

EC 1253- ELECTROMAGNETIC FIELDS

IMPORTANT FORMULAE USED

UNIT-I STATIC ELECTRIC FIELDS

1. Rectangular Coordinate system

$$dL = (dx)a_x + (dy)a_y + (dz)a_z$$

$$\text{Elemental length} = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

$$\text{Elemental area} = dx dy, dy dz, dz dx$$

$$\text{Volume} = dx dy dz$$

Right handed Cartesian coordinate system

$$a_x \times a_y = a_z$$

2. Circular Cylindrical coordinate system

$$dL = (d\rho)a_\rho + (\rho d\phi)a_\phi + (dz)a_z$$

$$\text{Elemental length} = \sqrt{(d\rho)^2 + (\rho d\phi)^2 + (dz)^2}$$

$$\text{Elemental area} = \rho d\rho d\phi, \rho d\phi dz, dz d\rho$$

$$\text{Volume} = \rho d\rho d\phi dz$$

Right handed Cylindrical coordinate system

$$a_\rho \times a_\phi = a_z$$

Rectangular to Cylindrical coordinate system

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$

Cylindrical to Rectangular coordinate system

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

Dot product of unit vectors in cylindrical and Cartesian coordinate system

\cdot	\mathbf{a}_ρ	\mathbf{a}_ϕ	\mathbf{a}_z
\mathbf{a}_x	$\cos \phi$	$-\sin \phi$	0
\mathbf{a}_y	$\sin \phi$	$\cos \phi$	0
\mathbf{a}_z	0	0	1

3. Spherical coordinate system

$$d\mathbf{L} = (dr)\mathbf{a}_r + (rd\theta)\mathbf{a}_\theta + (r \sin \theta d\phi)\mathbf{a}_\phi$$

$$\text{Elemental length} = \sqrt{(dr)^2 + (rd\theta)^2 + (r \sin \theta d\phi)^2}$$

$$\text{Elemental area } d\mathbf{s} = r dr d\theta, r \sin \theta dr d\phi, r^2 \sin \theta d\theta d\phi$$

$$\text{Volume } d\mathbf{v} = r^2 \sin \theta d\theta dr d\phi$$

$$\text{Right handed spherical coordinate system } \mathbf{a}_R \times \mathbf{a}_\theta = \mathbf{a}_\phi$$

Spherical to Cartesian coordinate system

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

Cartesian to Spherical coordinate system

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \tan^{-1}(y/x)$$

Dot products of unit vectors in spherical and Cartesian systems

\cdot	\mathbf{a}_R	\mathbf{a}_θ	\mathbf{a}_ϕ
\mathbf{a}_x	$\sin\theta \cos\phi$	$\cos\theta \cos\phi$	$-\sin\phi$
\mathbf{a}_y	$\sin\theta \sin\phi$	$\cos\theta \sin\phi$	$\cos\phi$
\mathbf{a}_z	$\cos\theta$	$-\sin\theta$	0

4. Coulomb's Law

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{12} \quad \text{Newton}$$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

5. Electric Field Intesity

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R \quad \text{Volt/meter}$$

6. Principle of superposition of Electric field

The total or resultant field at a point is the vector sum of the individual component fields at the point.

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$$

7. Electric field due to infinite Line charge

$$\mathbf{E} = E_\rho \mathbf{a}_\rho$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho} \mathbf{a}_\rho$$

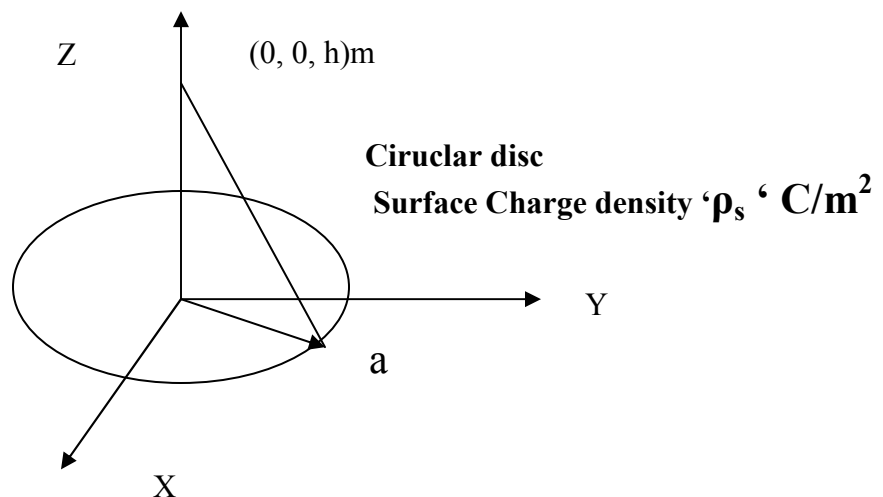
8. Electric field due to finite line charge

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho} \cos\theta \mathbf{a}_\rho \quad (\text{OR}) \quad \mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho} \left(\frac{L}{\sqrt{L^2 + \rho^2}} \right)$$

9. Electric field due to Infinite Uniform Charged sheet

$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0} \mathbf{a}_N$$

10. Electric field on the axis of a uniformly charged circular disc



$$\mathbf{E} = \left\{ \frac{\rho_S}{2\epsilon_0} \left[1 - \frac{h}{\left(h^2 + a^2 \right)^{\frac{1}{2}}} \right] \right\} \mathbf{a}_Z \text{ V/m}$$

11. Potential on the axis of a uniformly charged circular disc

$$V = \frac{\rho_S}{2\epsilon_0} \left[\left(h^2 + a^2 \right)^{\frac{1}{2}} - h \right] \text{ Volts}$$

12. Electric field due to Point Charge

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R$$

13. Electric field due to Line charge

$$\mathbf{E} = \int_{\text{line}} \frac{\rho_L d\mathbf{L}}{4\pi\epsilon_0 R^2} \cdot \mathbf{a}_R$$

$\rho_L =$ line chargedensity in C/m

14. Electric field due to Surface Charge density

$$\mathbf{E} = \int_{\text{Sur}} \frac{\rho_S d\mathbf{S}}{4\pi\epsilon_0 R^2} \cdot \mathbf{a}_R$$

$\rho_S =$ surface chargedensity in C/m²

15. Electric field due to Volume Charge density

$$\mathbf{E} = \int_{\text{Vol}} \frac{\rho_V d\mathbf{v}}{4\pi\epsilon_0 R^2} \cdot \mathbf{a}_R \quad \rho_V = \text{Volume Charge Density in C/m}^3$$

16. Electric Scalar Potential

$$V = \frac{Q}{4\pi\epsilon_0 R} \quad \text{Volts}$$

17. Potential Difference

$$V_{12} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \quad \text{Volts}$$

18. Relationship between potential and electric field intensity

$$\text{Potential difference} = V = - \int_{\text{initial}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L} \quad \text{Volts (or) joules/coulombs}$$

19. Work done by an external source in moving a charge Q from one point to another point in an electric field

$$W = -Q \int_{\text{initial}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L} \quad \text{joules}$$

20. Potential due to infinite uniformly charged line

$$V = \frac{\rho_L}{2\pi\epsilon_0} \ln\left(\frac{1}{\rho}\right)$$

21. Electric Flux Density

$$\mathbf{D} = \epsilon_0 \mathbf{E}$$

$$\mathbf{D} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R \quad \text{C/m}^2$$

22. Gauss's Law

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface

$$\Psi = \int d\Psi = \oint_{\text{closed surface}} \mathbf{D}_S \cdot d\mathbf{S} = \text{charge enclosed} = Q \text{ Coulombs}$$

23. Electric flux Intensity due to infinite line charge

$$\mathbf{D} = \frac{\rho_L}{2\pi\rho} \mathbf{a}_\rho$$

24. Electric flux density within two coaxial cylindrical conductors

$$\mathbf{D} = \frac{a\rho_S}{\rho} \mathbf{a}_\rho \quad (a < \rho < b)$$

a = Radius of the inner cylinder

b = Radius of the outer cylinder

25. Potential and Electric Field Intensity due to electrical dipole

$$V = \frac{Qd \cos \theta}{4\pi\epsilon_0 r^2}$$

$$\mathbf{E} = \frac{Qd}{4\pi\epsilon_0 r^3} ((2 \cos \theta) \mathbf{a}_r + (\sin \theta) \mathbf{a}_\theta)$$

26. Divergence

$$\text{Cartesian } \nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\text{Cylindrical } \nabla \cdot \mathbf{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

Spherical

$$\nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (D_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

27. Gradient

$$\text{Cartesian } \nabla V = \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z$$

$$\text{Cylindrical } \nabla V = \frac{\partial V}{\partial \rho} \mathbf{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \mathbf{a}_\phi + \frac{\partial V}{\partial z} \mathbf{a}_z$$

$$\text{Spherical } \nabla V = \frac{\partial V}{\partial r} \mathbf{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \mathbf{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \mathbf{a}_\phi$$

28. Curl

Cartesian

$$\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \mathbf{a}_z$$

Cylindrical

$$\nabla_{\mathbf{x}}\mathbf{H} = \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \mathbf{a}_\phi + \frac{1}{\rho} \left(\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right) \mathbf{a}_z$$

Spherical

$$\nabla_{\mathbf{x}}\mathbf{H} = \frac{1}{r \sin \theta} \left[\frac{\partial(H_\phi \sin \theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi} \right] \mathbf{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial(r H_\phi)}{\partial r} \right] \mathbf{a}_\theta + \frac{1}{r} \left[\frac{\partial(r H_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta} \right] \mathbf{a}_\phi$$

29. Stoke's Theorem

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_S (\nabla_{\mathbf{x}}\mathbf{H}) \cdot d\mathbf{S}$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_S (\nabla_{\mathbf{x}}\mathbf{H}) \cdot d\mathbf{S} = \int_S \mathbf{J} \cdot d\mathbf{S} = \oint \mathbf{H} \cdot d\mathbf{L} = I$$

30. Divergence Theorem

The Integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_{\text{vol}} \nabla \cdot \mathbf{D} dv$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q = \int_{\text{vol}} \rho_v dv = \int_{\text{vol}} \nabla \cdot \mathbf{D} dv$$

UNIT- II STATIC MAGNETIC FIELD

1. Biot-Savart Law in vector form

$$d\mathbf{H} = \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{I d\mathbf{L} \times \mathbf{R}}{4\pi R^2} \quad \text{amperes per meter (A/m)}$$

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

2. Biot –Savart law in Integral form

$$\mathbf{H} = \oint \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$$

3. Alternate forms of Biot-Savart law

$$\mathbf{H} = \int_S \frac{\mathbf{K} \times \mathbf{a}_R dS}{4\pi R^2} \quad \mathbf{K} - \text{Surface Current Density in A/m}$$

$$\mathbf{H} = \int_{\text{Vol}} \frac{\mathbf{J} \times \mathbf{a}_R dV}{4\pi R^2} \quad \mathbf{J} - \text{Current Density in A/m}^2$$

3. Magnetic field Intensity due to a finite wire carrying current I

$$\mathbf{H} = \frac{I}{4\pi\rho} (\sin \alpha_2 - \sin \alpha_1) \mathbf{a}_\phi$$

4. Magnetic field Intensity due to Infinite wire carrying current I

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi$$

5. Ampere's Circuital Law

Ampere's Circuital law states that the line integral of H about any closed path is exactly equal to the direct current enclosed by that path

$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

6. Magnetic field Intensity on the axis of Circular loop carrying current 'I'

case (1): The magnetic field intensity at a distance 'h' from the centre of the current carrying loop with Radius 'R'

$$\mathbf{H} = \frac{IR^2}{2(R^2 + h^2)^{3/2}} \mathbf{a}_z$$

case (2): The magnetic field Intensity at the centre of the loop h=0

$$\mathbf{H} = \frac{I}{2R} \mathbf{a}_z$$

case (3): At a large distance from the loop (h>>R)

$$\mathbf{H} = \frac{IR^2}{2h^3} \mathbf{a}_z$$

7. Magnetic field Intensity on the axis of rectangular loop carrying current 'I'

Case (1): Magnetic field intensity at the centre of the square loop

$$\mathbf{H} = \frac{2\sqrt{2}I}{\pi L} \mathbf{a}_z \quad \text{where 'L' -side of the square loop in 'm'}$$

Case(2): Magnetic field Intensity at a point (0,0,h)

$$\mathbf{H} = \left(\frac{2\sqrt{2}\mathbf{I}}{\pi} \right) \frac{L^2}{(L^2 + h^2)^{\frac{3}{2}}} \mathbf{a}_z$$

8. Magnetic Flux Density

$$\mathbf{B} = \mu_0 \mathbf{H} \quad \text{webers per square meter (Wb/ m}^2\text{)}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m permeability of free space

9. Magnetic flux

$$\phi = \int_S \mathbf{B} \cdot d\mathbf{S} \quad \text{Wb}$$

10. Gauss's law for Magnetic field

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

Application of divergence theorem

$$\nabla \cdot \mathbf{B} = 0$$

11. Lorentz force equation for moving charge

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

12. Applications of Lorentz force equation

Its solution is required in

- (1) Determining electron orbits in the magnetron (2) proton paths in the cyclotron
- (3) plasma characteristics in a magnetohydrodynamic (MHD) generator
- (4) Charged-particle motion in combined electric and magnetic fields

13. Force on a wire carrying a current 'I' placed in a magnetic field

$$\mathbf{F} = \oint \mathbf{I} d\mathbf{L} \times \mathbf{B} = -\mathbf{I} \oint \mathbf{B} \times d\mathbf{L}$$

Force in a uniform magnetic field $\mathbf{F} = I\mathbf{dL} \times \mathbf{B}$

(or)

Force in a uniform magnetic field $\mathbf{F} = \mathbf{BIL} \sin \theta$

14. Magnetic Dipole Moment

Defined as the product of the loop current and the vector area of the loop as the differential magnetic dipole moment

$$\mathbf{dm} = I\mathbf{dS}$$

(or) Ampere.m²

$$\mathbf{m} = I\mathbf{S}$$

15. Torque on a loop carrying current 'I'

$$\mathbf{T} = I\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B} \quad \text{Newton. meter(N.m)}$$

16. Torque expressed as a cross product

$$\mathbf{T} = \mathbf{R} \times \mathbf{F} \quad \text{where R- distance in meter, F- force in Newton}$$

17. Magnetic Vector Potential

$$\mathbf{A} = \oint \frac{\mu_0 I \mathbf{dL}}{4\pi R} \quad \text{webers per meter}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

UNIT- III ELECTRIC AND MAGNETIC FIELDS IN MATERIALS

1. Poisson's Equation

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\epsilon}$$

where ρ_v is volume charge density in C/ m³

$$\nabla \cdot \nabla V = \nabla^2 V = -\frac{\rho_v}{\epsilon}$$

2. Laplace's Equation

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{[Cartesian]}$$

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \left(\frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} \quad \text{[Cylindrical]}$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

[Spherical]

3. Dipole Moment

$$p = Qd \quad \text{C.m}$$

Q- is the positive one of the two bound charges composing the dipole

d-is the vector from the negative to the positive charge

4. Polarization

$$P = \lim_{\Delta v \rightarrow 0} \frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} p_i \quad \text{Coulombs per square meter}$$

5. Definition of Capacitance

Capacitance of two conductor system is defined as the ratio of the magnitudes of the total charge on either conductor to the potential difference between conductors.

$$C = \frac{Q}{V_0} \text{ Farads}$$

6. Capacitance of various geometries using Laplace's equation

(1) Capacitance of parallel plate capacitor $C = \frac{\epsilon S}{d}$

ϵ – permittivity

S – surface area

d – distance between plates

(2) Capacitance of Cylindrical capacitors $C = \frac{2\pi\epsilon L}{\ln(b/a)}$

L – length of the cylinder in 'm'

b – radius of the outer cylinder 'm'

a – radius of the inner cylinder 'm'

(3) Capacitance of Spherical capacitors $C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$

a – radius of the inner sphere in 'm'

b – radius of the outer sphere in 'm'

7. Energy Density

$$W = \frac{dW_E}{dv} = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} = \frac{1}{2} \epsilon_0 E^2 \quad \text{joules per cubic meter}$$

8. Energy

$$W_E = \frac{1}{2} \int_{\text{vol}} \mathbf{D} \cdot \mathbf{E} dv = \frac{1}{2} \int_{\text{vol}} \epsilon_0 E^2 dv \quad \text{joules}$$

9. Boundary conditions for electric fields

Boundary Conditions for Tangential components

$$\mathbf{E}_{\text{tan1}} = \mathbf{E}_{\text{tan2}} \quad (\text{Medium 1 and Medium 2 both are perfect dielectrics})$$

1) Tangential electric field intensity is continuous across the boundary.

$$V_1 = V_2$$

2) Potential difference between any two points on the boundary that are separated by a distance is the same above or below boundary.

$$\mathbf{E}_{\text{tan1}} = \mathbf{0} \quad (\text{Medium 1 is dielectric and Medium 2 is a conductor})$$

3) Tangential electric field at a dielectric conductor boundary is zero.

$$\frac{D_{\text{tan1}}}{D_{\text{tan2}}} = \frac{\epsilon_1}{\epsilon_2}$$

4) Tangential components of electric flux density 'D' is discontinuous

Boundary Conditions for Normal components

1) Flux leaving the top and bottom surfaces is the difference

$$D_{N1} - D_{N2} = \rho_s$$

2) There is no free charge available for perfect dielectrics. At the interface ' ρ_s ' is zero.

$$\mathbf{D}_{N1} - \mathbf{D}_{N2} = 0$$

$$\mathbf{D}_{N1} = \mathbf{D}_{N2}$$

3) Normal component of the flux density is continuous across the charge-free boundary between two dielectrics.

4) If medium '2' is a conductor $\mathbf{D}_{N1} = \rho_s$ $\mathbf{D}_{N2} = 0$

The normal component of the flux density at a dielectric-conductor boundary is equal to the surface charge density on the conductor.

5) $\frac{\mathbf{E}_{N1}}{\mathbf{E}_{N2}} = \frac{\epsilon_{r2}}{\epsilon_{r1}}$ (If medium 1 and medium 2 both are perfect dielectric)

10) Current 'I'

Electric charges in motion constitute a current. The unit of current is the ampere(A) defined as a rate of movement of charge passing a given reference point of one coulomb per second.

$$\mathbf{I} = \frac{dQ}{dt}$$

11) Current Density 'J'

Measured in amperes per square meter (A/m^2). Current density is a vector represented by 'J'. The relationship between current 'I' and current density 'J' is

$$\mathbf{I} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

12) Point form of ohm's law

$$\mathbf{J} = \sigma \mathbf{E} \quad \text{where } \sigma \text{ is conductivity measured in mhos per meter}$$

13) Ohm's Law

$$V = IR$$

where

V – potential difference in volts

$$R = \frac{L}{\sigma S}$$

I - current in amps

R- resistance in ohms

L- length in meters

S- area in m^2

14) Continuity equation for current

(1) Principle of conservation of charges

If the charge inside the closed surface is denoted by Q_i , then the rate of decrease is $-\frac{dQ_i}{dt}$ and the principle of conservation of charge requires

$$I = \oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{dQ_i}{dt}$$

(2) Point form of continuity equation

$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$ equation indicates that the current or charge per second, diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point

15) Definition of Inductance

Inductance 'L' is the ratio of the total magnetic flux linkage to the current 'I' through the inductor.

$$L = \frac{N\psi_m}{I} = \frac{\Lambda}{I}$$

N – number of turns

ψ_m – total magnetic flux in weber

Λ – flux linkage in weber / turns

16) Energy stored in an Inductor

$$W_m = \frac{1}{2}LI^2 \quad \text{joules}$$

L – inductance in Henry

17) Energy density in Magnetic fields

$$w_m = \lim_{\Delta v \rightarrow 0} \frac{\Delta W_m}{\Delta V} = \frac{1}{2}\mu H^2 \quad \text{joules / m}^3$$

18) Inductance of long solenoid

$$L = \frac{\mu N^2 A}{l} \quad \text{A- area, N- number of turns, l- length of the solenoid}$$

19) Inductance of toroid

$$L = \frac{\mu N^2 r^2}{2R} \quad r - \text{radius of coil, R - radius of Toroid}$$

20) Inductance of Coaxial Transmission Line

$$L = \frac{d\mu}{2\pi} \ln(b/a) \quad \begin{array}{l} d - \text{length, a - radius of inner cylinder,} \\ b - \text{radius of outer cylinder} \end{array}$$

21) Inductance of two wire transmission line

$$L = \frac{\mu d}{\pi} \ln(D/a)$$

a –radius of conductor, D – spacing between the centre, d-length of the line

22) Mutual Inductance between circuits 1 and 2

$$M_{12} = \frac{N_2 \phi_{12}}{I_1}$$
$$M_{21} = \frac{N_1 \phi_{21}}{I_2}$$

Another formula

$$M_{12} = \frac{1}{I_1 I_2} \int_{\text{vol}} (\mathbf{B}_1 \bullet \mathbf{H}_2) dv$$
$$M_{21} = \frac{1}{I_1 I_2} \int_{\text{vol}} (\mu \mathbf{H}_1 \bullet \mathbf{H}_2) dv$$

23) Magnetization - *magnetic dipole moment per unit volume*

$$\mathbf{M} = \lim_{\Delta v \rightarrow 0} \frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} \mathbf{m}_i$$

amperes per meter

$$\mathbf{m} = I_b d\mathbf{S}$$

magnetic dipole moment in amperes. Meter²

24) Permeability

$$\mu = \mu_0 \mu_R \quad \text{H/m}$$

$$\mu_0 = \text{free space permeability} = 4\pi \times 10^{-7} \quad \text{H/m}$$

$$\mu_R = \text{relative permeability} \quad \text{unitless}$$

25) Magnetic boundary conditions

Boundary condition on Normal Components

(1) The normal components of magnetic flux densities are continuous

$$\mathbf{B}_{N1} = \mathbf{B}_{N2}$$

(2) The normal components of Magnetic field Intensities are discontinuous

$$\mathbf{H}_{N2} = \frac{\mu_1}{\mu_2} \mathbf{H}_{N1}$$

Boundary condition on Tangential components

$$\mathbf{H}_{t1} - \mathbf{H}_{t2} = \mathbf{K} \text{ (surface current)}$$

$$(1) \quad \mathbf{H}_{t1} - \mathbf{H}_{t2} = \mathbf{a}_{N12} \times \mathbf{K}$$

$$(2) \quad \frac{\mathbf{B}_{t1}}{\mu_1} - \frac{\mathbf{B}_{t2}}{\mu_2} = \mathbf{K}$$

UNIT IV TIME VARYING ELECTRIC AND MAGNETIC FIELDS

1. Faraday's Law

$$\text{emf} = -\frac{d\phi}{dt} \quad \text{volts}$$

2. Lenz's Law

$$\text{emf} = -N \frac{d\phi}{dt}$$

3. Maxwell's equation from Faraday's Law

Integral form:

$$\text{emf} = \oint \mathbf{E} \cdot d\mathbf{L} = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

Point form (or) Differential form:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

4. Displacement current – current flow through the capacitor

$$\mathbf{I}_d = \int \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

5. Displacement current density

$$\mathbf{J}_d = \frac{d\mathbf{D}}{dt}$$

6. Maxwell's equation from Ampere's circuital law

Integral form:

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_S \left(\sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) \cdot d\mathbf{S}$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$$

Point form (or) Differential form:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

7. Maxwell's four equations in integral form and differential form

Point form (or) Differential form	Integral form
1. $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	1. $\oint \mathbf{H} \cdot d\mathbf{L} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$
2. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	2. $\oint \mathbf{E} \cdot d\mathbf{L} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$
3. $\nabla \cdot \mathbf{D} = \rho_v$	3. $\oint \mathbf{D} \cdot d\mathbf{S} = \int \rho_v dv$
4. $\nabla \cdot \mathbf{B} = 0$	4. $\oint \mathbf{B} \cdot d\mathbf{S} = 0$

8. Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \quad \text{watts per square meter}$$

\mathbf{E} – Electric field Intensity in v/m

\mathbf{H} - magnetic field intensity in A/m

9. Power flow in a co-axial cable

$$\mathbf{W} = \mathbf{VI} \quad \text{watts}$$

Power along the cable is the product of voltage and current

10. Instantaneous power flow (or) Instantaneous poynting vector

$$\tilde{\mathbf{P}} = \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}$$

11. Complex poynting vector

$$\mathbf{P} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^*$$

12. Average Poynting vector

$$\mathbf{P}_{\text{av}} = \frac{1}{2} \text{Re} \{ \mathbf{E} \times \mathbf{H}^* \}$$

UNIT V ELECTROMAGNETIC WAVES

1. Wave Equations for Free Space

$$\nabla^2 \mathbf{E} = \mu\epsilon \ddot{\mathbf{E}}$$
$$\nabla^2 \mathbf{H} = \mu\epsilon \ddot{\mathbf{H}}$$

2. Uniform Plane Wave Representation

$$\mathbf{E} = \mathbf{f}_1(\mathbf{x} - \mathbf{v}_0 \mathbf{t})$$

3. Relationship between 'E' and 'H'

$$\frac{\mathbf{E}}{\mathbf{H}} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

4. Wave Equation for a Conducting Medium

$$\nabla^2 \mathbf{E} - \mu\epsilon \ddot{\mathbf{E}} - \mu\sigma \dot{\mathbf{E}} = 0 \quad \text{and} \quad \nabla^2 \mathbf{H} - \mu\epsilon \ddot{\mathbf{H}} - \mu\sigma \dot{\mathbf{H}} = 0$$

5. Plane Waves in Lossy Dielectrics

From Equation '1'

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left(1 + \frac{\sigma^2}{2\omega^2\epsilon^2} - 1 \right)} = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}}$$

Phase Constant 'β'

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} + 1 \right)}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left(1 + \frac{\sigma^2}{2\omega^2\epsilon^2} + 1 \right)} = \omega \sqrt{\mu\epsilon} \left(1 + \frac{\sigma^2}{8\omega^2\epsilon^2} \right)$$

$$\eta = \sqrt{\frac{\mu}{\epsilon} \left(\frac{1}{1 + \frac{\sigma}{j\omega\epsilon}} \right)}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon} \left(1 + \frac{j\sigma}{2\omega\epsilon} \right)}$$

Velocity of Wave in Dielectric

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon} \left(1 + \frac{\sigma^2}{8\omega^2\epsilon^2} \right)} \cong v_0 \left(1 - \frac{\sigma^2}{8\omega^2\epsilon^2} \right)$$

where $v_0 = \frac{1}{\sqrt{\mu\epsilon}}$ is the velocity of the dielectric when conductivity is zero

6. Wave Propagation in Good Conductors

$\sigma / \omega \epsilon \gg 1$ for good Conductors

$$\gamma = \sqrt{(j\omega\mu\sigma)\left(1 + j\frac{\omega\epsilon}{\sigma}\right)} \cong \sqrt{j\omega\mu\sigma}$$

$$\gamma = \sqrt{\omega\mu\sigma} \angle 45^\circ$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

The Velocity of the wave in the conductor

$$v = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

The intrinsic impedance of the conductor is

$$\eta \cong \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

7. Depth of Penetration (or) Skin depth

$$\delta = \frac{1}{\alpha} \cong \sqrt{\frac{2}{\omega\mu\sigma}}$$

8. Maxwell's Equation in Phasor form

Maxwell's Equation in Point form (Phasor form)	Maxwell's Equation in Integral form (Phasor form)
$\nabla \times \mathbf{H} = \mathbf{j}\omega\mathbf{D} + \mathbf{J}$	$\oint \mathbf{H} \cdot d\mathbf{L} = \int_S (\mathbf{j}\omega\mathbf{D} + \mathbf{J}) \cdot d\mathbf{S}$
$\nabla \times \mathbf{E} = -\mathbf{j}\omega\mathbf{B}$	$\oint \mathbf{E} \cdot d\mathbf{L} = -\int_S \mathbf{j}\omega\mathbf{B} \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho_v dV$
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

9. Wave Equation in Phasor form

$$\nabla^2 \mathbf{E} = -\omega^2 \mu \epsilon \mathbf{E}$$

$$\nabla^2 \mathbf{H} = -\omega^2 \mu \epsilon \mathbf{H}$$

10. Wave Equation for Lossy medium (i.e. Partially conducting dielectric)

$$\nabla^2 \mathbf{E} + (\omega^2 \mu \epsilon - j\omega \mu \sigma) \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + (\omega^2 \mu \epsilon - j\omega \mu \sigma) \mathbf{H} = 0$$

11. Reflection by a Perfect Conductor – Normal Incidence

$$\mathbf{E}_T(\mathbf{x}, t) = 2E_i \sin \beta x \sin \omega t$$

$$\mathbf{H}_T(\mathbf{x}, t) = 2H_i \cos \beta x \cos \omega t$$

12. Reflection by a Perfect Dielectric – Normal Incidence

$$\frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$
$$\frac{\mathbf{E}_t}{\mathbf{E}_i} = \frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$
$$\frac{\mathbf{H}_r}{\mathbf{H}_i} = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$
$$\frac{\mathbf{H}_t}{\mathbf{H}_i} = \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

13. Reflection by a Perfect Dielectric – Oblique Incidence

(a) Perpendicular Polarization (or) Horizontal Polarization

$$\frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{\cos \theta_1 - \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2 \theta_1}}{\cos \theta_1 + \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2 \theta_1}}$$

(b) Parallel polarization (or) Vertical polarization

$$\frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos \theta_1 - \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2 \theta_1}}{\left(\frac{\epsilon_2}{\epsilon_1}\right) \cos \theta_1 + \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2 \theta_1}}$$

14 . Polarization

(a) Circular Polarization

$$\mathbf{E}(\mathbf{0}, t) = (\hat{x} + j\hat{y})\mathbf{E}_a$$

$$\mathbf{E}(\mathbf{0}, t) = (\hat{x} \cos \omega t - \hat{y} \sin \omega t)\mathbf{E}_a$$

Where

$$\mathbf{E}_x = \mathbf{E}_a \cos \omega t$$

$$\mathbf{E}_y = -\mathbf{E}_a \sin \omega t$$

$$\mathbf{E}_x^2 + \mathbf{E}_y^2 = \mathbf{E}_a^2$$

(b) Elliptical polarization

$$\mathbf{E}(\mathbf{0}, t) = (\hat{x} - j\hat{y})\mathbf{E}_a$$

$$\mathbf{E}(\mathbf{0}, t) = \hat{x}A + j\hat{y}B$$

$$\mathbf{E}(\mathbf{0}, t) = \hat{x}A \cos \omega t - \hat{y}B \sin \omega t$$

Where

$$\mathbf{E}_x = A \cos \omega t$$

$$\mathbf{E}_y = -B \sin \omega t$$

$$\frac{\mathbf{E}_x^2}{A^2} + \frac{\mathbf{E}_y^2}{B^2} = 1$$

15. Brewster Angle

$$\tan \theta_1 = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

At this angle, which is called the *Brewster angle*, there is no reflected wave when the incident wave is parallel (or vertically) polarized.

For perpendicular polarization there is no corresponding Brewster angle.

16. Snell's Law (or) Law of Sines

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

The angle of incidence is related to the angle of refraction by above equation, which in optics is known as the *law of sines* (or) *Snell's law*