Delta x \Delta y = J_z \Delta x \Delta y$$

$$\frac{\oint H \bullet dL}{\Delta x \Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = J_z$$

$$\lim_{\Delta x, \Delta y \to 0} \frac{\oint H \bullet dL}{\Delta x \Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = J_z \qquad \to (8)$$

$$\lim_{\Delta z,\Delta x \to 0} \frac{\oint H \bullet dL}{\Delta z \Delta x} = \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) = J_y \qquad \to (9)$$

$$\lim_{\Delta y,\Delta z \to 0} \frac{\oint H \bullet dL}{\Delta y \Delta z} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) = J_x \qquad \to (10)$$

$$(\mathrm{curl})_N = \lim_{\Delta S_N \to 0} \frac{\oint H \bullet dL}{\Delta S_N}$$

$$\mathrm{curl} \quad H = (8) + (9) + (10)$$

Cartesian Co-ordinate System

$$\mathrm{curl} \quad H = \nabla x H = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) a_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) a_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) a_z$$

Cylindrical Co-ordinate System

$$\mathrm{curl} \quad H = \nabla x H = \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) a_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) a_\phi + \frac{1}{\rho} \left(\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right) a_z$$

Spherical Co-ordinate System

$$\text{curl}\quad H = \nabla x H = \frac{1}{r\sin\theta}\left[\frac{\partial\left(H_\phi \sin\theta\right)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right]a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right]a_\theta$$

$$+\frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right]a_\phi$$

In General

$$\text{Curl}\,H \quad = \nabla x H = \begin{vmatrix} \frac{a_u}{h_2 h_3} & \frac{a_v}{h_3 h_1} & \frac{a_w}{h_1 h_2} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 H_u & h_2 H_v & h_3 H_w \end{vmatrix}$$

Note:

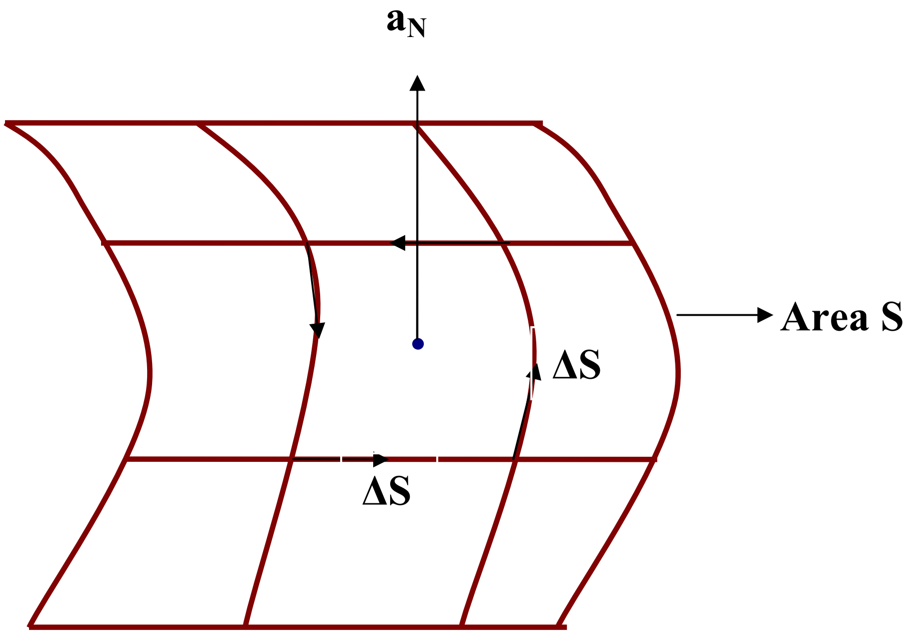
1) $\nabla x H = J \qquad ; \nabla x E = 0$

2) $\oint H \bullet dL = I; \quad \oint E \bullet dL = 0$

1.23 Stoke's Theorem

> *"The surface integral of curl of a vector field over an open surface is equal to the closed line integral of vector along contour bounding the surface".*

$$\oint H \bullet dL = \int_S (\nabla x H) \bullet dS$$



The Figure shows a surface of area S is broken up into incremental surface of area ΔS. Apply the definition of curl to one of these incremental surfaces.

$$\frac{\oint H \bullet dL_{\Delta S}}{\Delta S} = \left(\nabla x H\right)_N \qquad \rightarrow (1)$$

$$\frac{\oint H \bullet dL_{\Delta S}}{\Delta S} = \left(\nabla x H\right) \bullet a_N$$

$$\oint H \bullet dL_{\Delta S} = \left(\nabla x H\right) \bullet a_N \Delta S = \left(\nabla x H\right) \bullet \Delta S \qquad \rightarrow (2)$$

Where a_N is the unit vector in the direction of the right-hand normal to ΔS. The common part of contours of two adjacent elements is traversed in opposite direction. The net contribution of all the common parts in the interior to the line integral is zero. Therefore the sum of the closed line integrals about the perimeter of every ΔS is the same as the closed line integral about the perimeter of S because of cancellation of every interior path.

$$\oint H \bullet dL = \int_S \left(\nabla x H\right) \bullet dS \qquad \rightarrow (3)$$

Note: $$\oint H \bullet dL = \int_S J \bullet dS = \int_S \nabla x H \bullet dS = I$$

Solenoidal and Irrotational

Solenoidal => Divergence less field is solenoidal

Irrotational => Curl free field is irrotational

Case (I) : A vector field F is solenoidal if $\nabla \bullet F = 0$ and irrotational if $\nabla x F = 0$

Example: A Static electric field in a charge free region

$$\nabla \bullet E = 0; \quad \nabla x E = 0.$$

Case (II) : Solenoidal $\nabla \bullet F = 0$ but not irrotational $\nabla x F \neq 0$

Example: Steady magnetic field in a current carrying conductor.

Case (III) : Irrotational $\nabla x F = 0$ but not solenoidal $\nabla \bullet F \neq 0$

Example: A Static electric field in a charged region

$$\nabla \bullet D = \rho_v; \quad \nabla x D = 0.$$

Case (IV) : Neither solenoidal $\nabla \bullet F \neq 0$ nor irrotational $\nabla x F \neq 0$.

Example: Time varying electric field in a charged region

$$\nabla \bullet E = \frac{\rho_v}{\varepsilon_0}; \quad \nabla x E = -\frac{\partial B}{\partial t}$$

This soft copy of Electromagnetic Fields is prepared by

R. Senthilkumar

Lecturer in Department of Electronics and Communication Engineering

Institute of Road and Transport Technology

Erode – 638301

rsenthil_1976@yahoo.com

Dedicated to My beloved Principal Dr.B.Viswanathan and My beloved Head of the Department

Prof. L.Peter Stanley Bebbington