

# Cross-Sectional Analysis of the returns of iShares MSCI Index Funds Using Independent Component Analysis

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## Abstract

The term structure and the interactions between the returns of seven iShares MSCI Index Funds and the the returns of the Standard and Poor's Depository Receipts (SPY) are analyzed via statistical techniques known as multi-scale decomposition (MSD) and Independent Component Analysis (ICA). The approach closely follows ideas presented by Moody and Yang, 1999[9]. The basic idea is that MSD facilitates a time-domain term structure analysis via a causal decomposition of the return series, while the cross-sectional analysis based on ICA may lead to the discovery of nonlinear and non-Gaussian effects in the interactions between the return changes across multiple Fund Indexes. For the shake of completeness, similar analysis using Principal Component Analysis (PCA) are also reported. A little more than four years of data (from **02-Apr,1996** to **06-Oct,2000**) of the eight Indexes listed in the **Appendix A** were studied. We show the existence of an optimal value for the learning rate implicit in any ICA algorithm for which averaged mutual information between ICA outputs is minimal. This is contrary to the general recommendation of choosing a value around 0.01 for the learning rate (Makeig *et al.*, 1997[6]).

## 1 Introduction

The application of Multi-scale Decomposition (MSD) has been shown to be a useful technique to analyze price movements in the foreign exchange markets (Moody and Yang, 1999[9] and references there in). Specifically, MSD has shed some light to elucidate a number of interesting phenomena for single exchange rates such as: long term memory

and fractional integration effects, the existence of trends and mean reverting behaviors, nonlinear effects, and the presence of hierarchical effects in the term structure of volatility. Similarly, Independent Component Analysis (ICA) has also been applied to fully understand the behavior of multivariate financial time series with encouraging and optimistic results to pursue further research in this direction (Moody and Wu, 1996[7]; Back and Weigend, 1997[1]; Moody and Yang, 1999[9]).

The aim of this paper is to apply the above mentioned methodologies to investigate the term structure of the interactions between the returns of seven iShares MSCI Index Funds listed in the **Appendix A** (page 43) and the returns of the Standard and Poor's Depository Receipts Index (SPY). The analyzed data corresponds to the period ranging from **02-Apr,1996** to **06-Oct,2000**. The techniques used include a causal Multi-Scale Decomposition and Independent Component Analysis as described by Moody and Yang, 1999 [9]. In Fig. 1 we show the time evolution of the market level prices for each Index during the period in consideration. In **Appendix B**, starting on page 44, we report descriptive statistics for each one of this Indexes as well as results of the Kolmogorov-Smirnov test for normality performed on the data.

In general terms this report is divided in five major sections. A background introduction presenting a brief theoretical digression about Multiscale-Decomposition (MSD), Independent Component Analysis (ICA), and Principal Component Analysis (PCA) can be found in section 2. In addition, a description of the **data pre-processing** is also presented in section 2. After that, in section 3, starting on page 11, we present results obtained by the application of the ICA algorithm to the oscillators obtained via the application of MSD to the returns serie of each Index. This section also includes results of the cross-sectional interaction between the Indexes. Though we don't provide an exhaustive comparison between PCA and ICA, the results of the decomposition of the oscillators via PCA can be found in section 4, starting on page 27. Then, results obtained by the application on ICA directly to the returns series of each Index are presented in section 5. Finally, in section 6 we summarize our findings.

## 2 Background

### 2.1 Data acquisition and Pre-processing

The data for each Index was collected via *YAHOO Financial web site*, and consist of little more than 4 years of daily prices (from **02-Apr,1996** to **06-Oct,2000**). Preprocessing of the data includes dividends backward adjustment. No attempt was given to model missing values, and the level price series were adjusted so each one contains the same number of entries matching the corresponding day. Additional data pre-processing as required by the MSD and ICA algorithms were also given. Details are presented in the respective section. At this point one should mention, however, that non-stationary daily market level prices  $p(t)$  of each Index were transformed by taking log differences between successive values  $r(t) = Ln[p(t)] - Ln[p(t-1)]$ . This is nothing more than the standard definition of

**returns** associated with geometric growth. A plot presenting the daily behaviour of such returns for the Indexes under consideration is shown in Fig. 2. The time period goes from **02-Apr,1996** to **06-Oct,2000**.

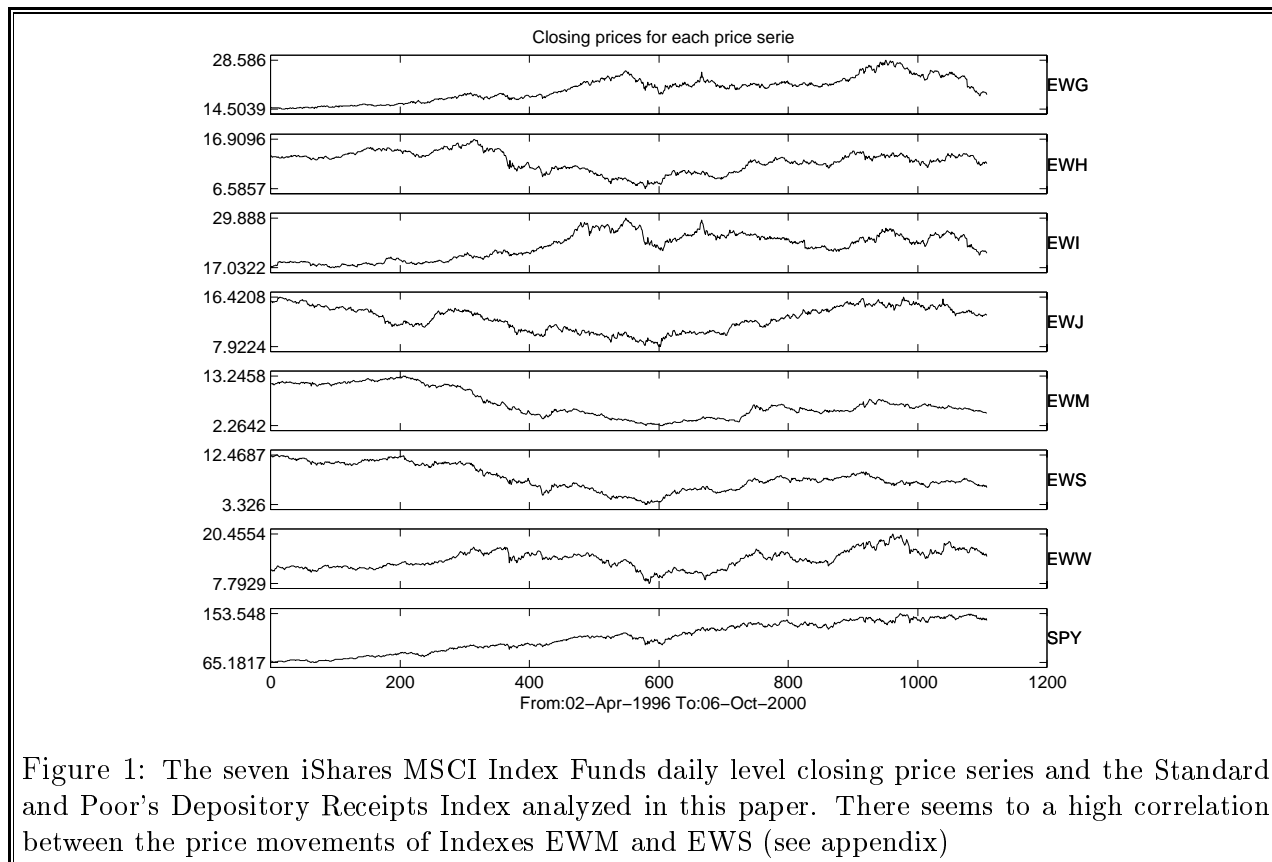


Figure 1: The seven iShares MSCI Index Funds daily level closing price series and the Standard and Poor's Depository Receipts Index analyzed in this paper. There seems to a high correlation between the price movements of Indexes EWM and EWS (see appendix)

## 2.2 Multi-Scale Decomposition (MSD)

The interactions between different Indexes of the MSCI series is analyzed by decomposing each return series into returns movements on a hierarchy of time scales.

As described in Moody and Yang, 1999[9], a term structure of return movements on several time scales can be constructed via a multi-scale decomposition of the original return series. This proposed decomposition makes use of causal, overlapping band pass filters applied to the return series. These filters are constructed by taking differences of exponential moving averages (EMAs) on several time scales  $\tau$  as described below. The resulting "oscillators" are mean reverting by construction. This approach is motivated by a desire to characterize return movements on a spectrum of time scales using a transformation that (1) is causal, (2) can be recursively updated and (3) yields a complete representation of the original return series. The recipe is as follows (additional details can be found in Moody and Yang, 1999[9]):

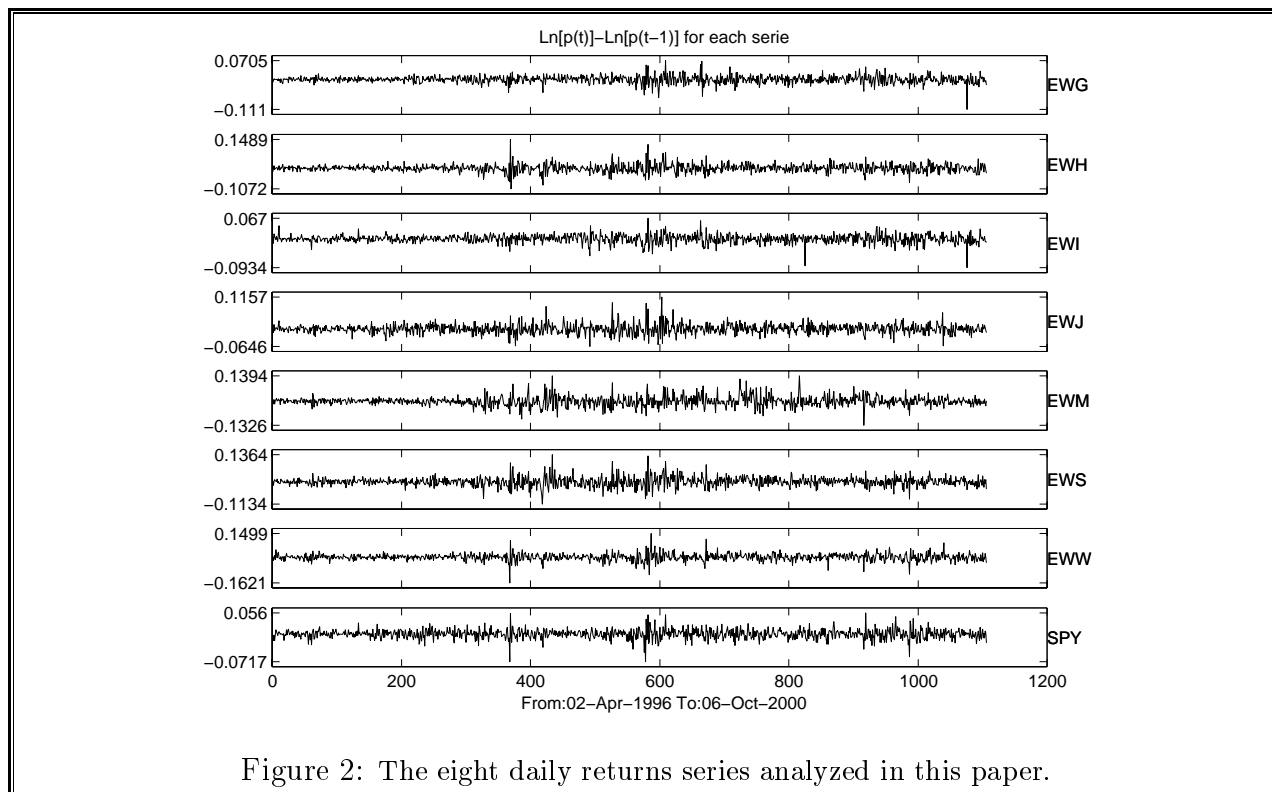


Figure 2: The eight daily returns series analyzed in this paper.

- Define a set of integer time constants  $(\tau_0, \tau_1, \dots, \tau_K)$  measuring the number of desired time intervals.
- Define  $\eta_k = 1/\tau_k$ ,  $k = 0, 1, \dots, K$
- Given a process  $x_t$ , we define  $K + 1$  EMAs of  $x_t$ , for  $k = 0, 1, \dots, K$  in the following way:

$$M_{t,k} = (1 - \eta_k)M_{t-1,k} + \eta_k x_t \quad (1)$$

$$M_{0,k} = x_0 \quad (2)$$

- Define  $N_{t,j} = M_{t,j-1} - M_{t,j}$ ,  $j = 1, \dots, K$  which is an output of the band pass filter.  $N_{t,j}$  is called an oscillator in technical analysis. In this report,  $N_{t,j}$  is called the scale  $j$  oscillator.
- By definition,  $\eta_0 = 1$  when  $\tau_0 = 1$  and  $M_{t,0} = x_t$ . We thus obtain a multi-scale decomposition of  $x_t$ :

$$x_t = \sum_{j=1}^K N_{t,j} + M_{t,K} \quad (3)$$

In this paper, we choose the following time intervals:

$$(\tau_0, \tau_1, \dots, \tau_K) = (1, 5, 21, 63) \quad (4)$$

These time constants correspond to the time scales of one day, one week(5), one month(21), and three months(63).

The multi-scale decomposition of the Index EWH is shown in Fig. 3.

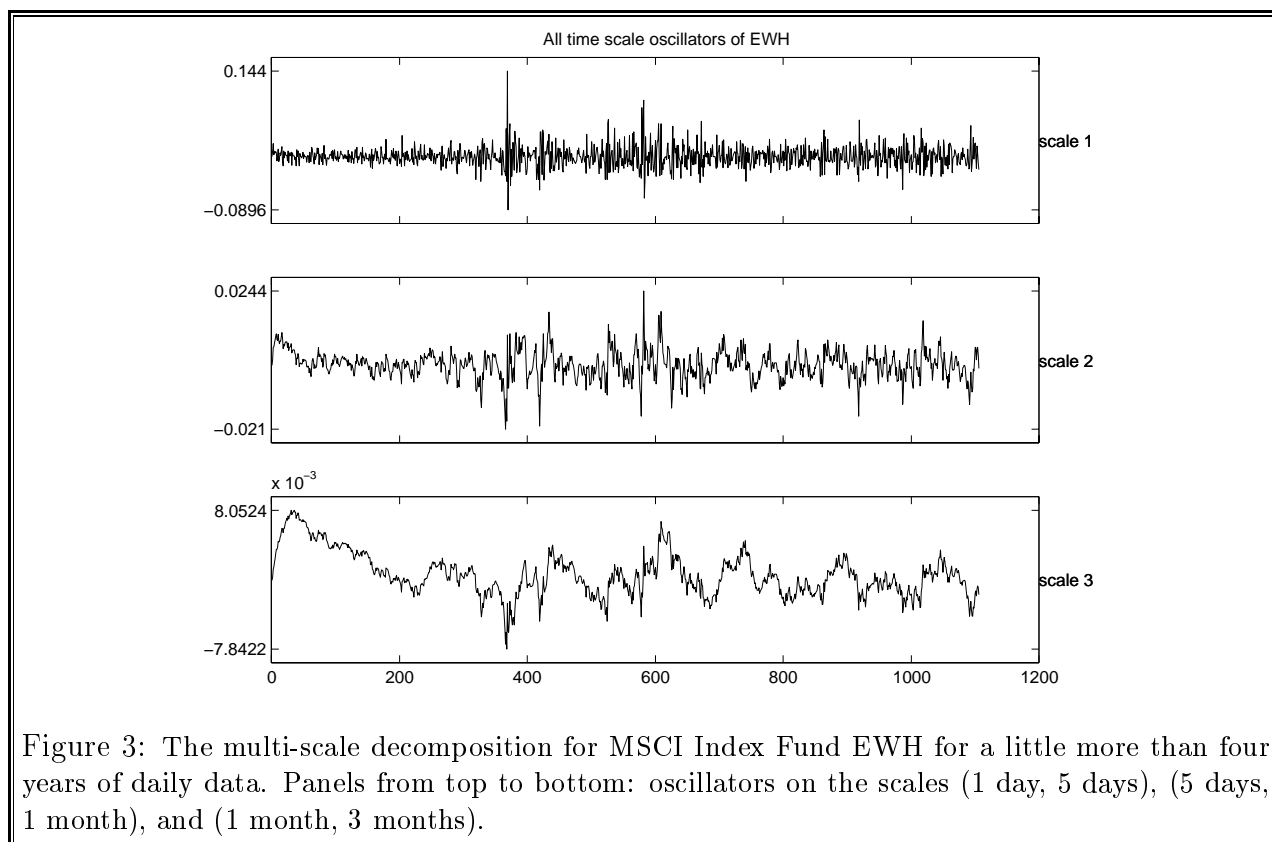


Figure 3: The multi-scale decomposition for MSCI Index Fund EWH for a little more than four years of daily data. Panels from top to bottom: oscillators on the scales (1 day, 5 days), (5 days, 1 month), and (1 month, 3 months).

### 2.3 Mutual Information (MI)

Let's recall that a gaussian process can be completely defined by second order statistics, namely mean and variance. Consequently, higher order statistics is necessary to uniquely characterize non-gaussian processes.

We make use of second order statistics correlation coefficient (CC) and the high order statistics known as mutual information (MI) to measure the dependency between two random variables. Though hard to compute, MI is a natural measure of the independence between random variables. In fact, MI takes into account the whole dependence structure

of the variables, and not only the covariance.

The mutual information is defined by the entropies  $H(X)$ ,  $H(Y)$  and  $H(X, Y)$  (Papoulis, 1991[10]):

$$MI(X; Y) = H(X) - H(X|Y) \quad (5)$$

$$H(X|Y) = H(X, Y) - H(Y) \quad (6)$$

$$MI(X; X) = H(X) \quad (7)$$

The mutual information is always non-negative and zero if and only if the variables are statistically independent (Papoulis, 1991[10]).

## 2.4 Independent Component Analysis (ICA)

Independent Component Analysis is a relatively new statistical technique to transform a set of variables into a new set of variables, so that statistical dependency among the transformed variables is minimized (Comon, 1994[3]; Lee, 1998[5]; Hyvärinen and Oja, 2000[4]). This methodology is an alternative to Principal Component Analysis (PCA) to analyze financial data. By performing ICA one could find bases that reduce statistical dependencies among the transformed outputs, even when the input data contains non-Gaussian innovations and nonlinearities (Moody and Wu, 1996[8]). These basis are typically *non-orthogonal* when the joint distribution of returns is non-Gaussian. ICA is sensitive to higher order moments, while PCA has an implicit assumption that the return series are jointly Gaussian. Other applications of ICA to financial data include Wu and Moody, 1997[12]; Back and Weigend, 1997[1].

ICA algorithms are nonlinear estimators, and a variety of different forms have been proposed (Hyvärinen and Oja, 2000[4] and references there in). The version of ICA that we use here discovers a non-orthogonal basis that minimizes mutual information between basis vectors (Moody and Yang, 1999[9]; Yang and Amari, 1997[13]):

$$\begin{aligned} \mathbf{y}_t &= \mathbf{W}_t \mathbf{x}_t \\ \mathbf{W}_{t+1} &= \mathbf{W}_t + \eta(\mathbf{I} - f(\mathbf{y}_t)\mathbf{y}_t)\mathbf{W}_t \end{aligned} \quad (8)$$

where  $\eta > 0$  is a learning rate, the matrix  $\mathbf{W}_t$  which transforms the input vector  $\mathbf{x}_t$  to the output vector  $\mathbf{y}_t$  is updated by the above equation. There are many choices for the nonlinear function  $f(\cdot)$  in the ICA algorithm. We choose  $f(y) = 2\tanh(y)$ . The iterative learning process performs a nonlinear optimization. After the learning phase, the components of  $\mathbf{y}_t$  become statistically independent under certain assumptions.

Different from some conventional ICA algorithms, the algorithm (8) does not need to compute the matrix inverse. It is more stable and robust due to the equivariant property (Cardoso and Laheld, 1996[2]). At convergence, we obtain the ICA transform  $\mathbf{W} = \mathbf{W}_\infty$ .

Let  $\mathbf{w}_i^T$  be the  $i$ -th row vector of  $\mathbf{W}$ ,  $\mathbf{v}_j$  be the  $j$ -th column vector of  $\mathbf{W}^{-1}$ . Then, we have two equations:

$$y_i(t) = \mathbf{w}_i^T \mathbf{x}_t \tag{9}$$

$$\mathbf{x}_t = y_1(t)\mathbf{v}_1 + \cdots + y_n(t)\mathbf{v}_n \tag{10}$$

The first equation shows how each ICA component is formed from the observation vector  $\mathbf{x}(t)$  and the second equation gives the decomposition of the observation vector in terms of the ICA components  $y_i(t)$  and the vectors  $\mathbf{v}_i$ .

Due to the overlap between the frequency responses of the band pass filters, the oscillators are not statistically independent.

To reduce the dependency between oscillators, we apply the ICA algorithm to process the oscillators. The outputs of the ICA are the new representation of the oscillators.

A glance to the two left panels of Figs. 4 and 5 shows that oscillators on adjacent scales (one and two, and two and three) are not statistically independent. Instead, they have positive correlations (refers to the middle bar plot on the left panels of Figs. 4 and 5).

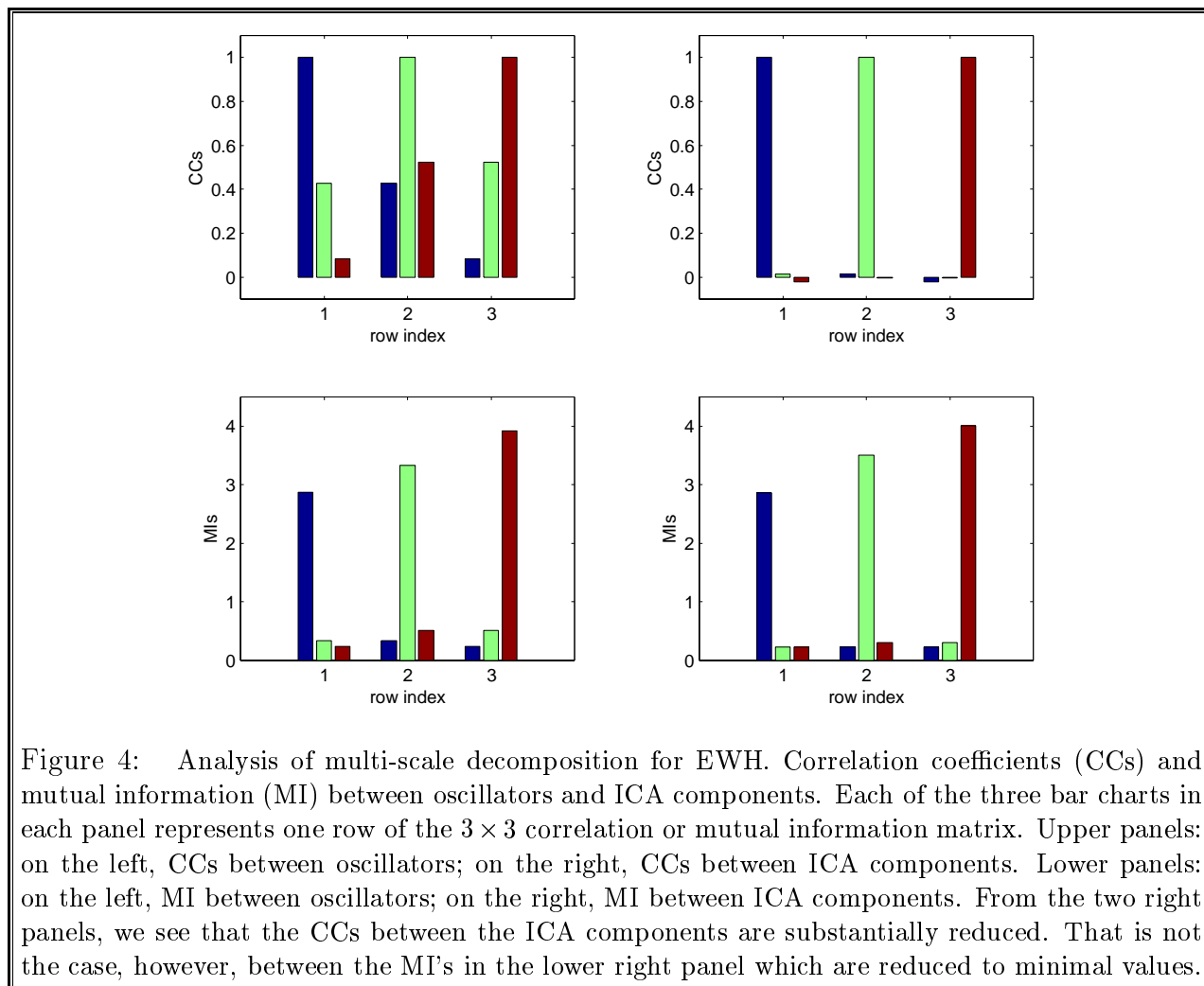
As shown in the two right panels of Figs. 4 and 5, the statistical dependency between oscillators is substantially reduced after ICA, while MI is reduced to minimum values. This, by definition, is an indication that through the application of ICA one was able to find a non-orthogonal coordinate basis with minimum statistical dependencies.

#### 2.4.1 Data Pre-processing before ICA

Before leaving this section, one should mention that standard data pre-processing technique known as *centering*, consisting in subtracting the mean from the original data so that the input to ICA has zero mean, was performed before the application of ICA algorithm. We also transformed the input to ICA to have unit variance.

Another common standard pre-processing artifact, performed after *centering*, is to transform the zero-mean observations to a new set of observations whose components are uncorrelated and their variances equal to unity. That is, the new transformed set has covariance matrix equals to the identity matrix. This pre-processing stage is known as *whitening*. The idea of performing *whitening* is grounded on the assumption that with the first and second-order statistics removed, ICA algorithms would somehow concentrate its attention on minimizing higher order statistics. Nevertheless, in order to be able to observe the mixing of the initial input to ICA we decided not to perform *whitening* on our data. Nonetheless, for the shake of completeness we present in the next section some results on the the effect of performing *whitening* in the data before ICA.

The basic intention of any kind of pre-processing strategies is some how to simplify the ICA computations by reducing the number of parameters to be estimated. That will certainly have an impact in computational time if that is an issue.



#### 2.4.2 Choosing the optimal learning rate ( $\eta$ )

The ICA algorithm presented by equation 8 depends on the selection of the parameter ( $\eta$ ) or learning rate. In this work the optimal value of  $\eta$  was chosen so that the averaged mutual information between the ICA outputs was minimal.

Figures 6 to 11 show the effect of the learning rate on the averaged mutual information and the correlation coefficient when applying *whitening* to the ICA input data and when *whitening* was not applied to the data before ICA. The left vertical axis of each plot (figures 6 to 11) represents the sum of the absolute value of the off-diagonal elements of the correlation among the ICA outputs, obtained after applying ICA to each oscillator at different values of the learning rate ( $\eta$ ). Similarly, the right vertical axis on each plot represents the sum of the off-diagonal elements of the *normalized* mutual information,  $MI(X, Y)/H(X)$  (see equation 5), among the ICA outputs obtained from each oscillator at different values

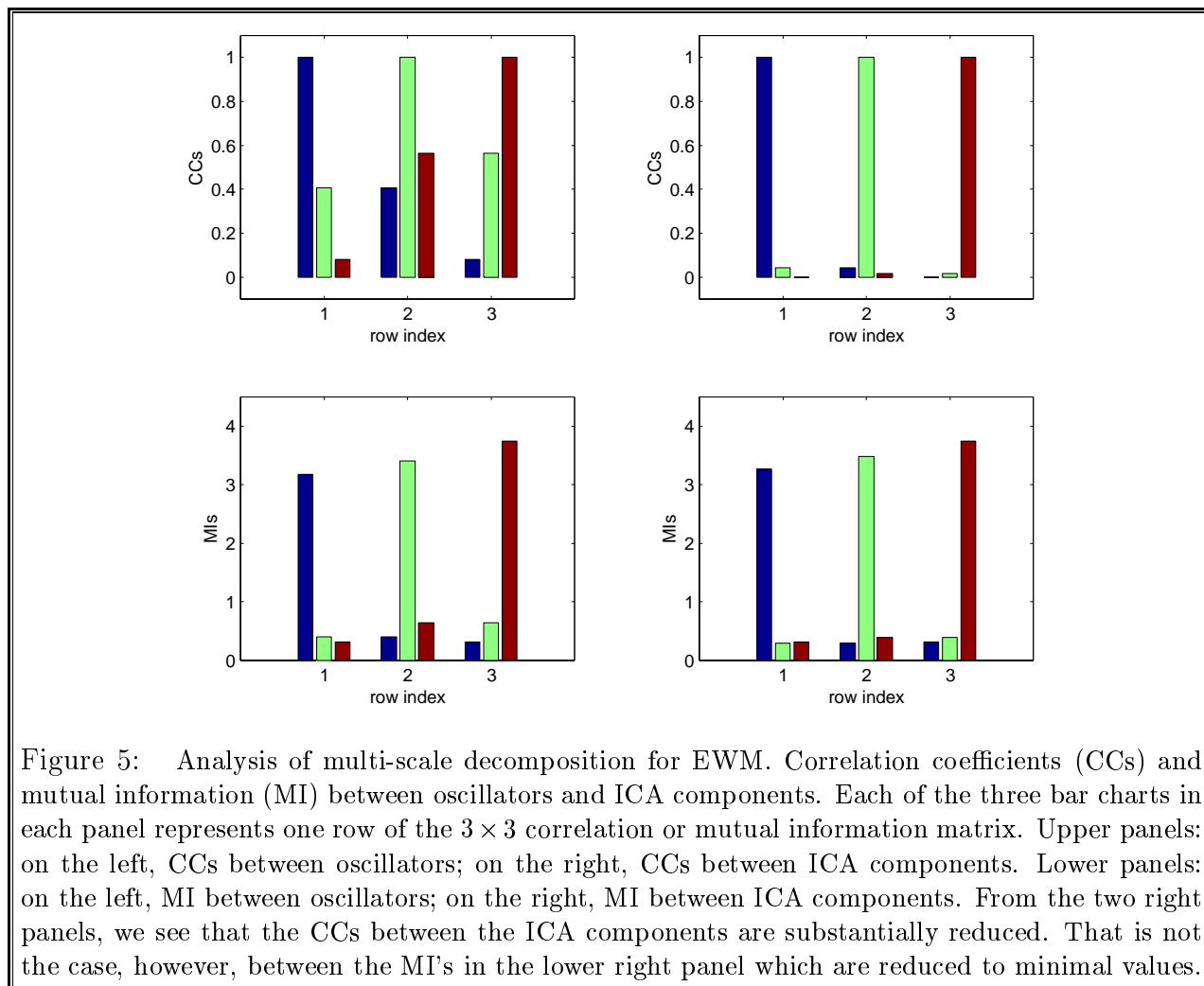


Figure 5: Analysis of multi-scale decomposition for EWM. Correlation coefficients (CCs) and mutual information (MI) between oscillators and ICA components. Each of the three bar charts in each panel represents one row of the  $3 \times 3$  correlation or mutual information matrix. Upper panels: on the left, CCs between oscillators; on the right, CCs between ICA components. Lower panels: on the left, MI between oscillators; on the right, MI between ICA components. From the two right panels, we see that the CCs between the ICA components are substantially reduced. That is not the case, however, between the MI's in the lower right panel which are reduced to minimal values.

of the learning rate.

A first observation from Figures 6 to 11 is the existence of an optimal learning rate ( $\eta$ ) at which the averaged mutual information among ICA components is minimal. Also, such optimal value of  $\eta$  depends on whether *whitening* is or not performed on the input data before ICA.

When *whitening* is performed on the input data, the ICA algorithm introduces correlation among the ICA outputs as the learning rate increase ( $\eta$ ). Correlation goes to zero as ( $\eta$ ) decrease. On the other hand, averaged mutual information reached a minimum value at a particular optimal value of the learning rate ( $\eta_{\text{optimal}}$ ). At values of the learning rate above or below  $\eta_{\text{optimal}}$  the averaged mutual information between ICA outputs increase. These findings are against the general recommendation of choosing a learning rate less than 0.01 when performing ICA data decomposition (Makeig *et al.*, 1997[6]).

When *whitening* is not performed on the input to ICA, it is striking to see how both average correlation and average mutual information among ICA outputs increase when  $\eta < \eta_{\text{optimal}}$  or when  $\eta > \eta_{\text{optimal}}$ .

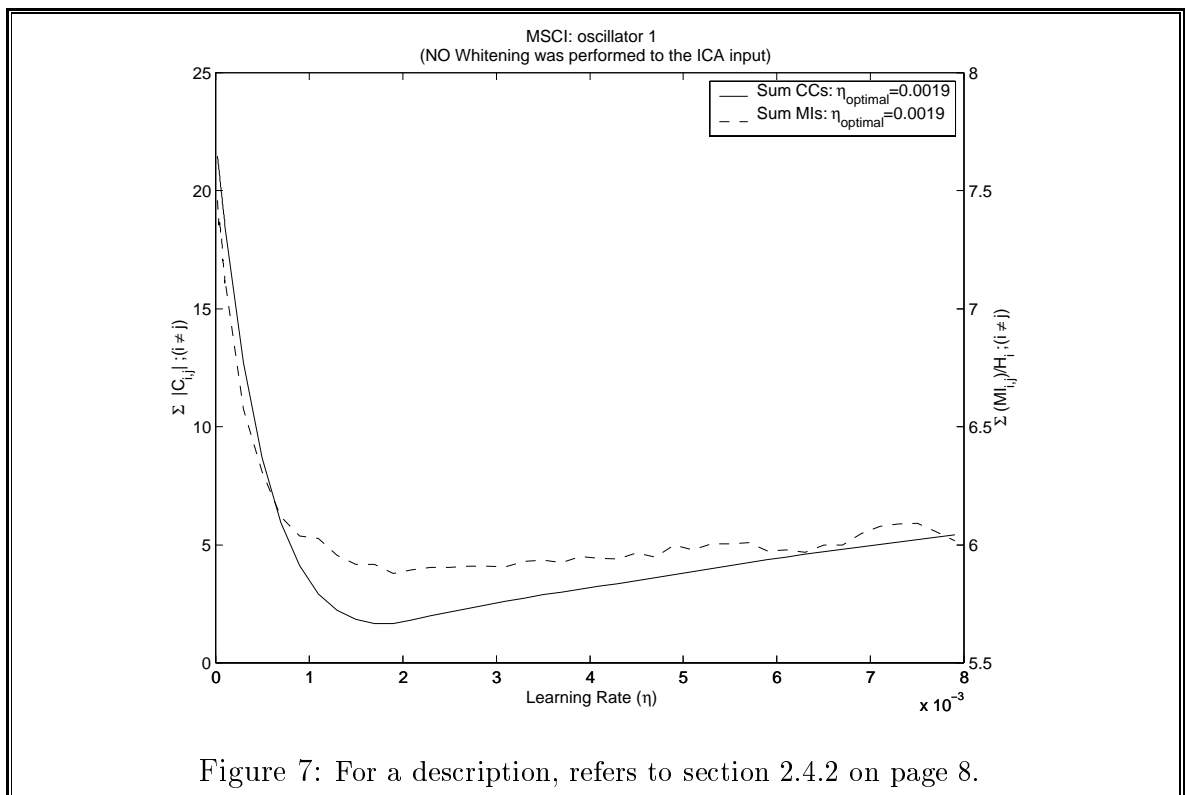
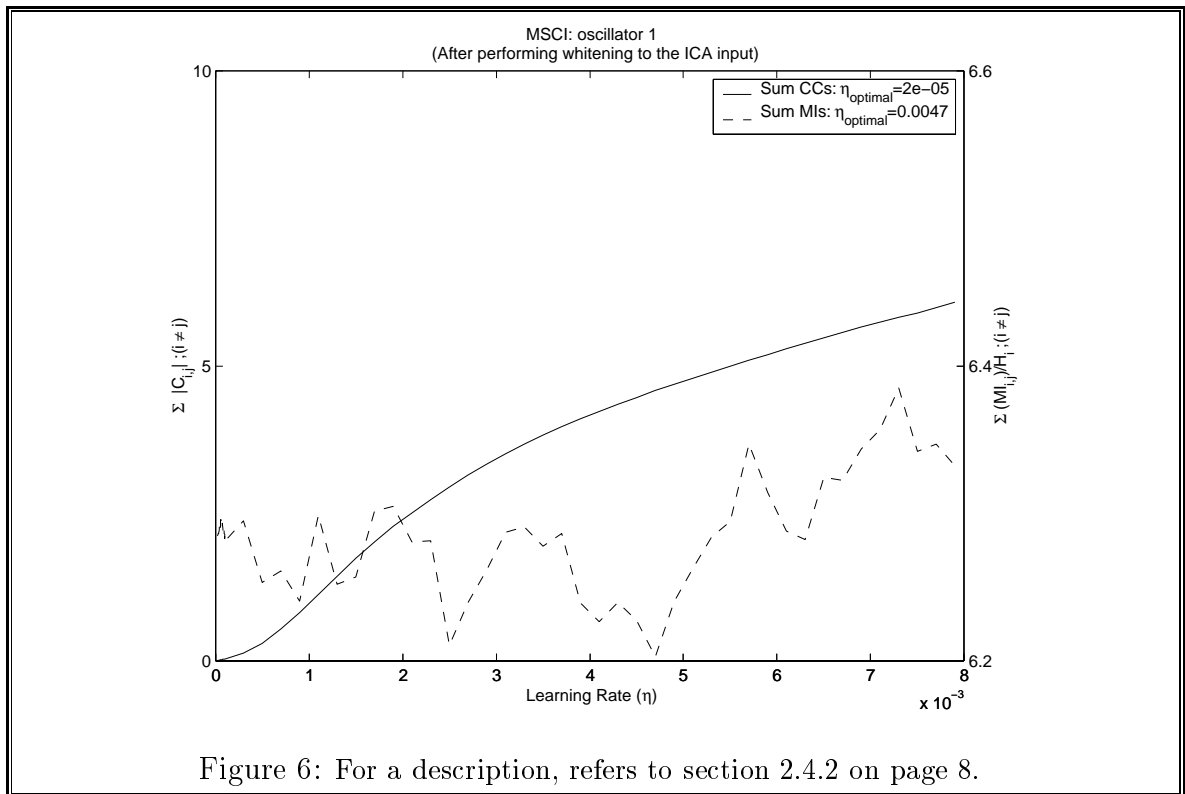
Let's also notice that when no *whitening* is performed in the input data, for the case of low frequency oscillators, namely oscillators 1 and 2, the optimal value for the learning rate minimize both averaged correlation and averaged mutual information among ICA outputs. That is not the case, however, for oscillator 3 where the minimum of both quantities happen at different values of the learning rates.

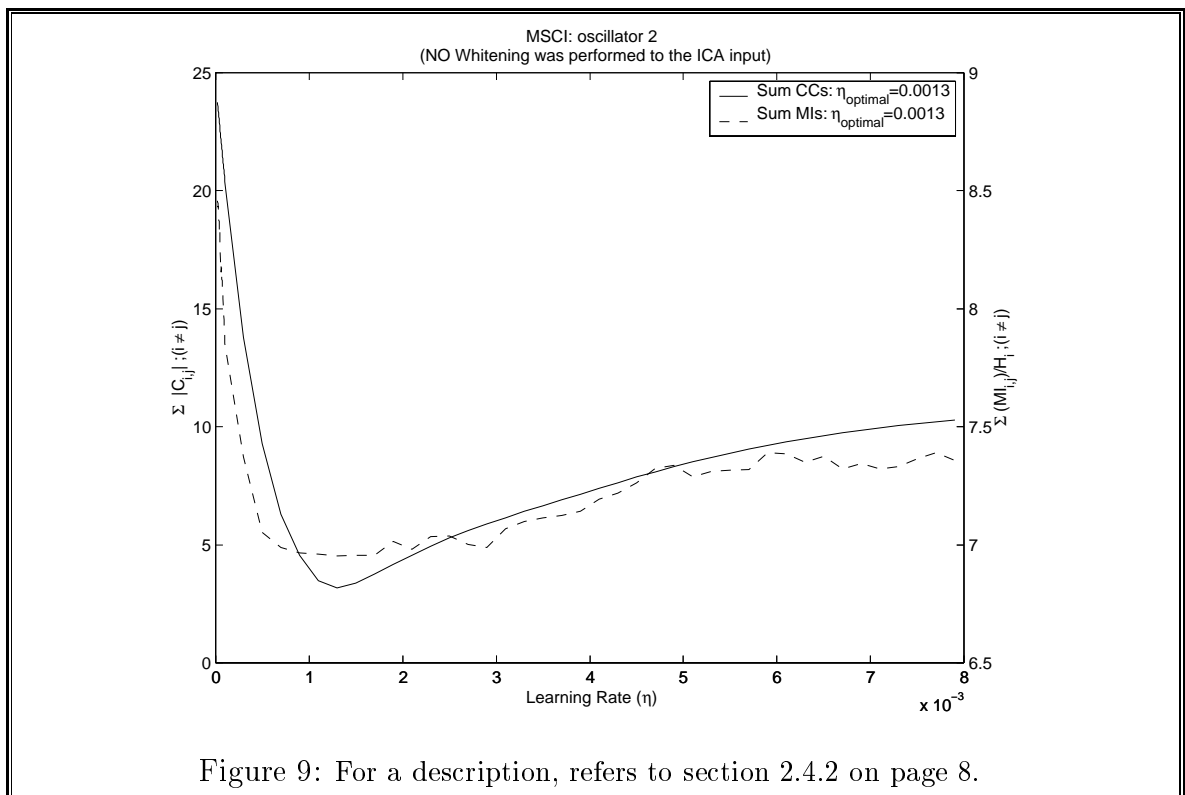
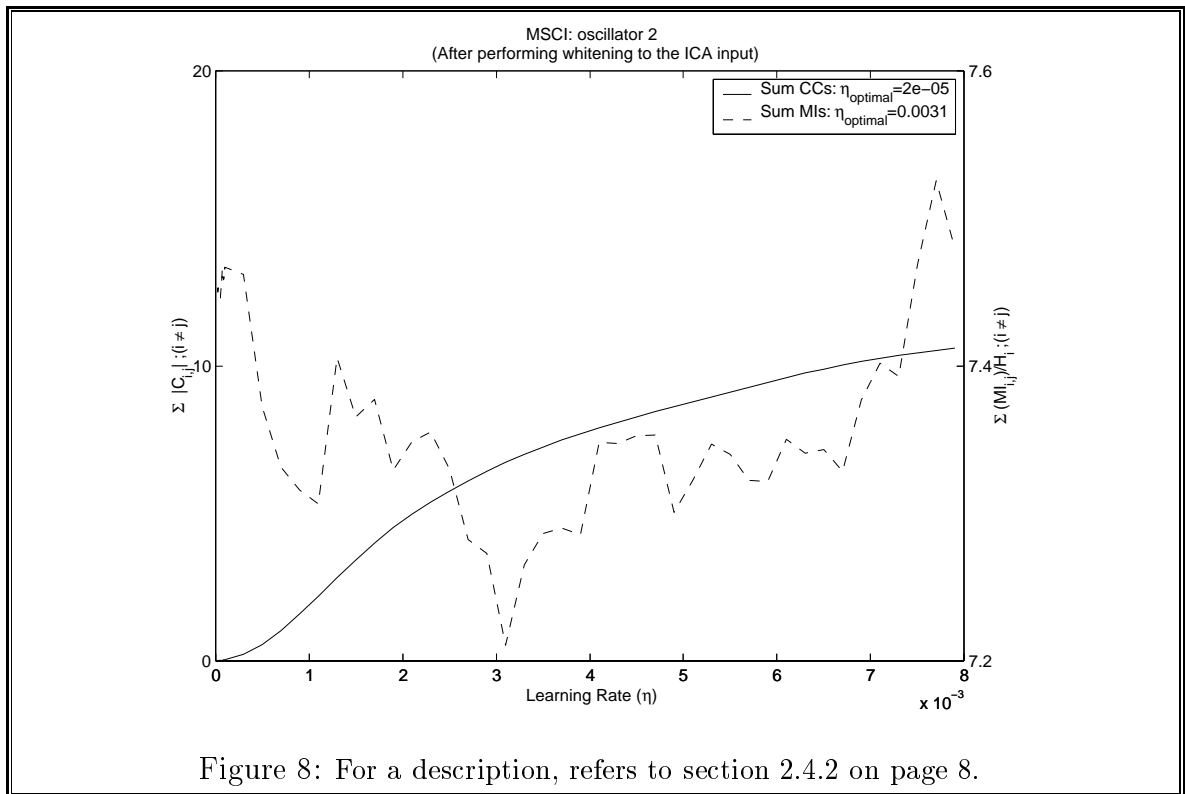
We should also notice that when *whitening* is performed, the mutual information curve is very wiggly in a small range of variation. A smooth curve is obtained, however, when no *whitening* is performed, and the range of variation is more wider.

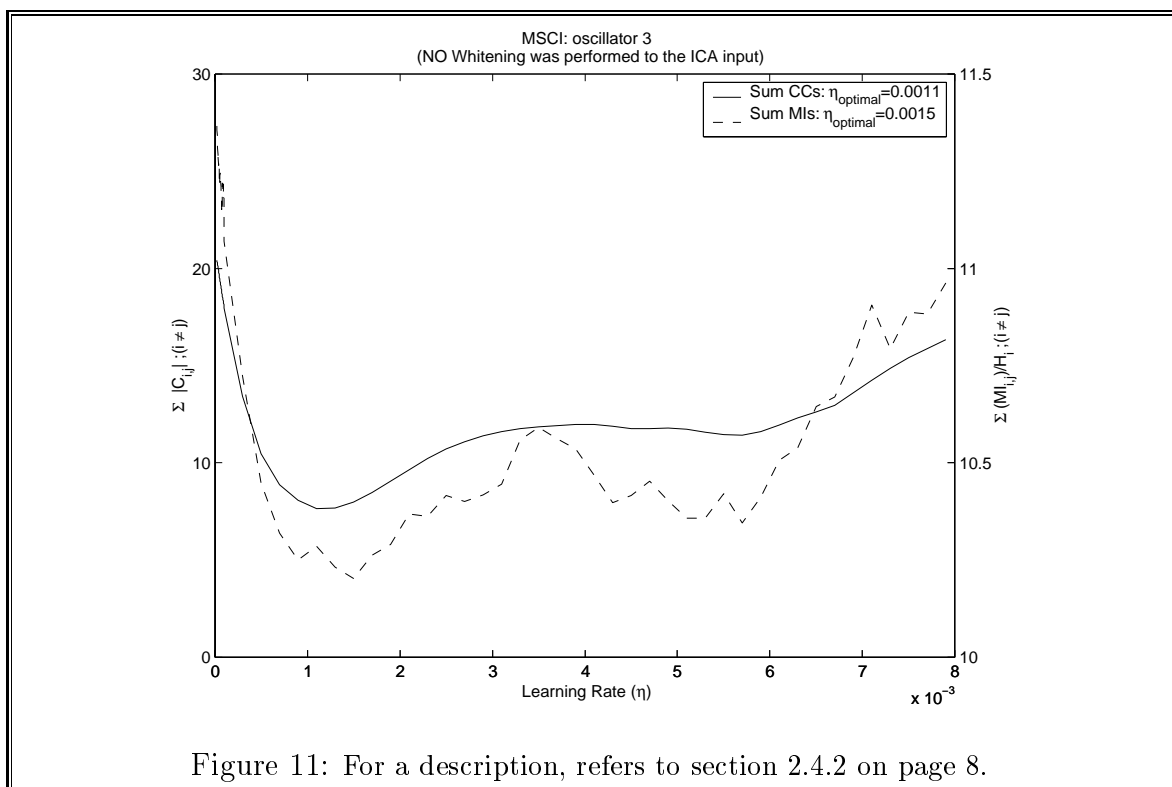
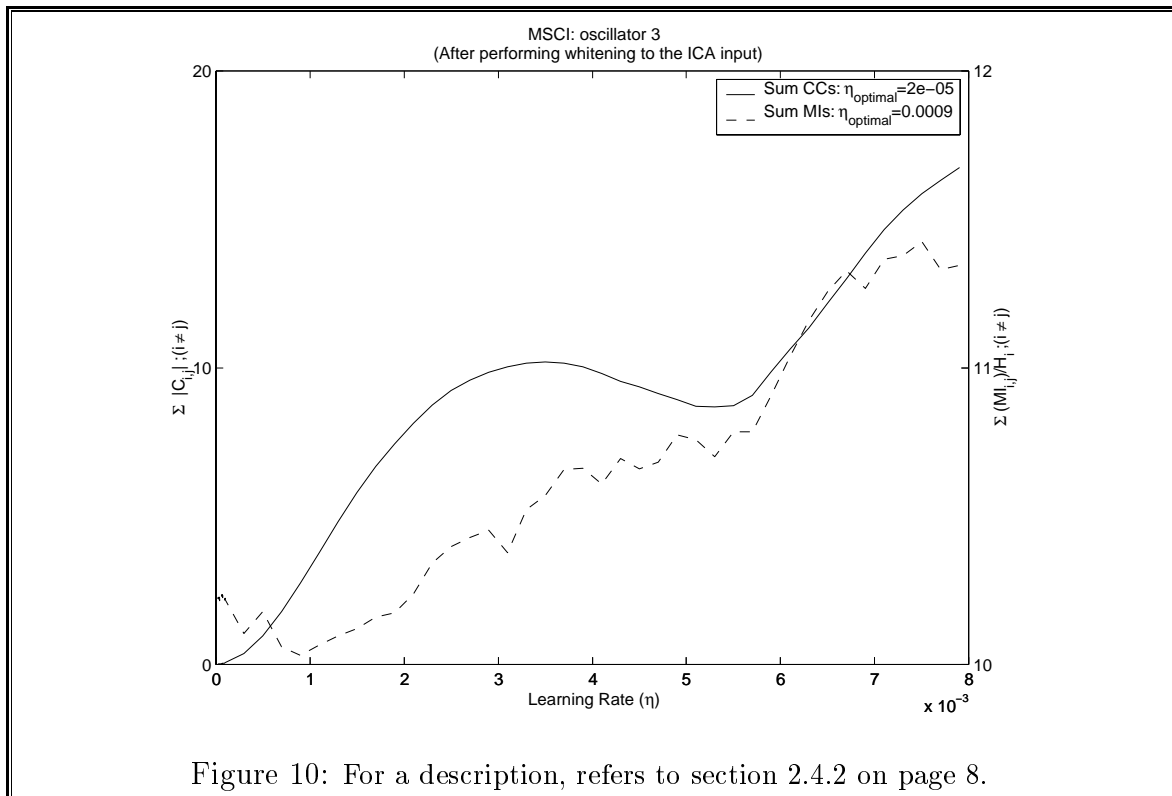
### 3 Cross-Sectional Analysis of returns Interactions

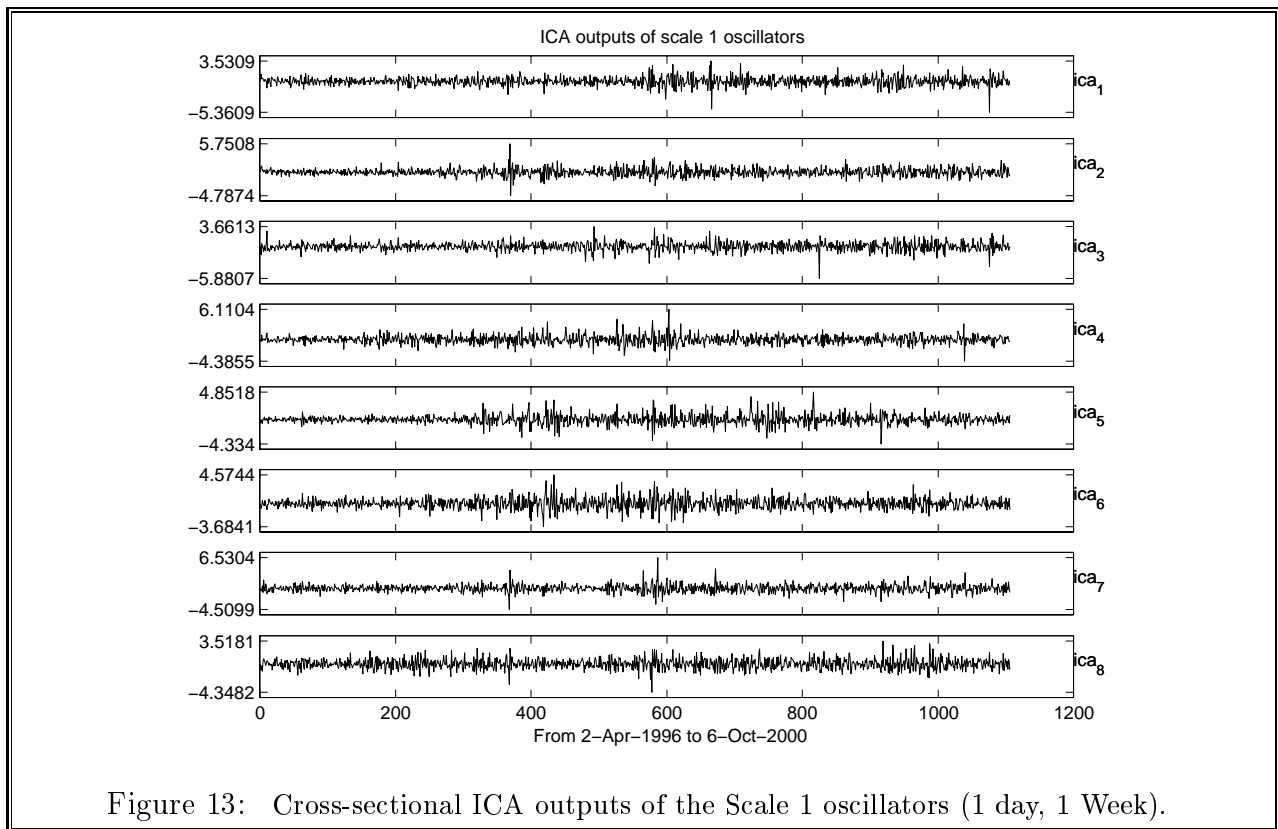
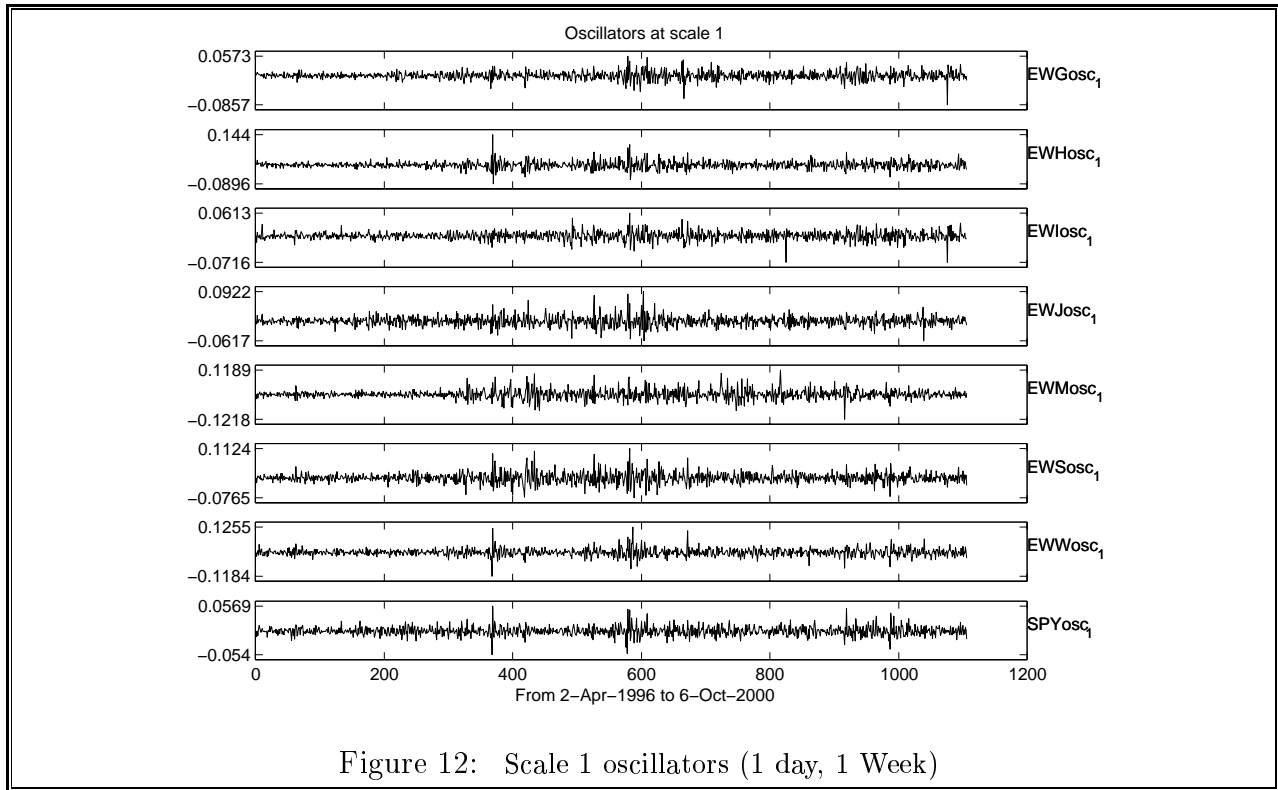
As indicated earlier, for each Index we decompose the natural logarithm of the returns as defined in section 2.1, page 2. On each time scale, eight oscillators are obtained corresponding to the eight Indexes considered here. The oscillators on each scale are shown from Fig. 12 to Fig. 16. The corresponding ICA outputs are displayed from Fig. 13 to 17. The goal is to study the interaction between the evolution of the returns of each Fund on a spectrum of time scales (see eq. 4, on page 5).

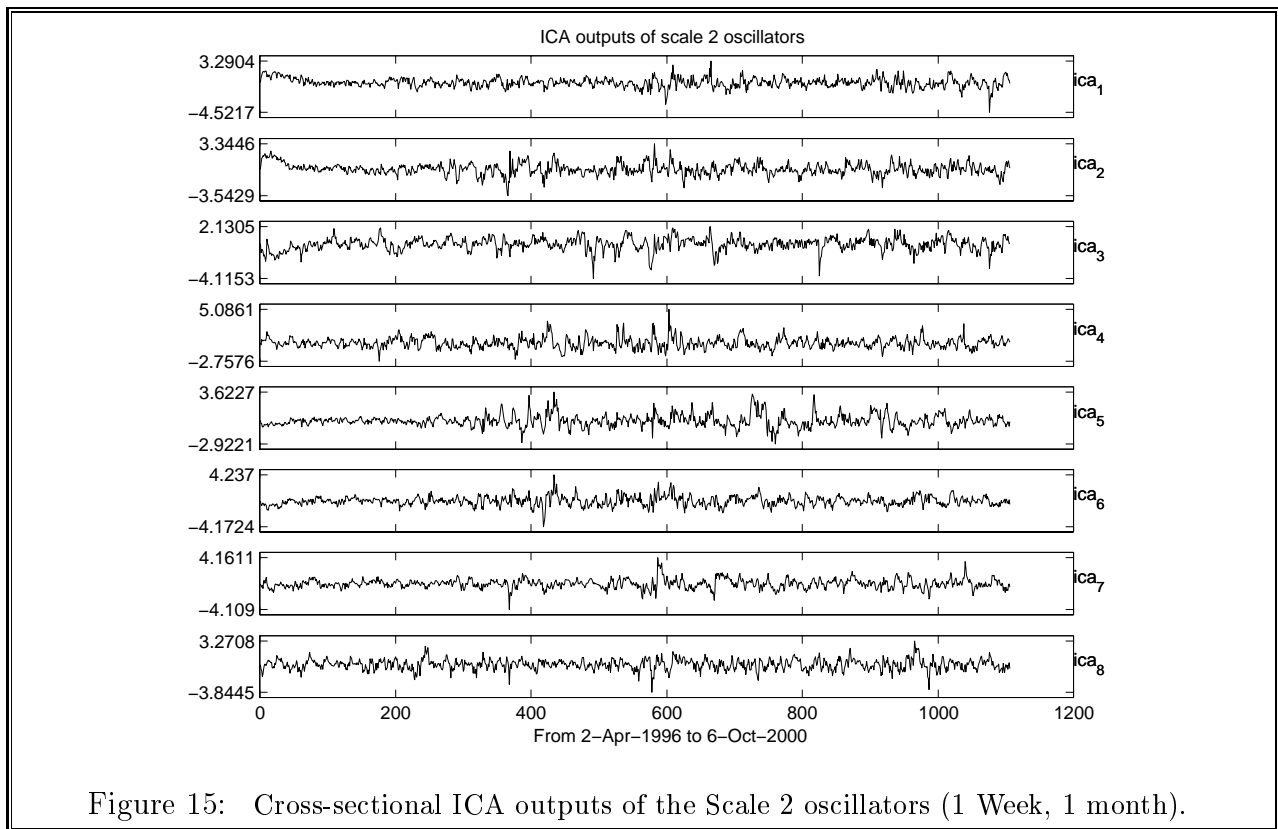
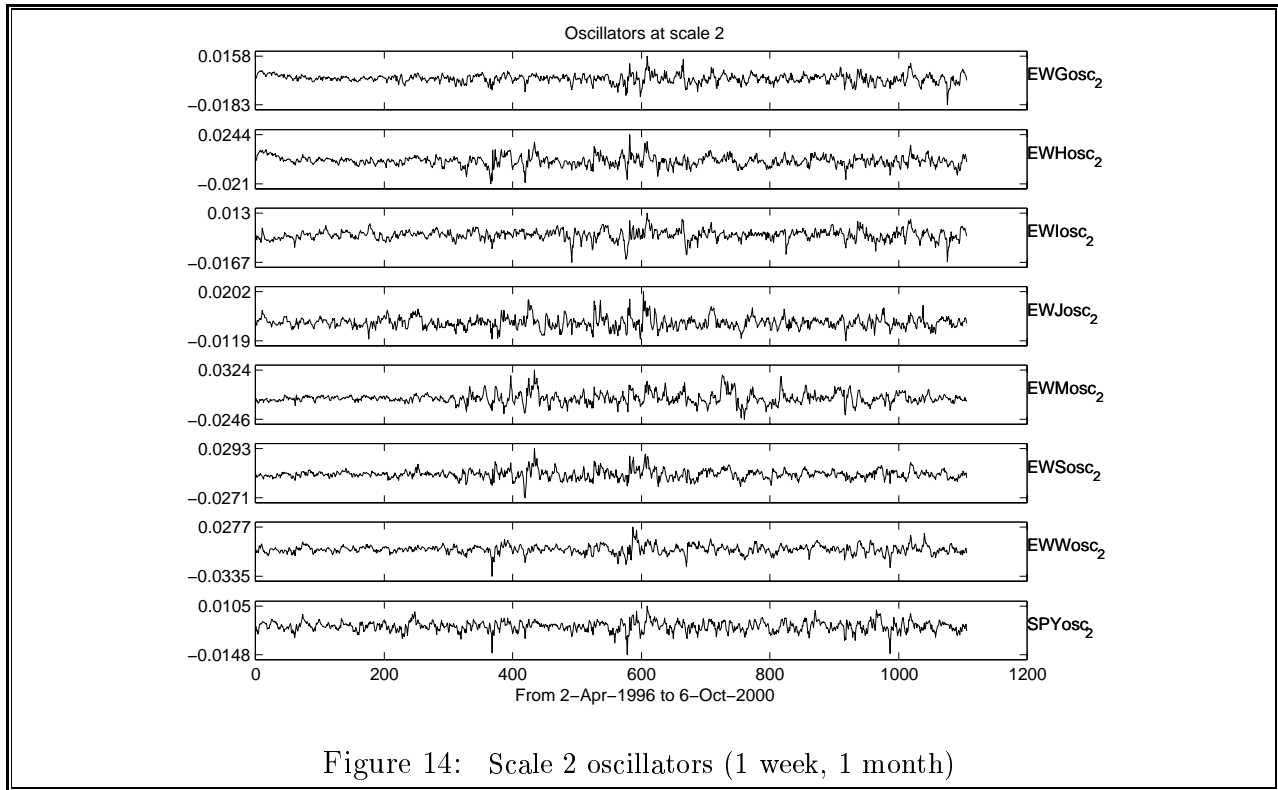
Second order statistical dependency between the returns are shown in Fig. 18. Higher order statistical dependencies as measured by the Mutual Information (MI) coefficients at all time scales is shown in Fig. 19. At all time scales, the independent components found by ICA correspond to linear combinations of return movements that have reduced cross-correlation and mutual information relative to the original values. A strong statistical dependency between the Indexes is observed at all three time scales (left panel of Fig. 18 - 19) before ICA. Less mixing is observed, however, after performing ICA.

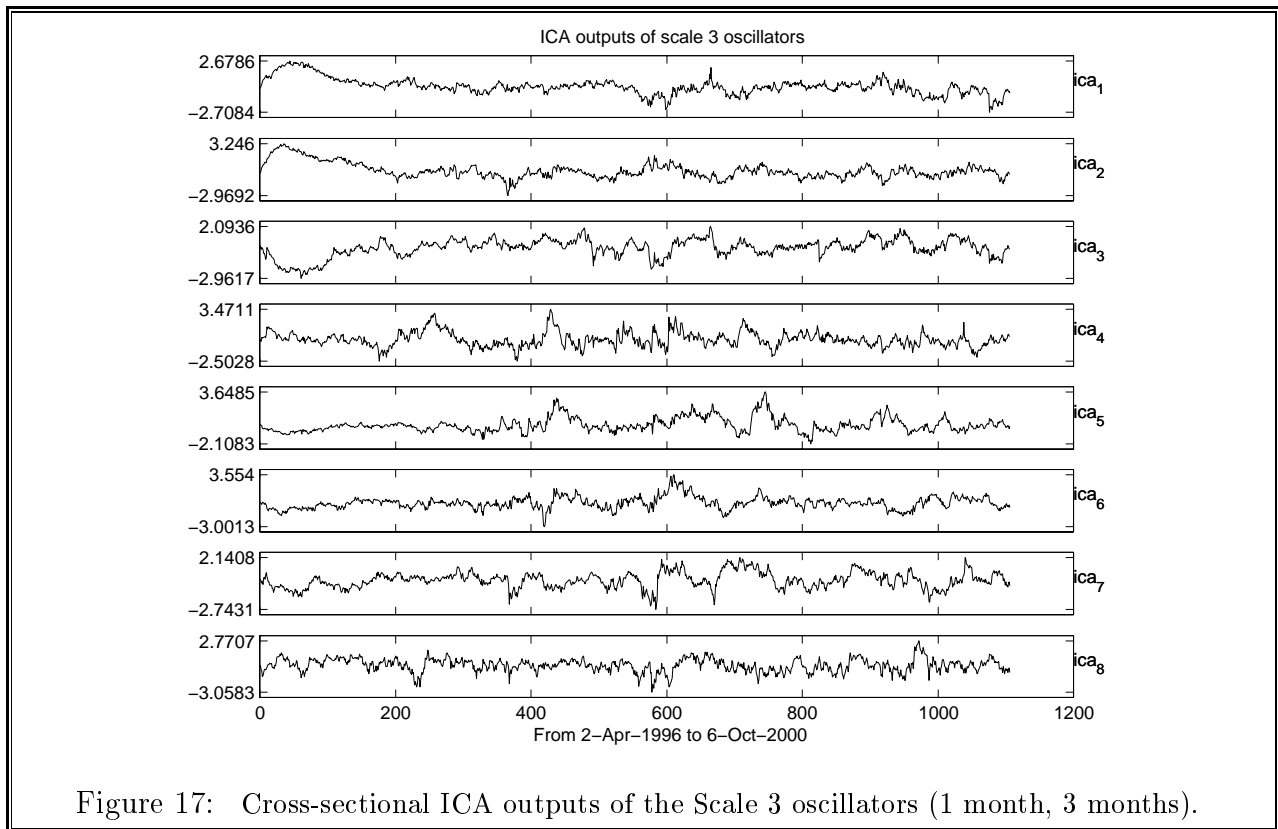
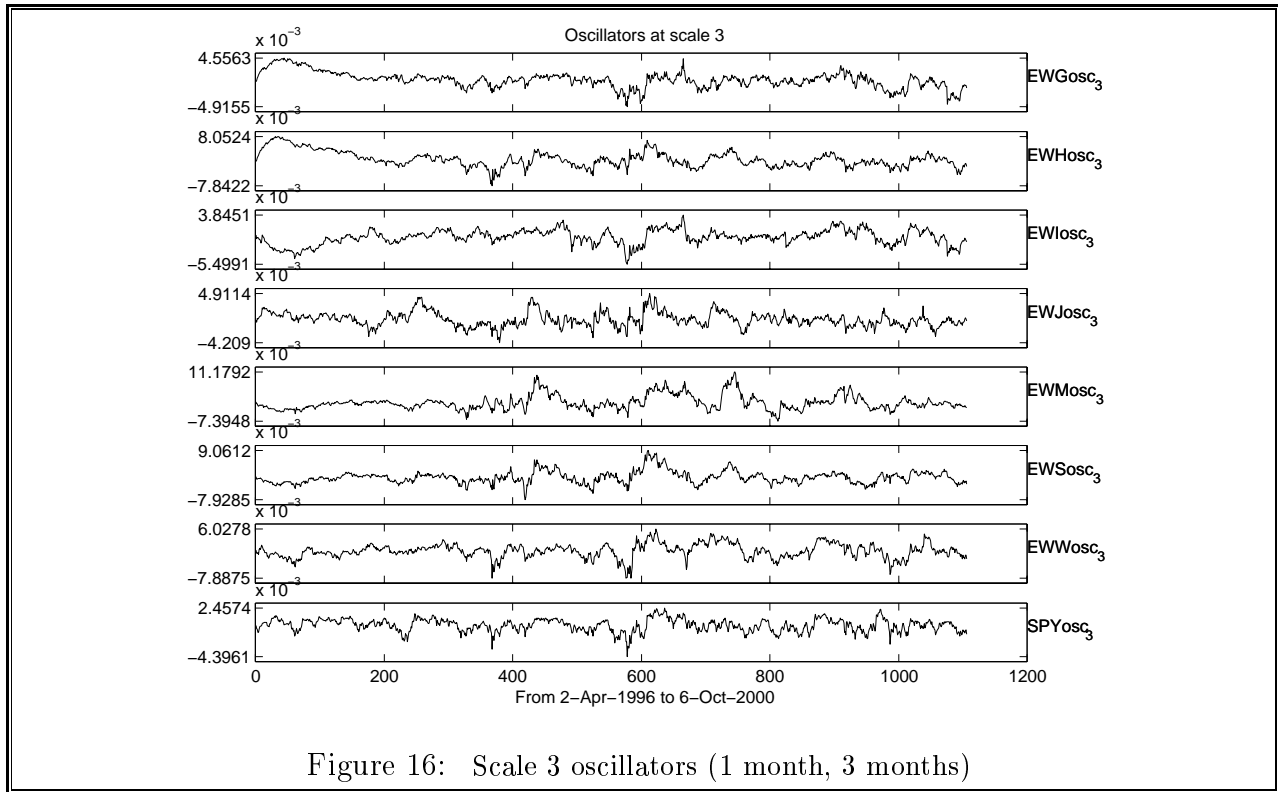












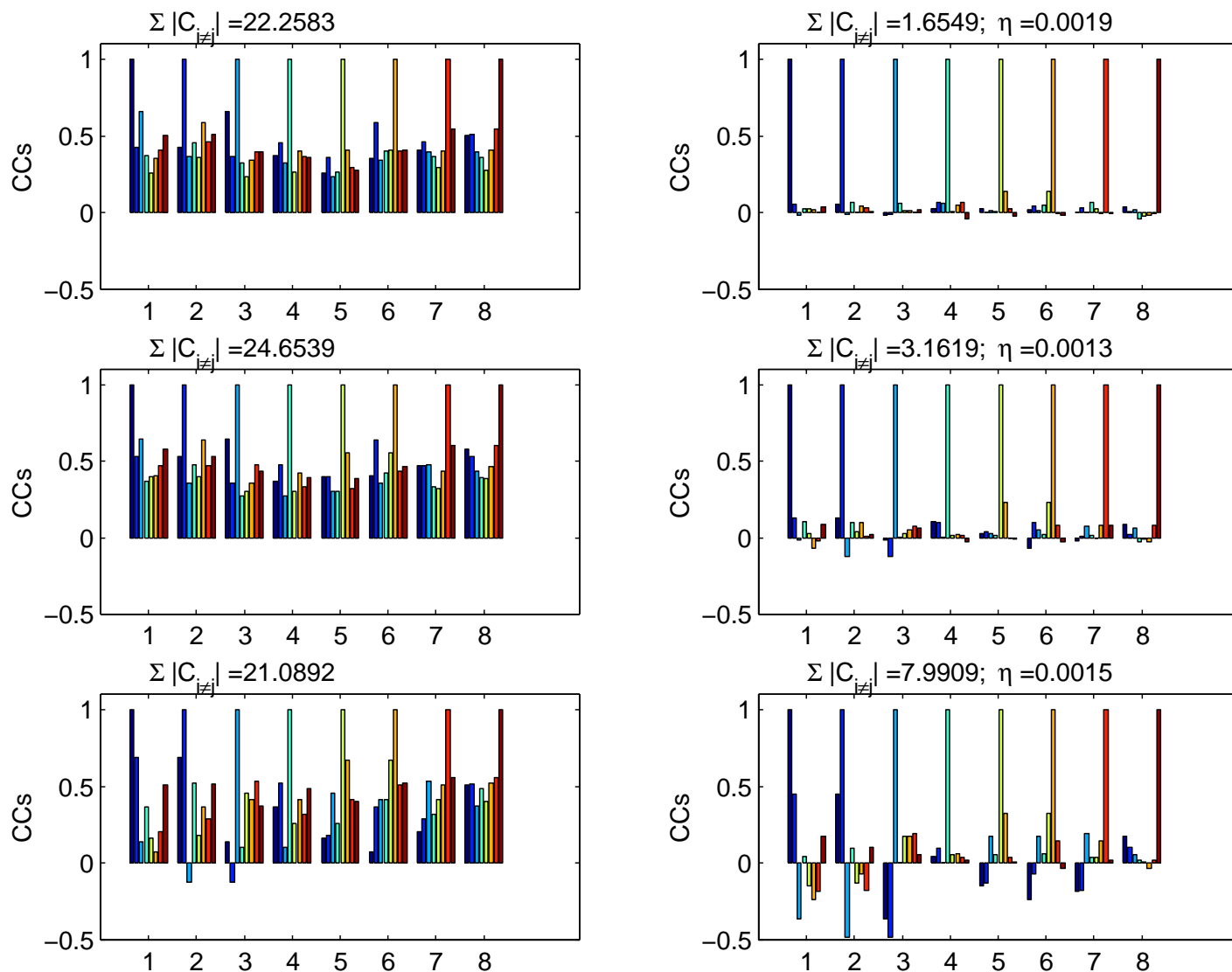


Figure 18: Cross-sectional interactions between the Indexes at time scales one (top), two (middle), and three (bottom). Correlation Coefficients (CCs) between oscillators (left) and ICA components (right). We see that the second order correlations between the eight ICA components are reduced substantially (right side panels). However, some off-diagonal elements remain non-zero, indicating the presence of nongaussian or nonlinear effects.

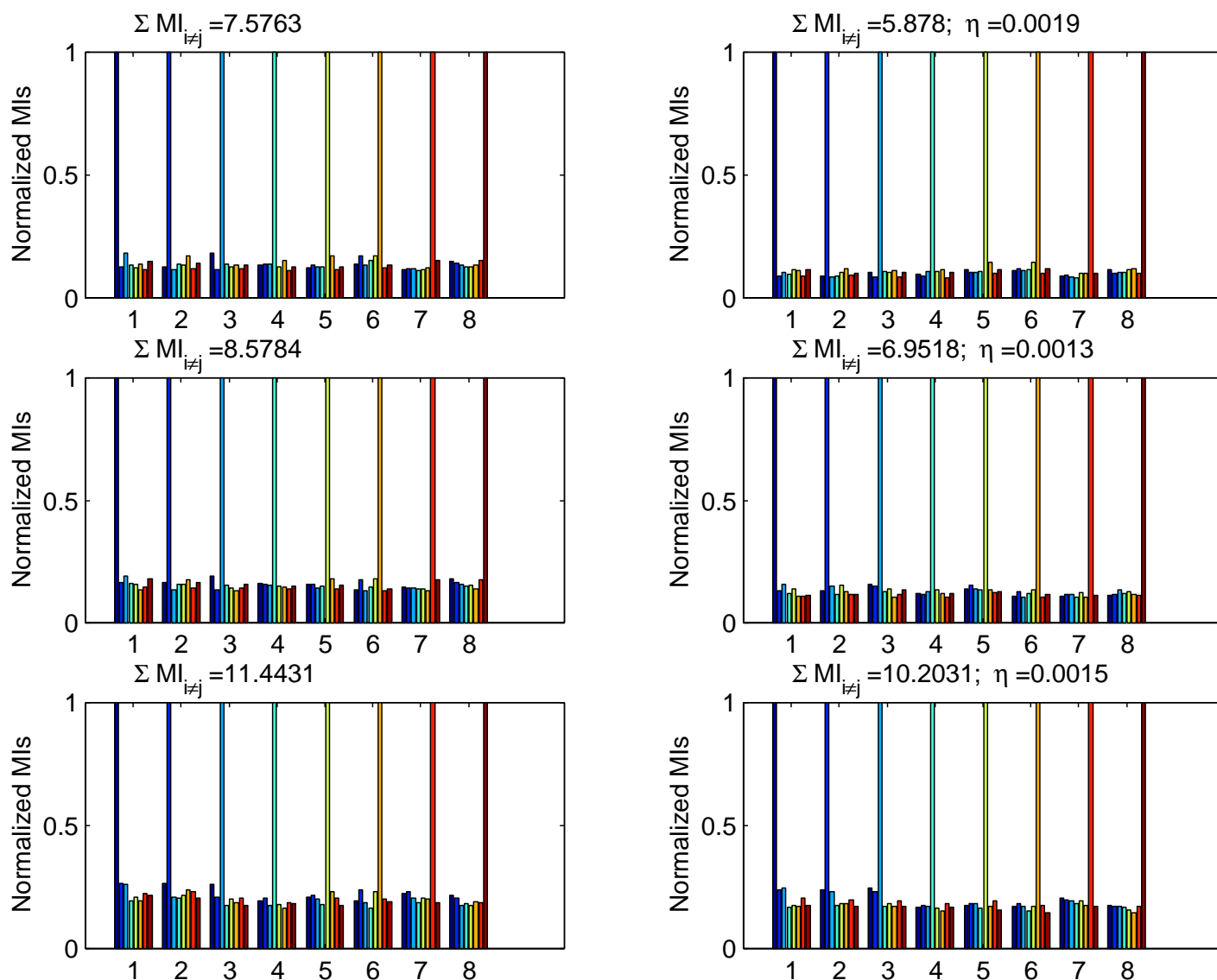


Figure 19: Normalized Mutual information (MI) between oscillators and ICA components. Each of the seven bar charts in each panel represents one row of the  $8 \times 8$  mutual information matrix. All panels on the left, MI at all three time scales considered here; All panels on the right, MI of the ICA components of the respective scale. We see that the MI between the ICA components are reduced to minimal values (panels on right).

### 3.1 ICA transforming Matrix

The mixing between the returns after performing ICA can be observed by looking at each row of the mixing matrix. The following table shows the transforming matrix and its normalization.

#### Matrix W1

```
-----
ica_1:  1.0435  -0.0677  -0.3086  -0.0490  -0.0157  -0.0200  -0.0567  -0.1311
ica_2: -0.0167   1.0541  -0.0704  -0.0769  -0.0641  -0.1975  -0.0821  -0.1583
ica_3: -0.3411  -0.0410   1.0140  -0.0053  -0.0311  -0.0261  -0.0804  -0.0349
ica_4: -0.0803  -0.1071  -0.0269   1.0106  -0.0352  -0.0748  -0.0311  -0.1118
ica_5: -0.0231  -0.0932  -0.0098  -0.0553   0.9630  -0.0644  -0.0479  -0.0689
ica_6: -0.0214  -0.2385  -0.0761  -0.0664  -0.0589   1.0988  -0.0870  -0.0703
ica_7: -0.0502  -0.0801  -0.0912  -0.0391  -0.0302  -0.0759   1.0056  -0.1858
ica_8: -0.1378  -0.1220  -0.0231  -0.0475  -0.0175  -0.0718  -0.2032   1.0260
-----
```

#### Matrix W2

```
-----
ica_1:  0.9980  -0.0817  -0.2757  -0.0085  -0.0771  -0.0424  -0.0546  -0.1234
ica_2: -0.0789   1.0264  -0.0740  -0.0893  -0.0263  -0.2157  -0.0809  -0.1102
ica_3: -0.3183  -0.0517   0.9821  -0.0052  -0.0077  -0.0327  -0.0963   0.0039
ica_4: -0.0109  -0.0797  -0.0377   0.9146  -0.0262  -0.0804  -0.0320  -0.1196
ica_5: -0.0657  -0.0265  -0.0245  -0.0522   0.8916  -0.0990  -0.0357  -0.0656
ica_6: -0.0441  -0.1900  -0.0357  -0.0905  -0.1046   1.0035  -0.0407  -0.0882
ica_7: -0.0653  -0.0858  -0.1104  -0.0293  -0.0351  -0.0317   0.9368  -0.1604
ica_8: -0.1353  -0.0923  -0.0209  -0.0575  -0.0642  -0.0817  -0.1834   0.9775
-----
```

#### Matrix W3

```
-----
ica_1:  1.0798  -0.2755  -0.2298  -0.0555  -0.0925   0.1286   0.0052  -0.1043
ica_2: -0.3070   1.3061   0.1317  -0.1218   0.0229  -0.3185  -0.1879  -0.0509
ica_3: -0.3397   0.1113   1.0296   0.0404  -0.0601  -0.1023  -0.1902   0.0223
ica_4: -0.1001  -0.0891   0.0792   1.0277  -0.0010  -0.1823  -0.0656  -0.1286
ica_5: -0.1219   0.0077  -0.0821   0.0220   0.9767  -0.1661  -0.0925  -0.0020
ica_6:  0.1149  -0.1849  -0.1012  -0.0383  -0.2312   1.1064  -0.0661  -0.1618
ica_7: -0.0198  -0.1553  -0.1729   0.0095  -0.0649  -0.0335   0.9550  -0.1349
ica_8: -0.1515  -0.0625  -0.0414  -0.1098  -0.0574  -0.1882  -0.1954   1.0633
-----
```

## Matrix W1\_Normalized

```

-----
ica_1:  0.9478  -0.0615  -0.2803  -0.0445  -0.0143  -0.0182  -0.0515  -0.1191
ica_2: -0.0153   0.9634  -0.0643  -0.0703  -0.0586  -0.1805  -0.0751  -0.1447
ica_3: -0.3173  -0.0381   0.9433  -0.0049  -0.0289  -0.0243  -0.0748  -0.0325
ica_4: -0.0780  -0.1040  -0.0261   0.9815  -0.0342  -0.0726  -0.0302  -0.1085
ica_5: -0.0237  -0.0955  -0.0100  -0.0567   0.9875  -0.0661  -0.0491  -0.0707
ica_6: -0.0188  -0.2099  -0.0670  -0.0584  -0.0518   0.9671  -0.0766  -0.0618
ica_7: -0.0485  -0.0774  -0.0881  -0.0378  -0.0291  -0.0734   0.9716  -0.1795
ica_8: -0.1293  -0.1145  -0.0216  -0.0446  -0.0164  -0.0674  -0.1906   0.9626
-----

```

## Matrix W2\_Normalized

```

-----
ica_1:  0.9495  -0.0777  -0.2623  -0.0081  -0.0734  -0.0403  -0.0520  -0.1174
ica_2: -0.0739   0.9617  -0.0693  -0.0837  -0.0246  -0.2021  -0.0758  -0.1033
ica_3: -0.3064  -0.0497   0.9455  -0.0050  -0.0074  -0.0315  -0.0927   0.0038
ica_4: -0.0117  -0.0856  -0.0405   0.9823  -0.0282  -0.0864  -0.0343  -0.1284
ica_5: -0.0726  -0.0292  -0.0271  -0.0577   0.9854  -0.1094  -0.0394  -0.0726
ica_6: -0.0425  -0.1833  -0.0344  -0.0873  -0.1009   0.9679  -0.0393  -0.0851
ica_7: -0.0677  -0.0889  -0.1144  -0.0304  -0.0364  -0.0328   0.9713  -0.1663
ica_8: -0.1333  -0.0909  -0.0206  -0.0566  -0.0633  -0.0805  -0.1807   0.9629
-----

```

## Matrix W3\_Normalized

```

-----
ica_1:  0.9350  -0.2385  -0.1990  -0.0481  -0.0801   0.1113   0.0045  -0.0903
ica_2: -0.2186   0.9300   0.0938  -0.0867   0.0163  -0.2268  -0.1338  -0.0363
ica_3: -0.3051   0.0999   0.9245   0.0363  -0.0540  -0.0919  -0.1708   0.0200
ica_4: -0.0940  -0.0836   0.0743   0.9649  -0.0010  -0.1712  -0.0616  -0.1207
ica_5: -0.1212   0.0076  -0.0816   0.0219   0.9708  -0.1651  -0.0919  -0.0020
ica_6:  0.0982  -0.1581  -0.0866  -0.0328  -0.1977   0.9462  -0.0565  -0.1384
ica_7: -0.0199  -0.1561  -0.1737   0.0095  -0.0653  -0.0337   0.9598  -0.1356
ica_8: -0.1356  -0.0560  -0.0370  -0.0982  -0.0514  -0.1684  -0.1749   0.9518
-----

```

### 3.2 Angles between rows in the ICA transforming Matrix

The acute angles between row vectors in each ICA transforming matrix are invariant to the scaling of the row vectors. These angles are shown below to indicate the structure of each ICA matrix. The angles less than 90 degrees indicate that the row vectors in the ICA transforming matrices are not orthogonal. In contrast, standard PCA is constrained to find orthogonal bases. The ICA can find non-orthogonal, but statistically independent bases. This is important when data is not jointly gaussian. In the following table, the rows and columns correspond to EWG, EWH, EWI, EWJ, EWM, EWS, EWW, and SPY; W1, W2, and W3 are the ICA transform matrices at the time scales one, two, and three.

## Angles between rows in W1:

ica_1:	0.0000	88.4308	56.3606	84.9723	89.2444	89.3734	87.6034	77.8292
ica_2:	0.0000	0.0000	85.6536	82.3466	83.1636	69.4914	84.4058	77.5466
ica_3:	0.0000	0.0000	0.0000	89.5518	88.9100	86.3764	82.5999	89.4090
ica_4:	0.0000	0.0000	0.0000	0.0000	86.3666	84.7373	88.4807	83.4263
ica_5:	0.0000	0.0000	0.0000	0.0000	0.0000	85.2710	87.2823	86.9241
ica_6:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	83.8283	85.4847
ica_7:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	70.5004
ica_8:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

## Angles between rows in W2:

ica_1:	0.0000	84.2577	58.0722	88.8181	83.3171	87.9383	86.9012	77.9128
ica_2:	0.0000	0.0000	85.5774	82.7123	89.5561	69.8327	83.1676	81.9882
ica_3:	0.0000	0.0000	0.0000	88.2968	89.8824	87.9124	80.0790	87.1603
ica_4:	0.0000	0.0000	0.0000	0.0000	86.5664	82.1485	88.5730	81.1303
ica_5:	0.0000	0.0000	0.0000	0.0000	0.0000	79.4425	87.3589	84.2527
ica_6:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	88.4919	83.2019
ica_7:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	71.9710
ica_8:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

## Angles between rows in W3:

ica_1:	0.0000	62.3125	59.7830	82.1021	78.6792	73.6329	85.9411	78.3034
ica_2:	0.0000	0.0000	73.4243	85.2673	84.8570	67.7267	74.0491	89.4905
ica_3:	0.0000	0.0000	0.0000	81.4942	86.6457	78.6761	70.7438	86.2667
ica_4:	0.0000	0.0000	0.0000	0.0000	86.6163	79.8826	88.5258	81.0610
ica_5:	0.0000	0.0000	0.0000	0.0000	0.0000	69.5470	82.5215	89.4862
ica_6:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	89.0245	74.3911
ica_7:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	74.2749
ica_8:	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The interactions of the various returns when the angles between the columns of the inverse matrix  $W^{-1}$  is considered are shown below. These columns are the ICA vector coordinates in the original bases of observed return rates. Small angles between ICA vectors indicate that the statistically-independent bases are *non-orthogonal* in the original space. This is typical for distributions that have nonlinear or nongaussian mixing. One possible source of such nonlinear effects are amplitude-dependent correlations.

Angles between columns in the inverse of W1:

```
-----
ica_1: 0.0000 68.9859 49.1337 73.8418 79.3477 73.3381 67.3849 62.3966
ica_2: 0.0000 0.0000 69.1139 69.3982 72.8147 57.4516 64.7464 59.6695
ica_3: 0.0000 0.0000 0.0000 76.9283 79.6551 72.3714 67.2289 67.6488
ica_4: 0.0000 0.0000 0.0000 0.0000 78.7741 72.5572 74.6275 70.1110
ica_5: 0.0000 0.0000 0.0000 0.0000 0.0000 75.2013 76.9696 75.4620
ica_6: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 67.8113 66.2344
ica_7: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 57.0080
ica_8: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
-----
```

Angles between columns in the inverse of W2:

```
-----
ica_1: 0.0000 64.5458 49.2033 76.2840 71.0124 68.3716 64.8620 61.5095
ica_2: 0.0000 0.0000 67.4179 69.5328 73.4154 57.1678 65.6847 62.1824
ica_3: 0.0000 0.0000 0.0000 77.3835 75.6029 71.1880 65.2379 69.0002
ica_4: 0.0000 0.0000 0.0000 0.0000 76.8591 69.6098 75.0900 69.2440
ica_5: 0.0000 0.0000 0.0000 0.0000 0.0000 68.9135 74.4195 70.5165
ica_6: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 69.9505 64.4660
ica_7: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 58.8044
ica_8: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
-----
```

Angles between columns in the inverse of W3:

```
-----
ica_1: 0.0000 58.0201 56.6721 74.6937 68.0712 74.2880 65.0127 65.0953
ica_2: 0.0000 0.0000 78.4805 69.3031 73.0707 58.7631 61.2547 64.3882
ica_3: 0.0000 0.0000 0.0000 87.1714 69.0328 72.9510 62.1627 73.7956
ica_4: 0.0000 0.0000 0.0000 0.0000 81.6845 69.9269 76.3330 69.1869
ica_5: 0.0000 0.0000 0.0000 0.0000 0.0000 61.6375 67.2984 71.4262
ica_6: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 64.0512 60.2717
ica_7: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 59.9826
ica_8: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
-----
```

It is important to note that the independent components found vary with time scale. The presence of a “term structure of returns interactions” has implications for both trading and risk management. Part of this term structure may reflect geographical or time zone effects.

### 3.3 Stability of ICA outputs for each oscillator

We also performed some stability study on each oscillator. This is done by dividing the corresponding data set for each oscillator in three parts and performing ICA on each one of the resulting set. Stability results for Index EWH are show from Fig. 20 to Fig. 23.

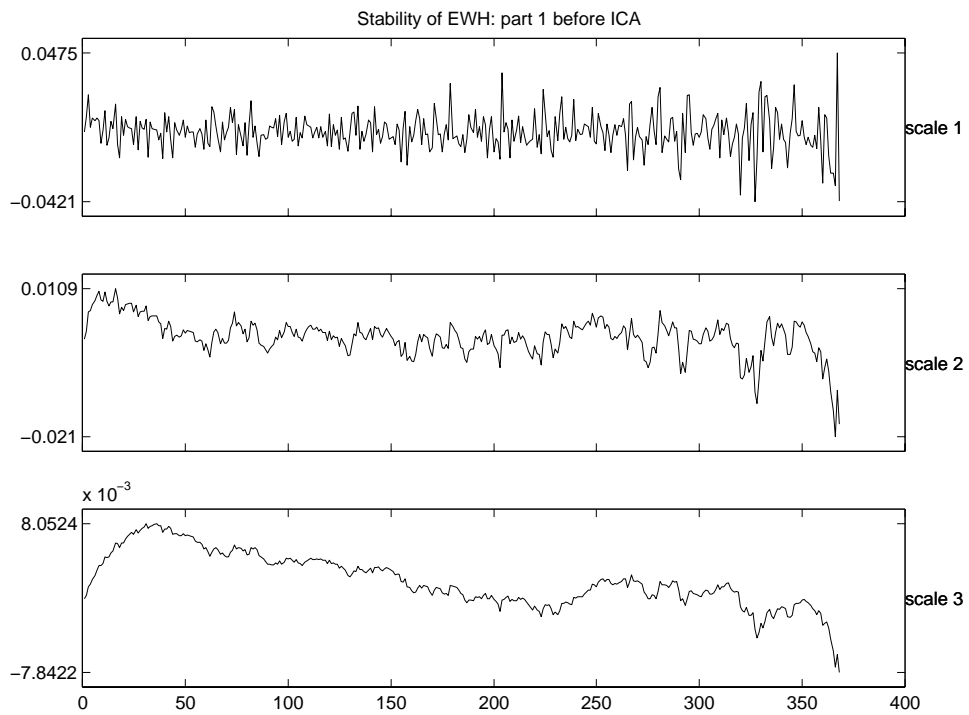


Figure 20:

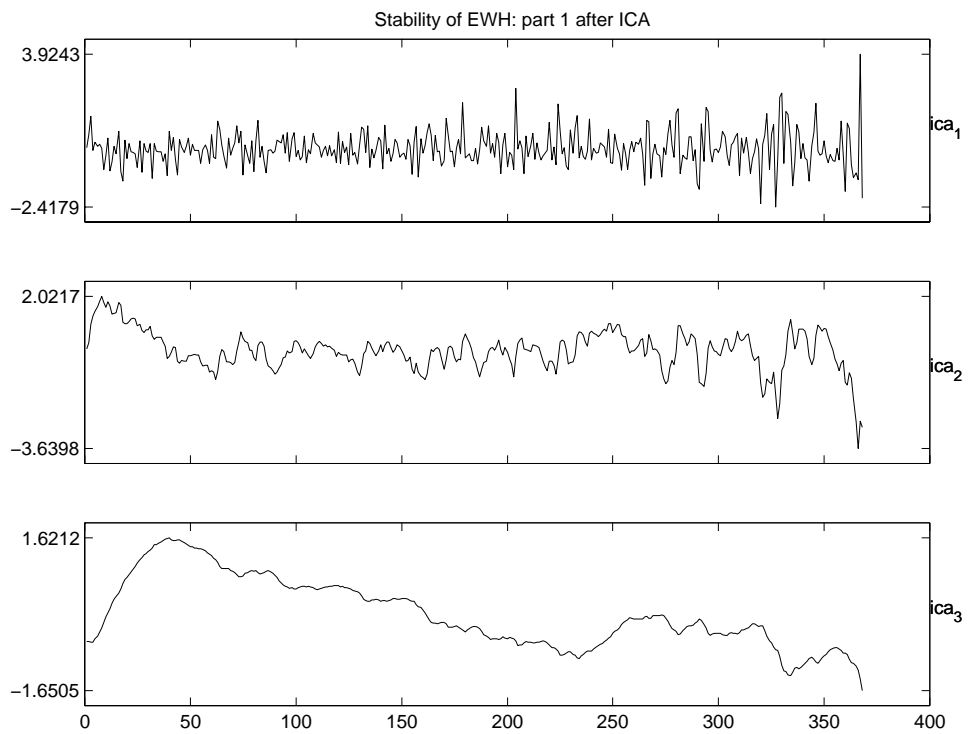


Figure 21:

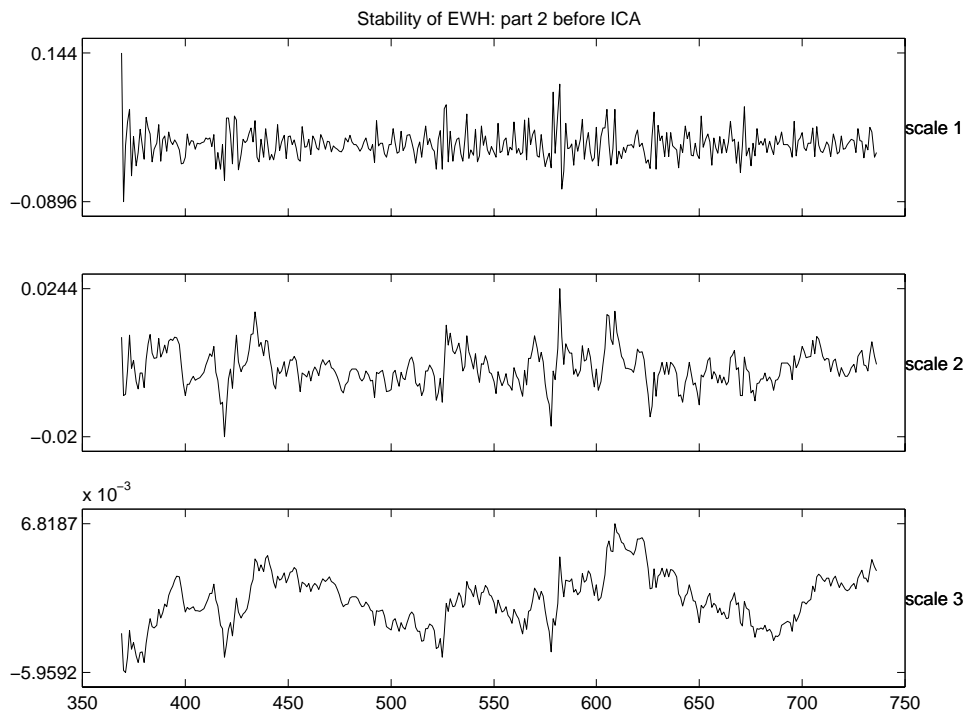


Figure 22:

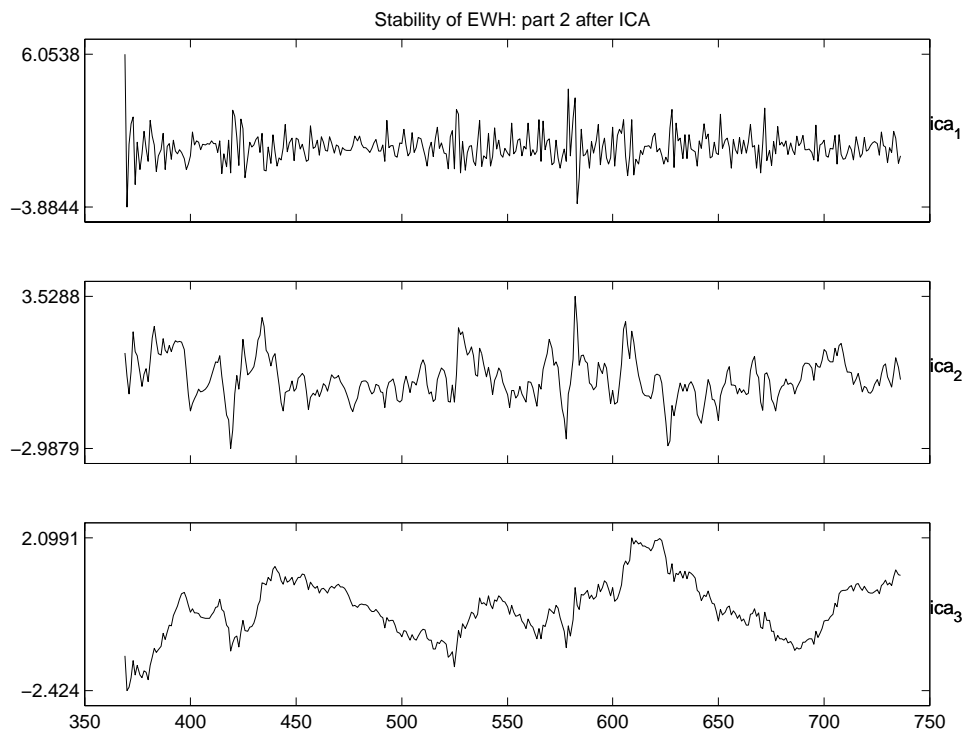


Figure 23:

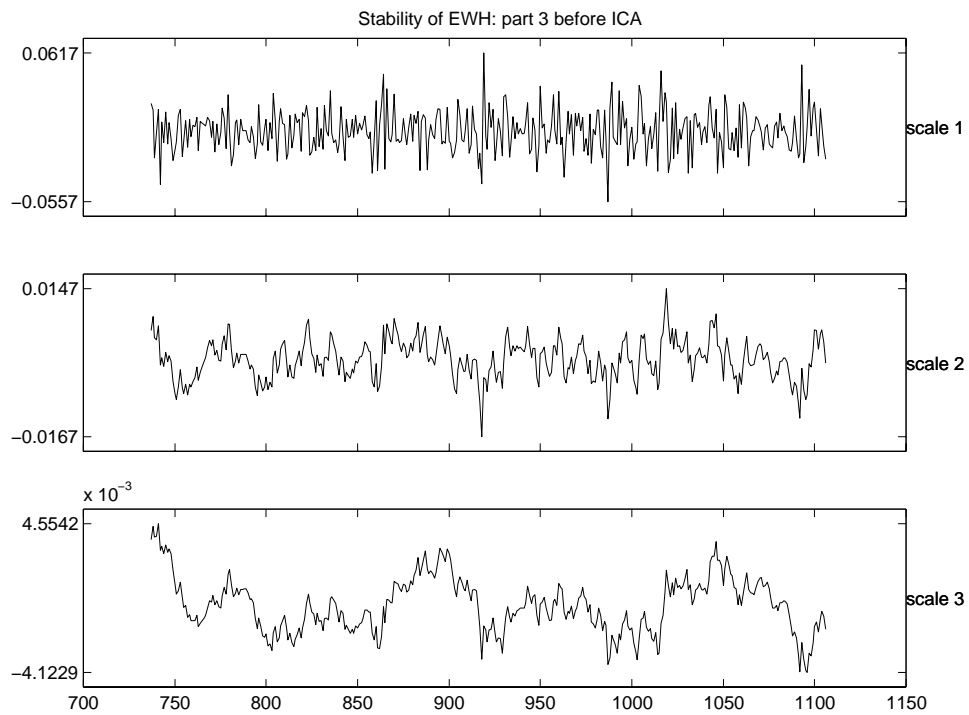


Figure 24:

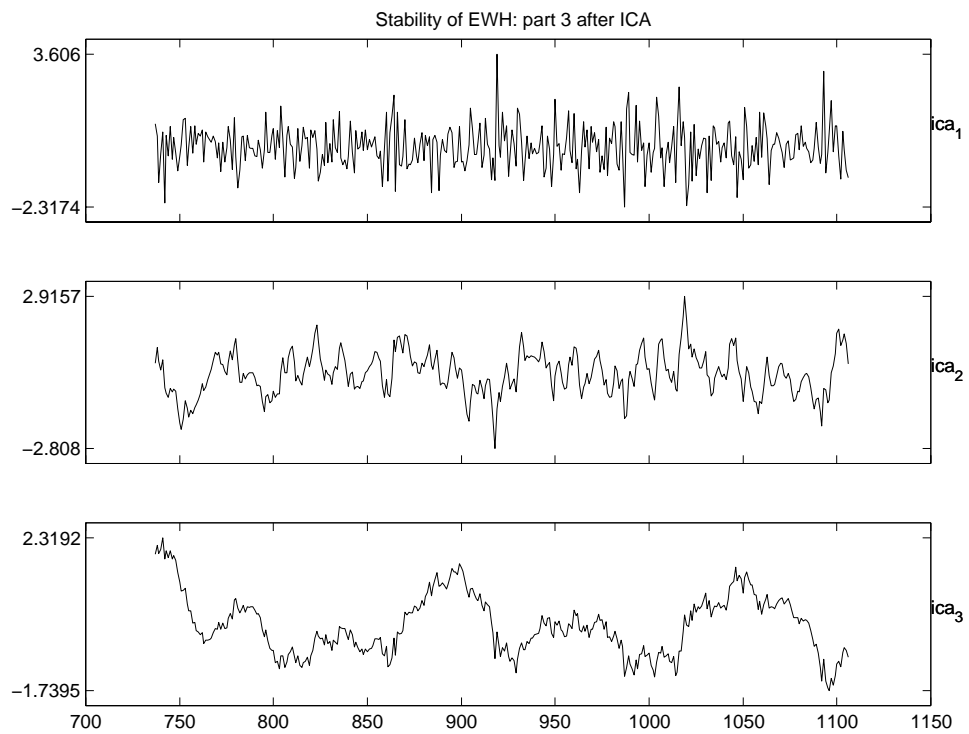


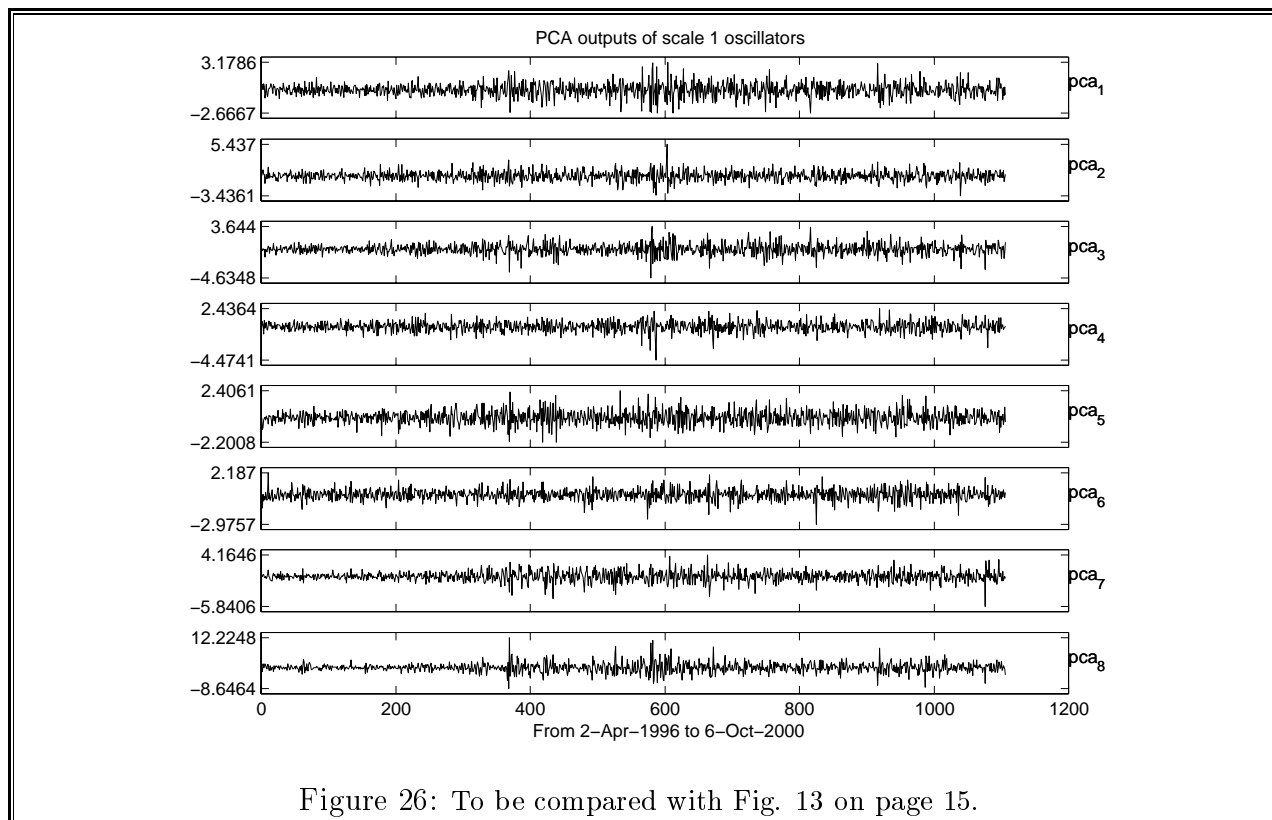
Figure 25:

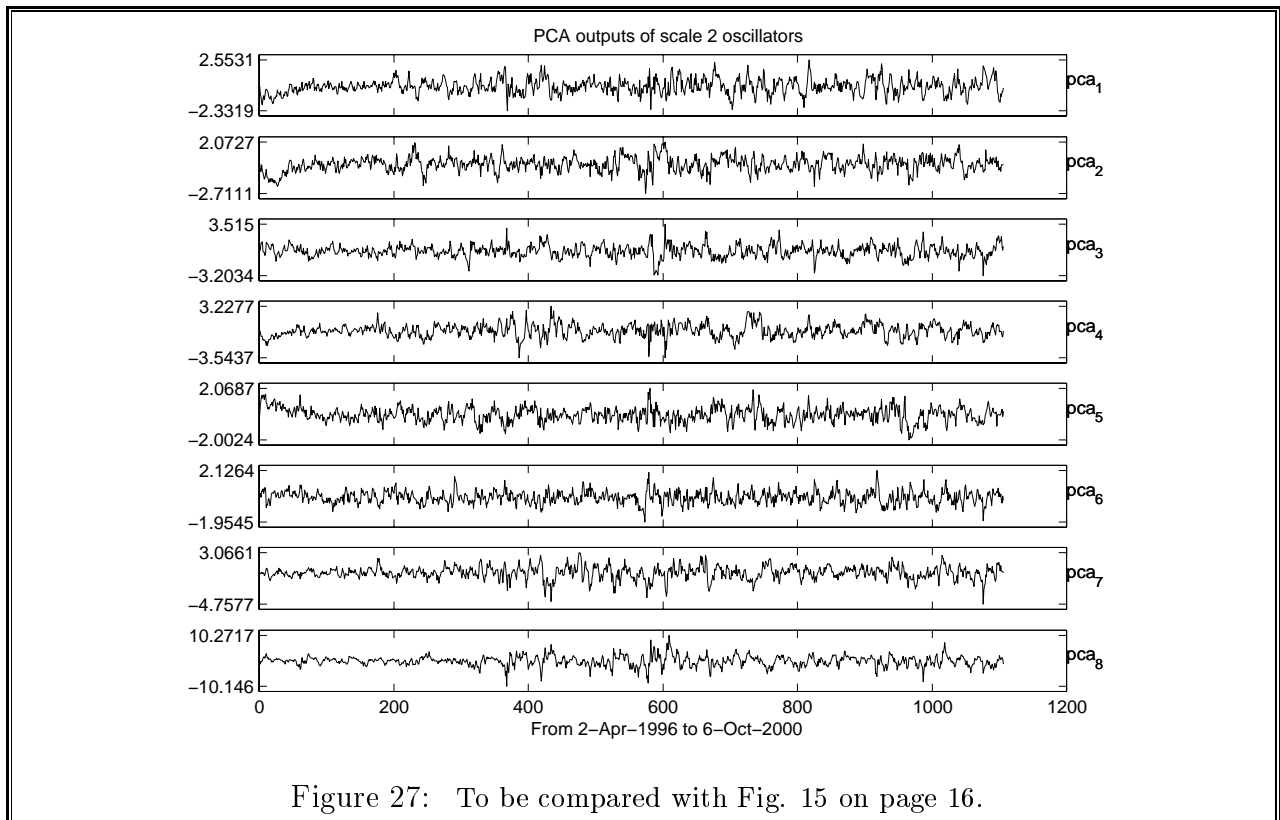
## 4 Principal Component Analysis (PCA) Results

In this section we present some results obtained after performing PCA in the data set.

### 4.1 Performing PCA on each oscillator

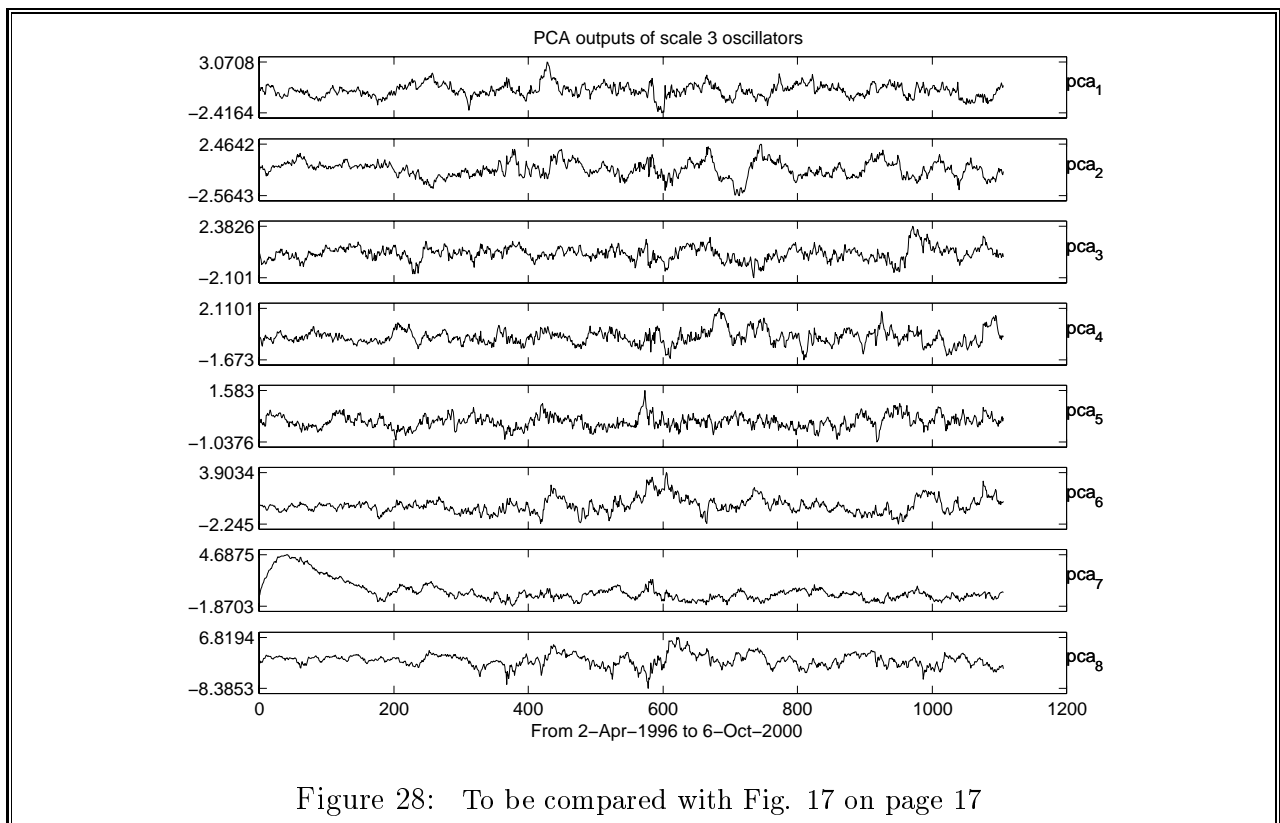
The results of performing PCA on each oscillator are shown from Fig. 26 to 28. On Fig. 29 and 30 we show the second order statistics and high order statistics before and after PCA.





## 4.2 PCA transforming Matrix

The following table shows PCA transforming matrix and its normalization.



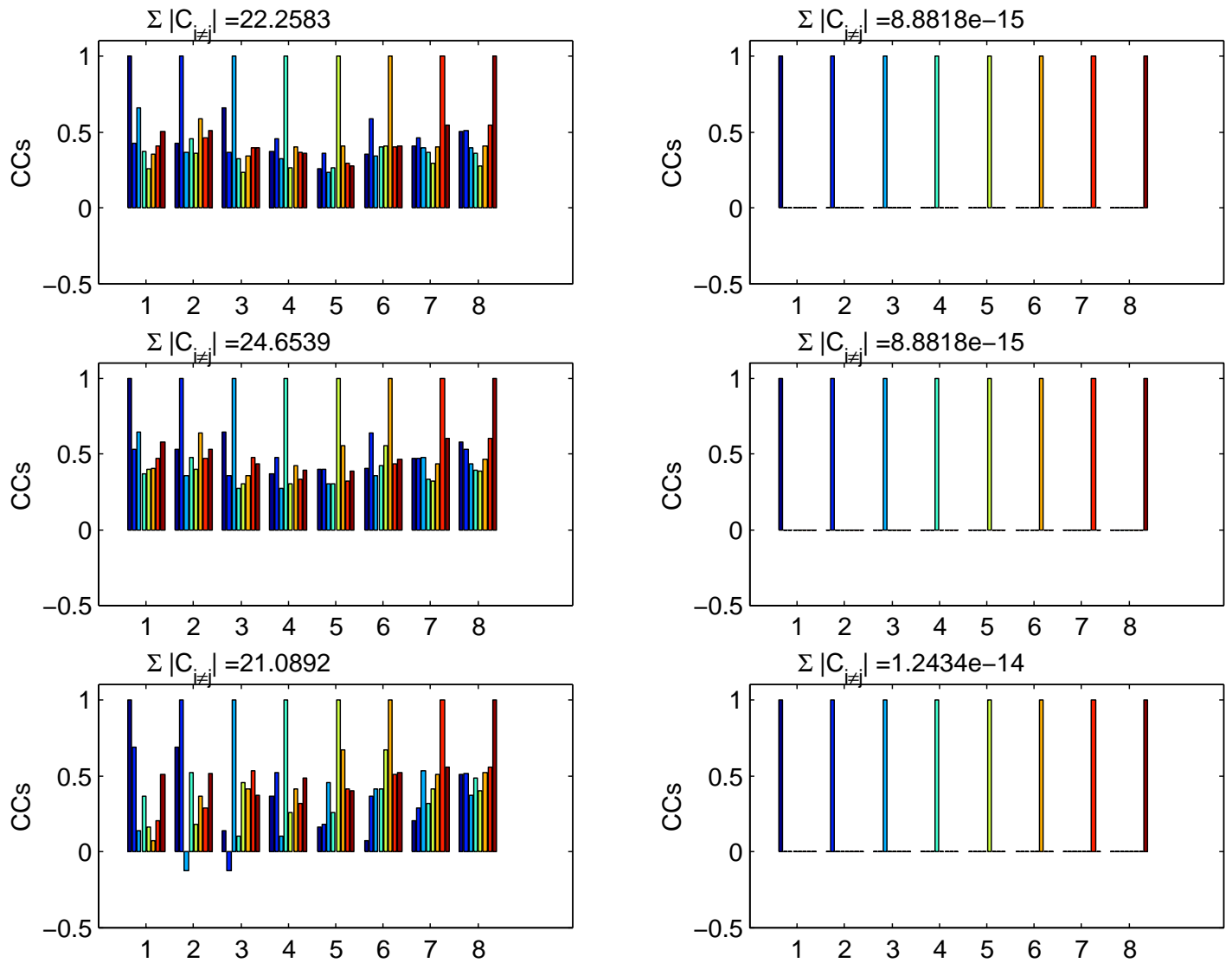


Figure 29: To be compared with Fig. 18 on page 18

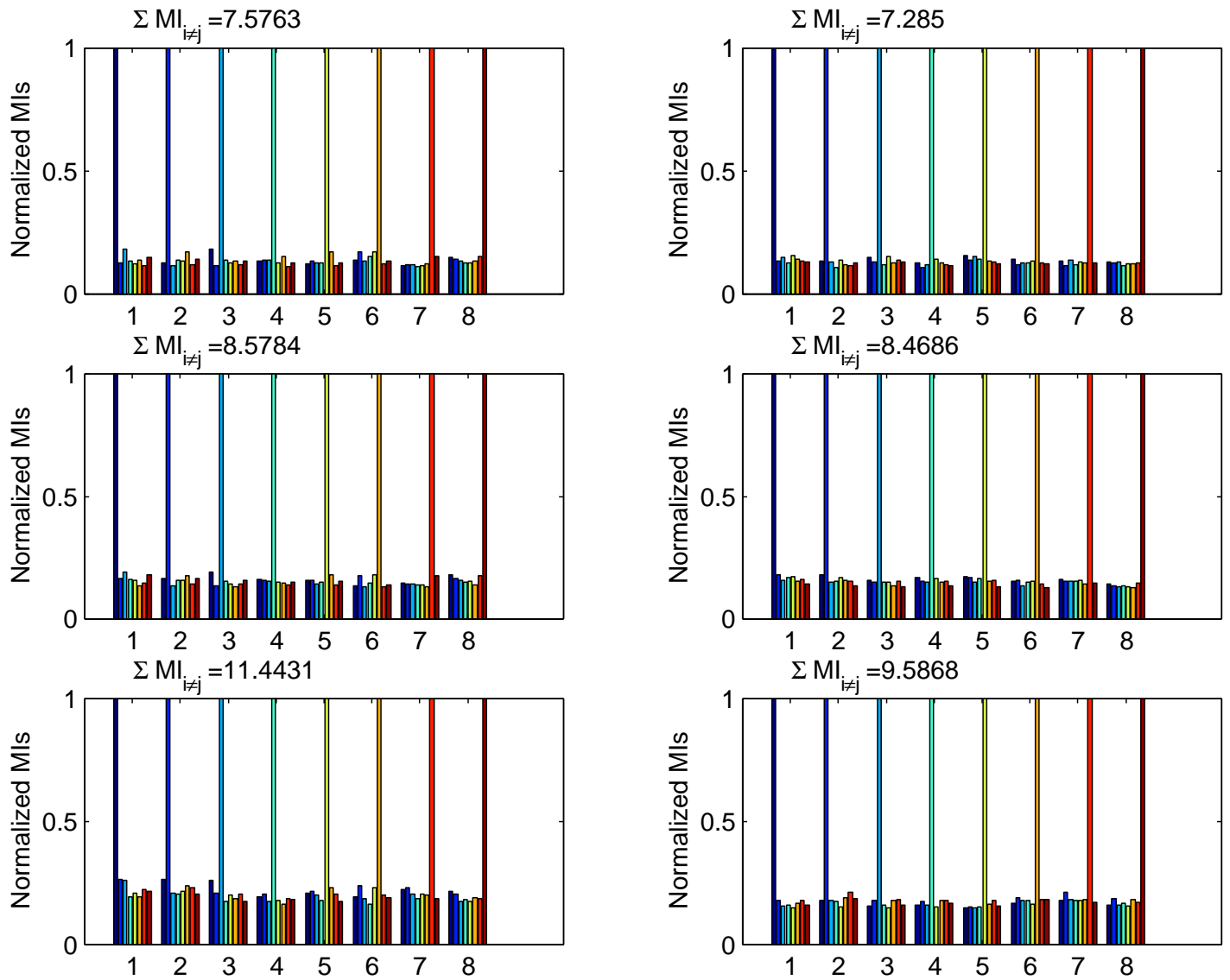


Figure 30: To be compared with Fig. 19 on page 19

## Matrix V1

```

-----
pca_1:  0.0497   0.4057   0.0962  -0.4981  -0.3831   0.5566  -0.3386  -0.0659
pca_2:  0.0856   0.0673   0.1201   0.7368  -0.2205   0.1043  -0.4571  -0.4030
pca_3:  0.2502  -0.1939   0.3732  -0.2706   0.6754   0.0261  -0.3485  -0.3302
pca_4:  0.2106   0.1702  -0.2779   0.0340   0.1286  -0.2793  -0.6176   0.6102
pca_5:  0.0105  -0.7403  -0.0478   0.0925  -0.0658   0.5586  -0.1078   0.3363
pca_6: -0.7137   0.0474   0.6043   0.0506   0.0210  -0.0852  -0.1708   0.2893
pca_7:  0.4840  -0.2445   0.5217  -0.1325  -0.5036  -0.3860   0.0350   0.1110
pca_8:  0.3734   0.3928   0.3465   0.3256   0.2712   0.3638   0.3627   0.3778
-----

```

## Matrix V2

```

-----
pca_1: -0.1514  -0.6170  -0.0613   0.4095   0.4523  -0.3262   0.2774   0.1925
pca_2: -0.4076  -0.1381   0.4186   0.1108  -0.1375   0.3642   0.4275  -0.5414
pca_3:  0.3373  -0.0768   0.3868   0.4586   0.1028  -0.0979  -0.5860  -0.3961
pca_4:  0.0834  -0.1923   0.1968  -0.6498   0.6520   0.1857  -0.1292  -0.1372
pca_5:  0.2552   0.3863  -0.2375  -0.0193   0.2360  -0.4716   0.4060  -0.5348
pca_6:  0.5908  -0.4464  -0.4173   0.0153  -0.1944   0.4211   0.1600  -0.1914
pca_7:  0.3606  -0.2479   0.5408  -0.3113  -0.3891  -0.4249   0.2502   0.1569
pca_8:  0.3822   0.3840   0.3311   0.3002   0.3097   0.3697   0.3566   0.3833
-----

```

## Matrix V3

```

-----
pca_1:  0.1729  -0.3297   0.3782   0.6488   0.1276  -0.1935  -0.4807  -0.1127
pca_2:  0.3917   0.1238   0.0025  -0.4990   0.5881   0.0814  -0.4658  -0.1127
pca_3: -0.1593  -0.2219  -0.0702  -0.1364  -0.1620   0.1220  -0.4294   0.8224
pca_4: -0.0060  -0.2970  -0.4351   0.1064   0.5190  -0.5414   0.3023   0.2395
pca_5: -0.5216   0.5912   0.3286  -0.0139   0.1748  -0.4535  -0.1495   0.1078
pca_6: -0.4672   0.1199  -0.4996   0.3473   0.2894   0.4695  -0.2439  -0.1831
pca_7:  0.4627   0.5120  -0.4807   0.2533  -0.3269  -0.2523  -0.2259   0.0839
pca_8:  0.2947   0.3354   0.2722   0.3394   0.3480   0.3978   0.3786   0.4343
-----

```

## Matrix V1\_Normalized

```

-----
pca_1:  0.0497   0.4057   0.0962  -0.4981  -0.3831   0.5566  -0.3386  -0.0659
pca_2:  0.0856   0.0673   0.1201   0.7368  -0.2205   0.1043  -0.4571  -0.4030
pca_3:  0.2502  -0.1939   0.3732  -0.2706   0.6754   0.0261  -0.3485  -0.3302
pca_4:  0.2106   0.1702  -0.2779   0.0340   0.1286  -0.2793  -0.6176   0.6102
pca_5:  0.0105  -0.7403  -0.0478   0.0925  -0.0658   0.5586  -0.1078   0.3363
pca_6: -0.7137   0.0474   0.6043   0.0506   0.0210  -0.0852  -0.1708   0.2893
pca_7:  0.4840  -0.2445   0.5217  -0.1325  -0.5036  -0.3860   0.0350   0.1110
pca_8:  0.3734   0.3928   0.3465   0.3256   0.2712   0.3638   0.3627   0.3778
-----

```

## Matrix V2\_Normalized

```

-----
pca_1: -0.1514  -0.6170  -0.0613   0.4095   0.4523  -0.3262   0.2774   0.1925
pca_2: -0.4076  -0.1381   0.4186   0.1108  -0.1375   0.3642   0.4275  -0.5414
pca_3:  0.3373  -0.0768   0.3868   0.4586   0.1028  -0.0979  -0.5860  -0.3961
pca_4:  0.0834  -0.1923   0.1968  -0.6498   0.6520   0.1857  -0.1292  -0.1372
pca_5:  0.2552   0.3863  -0.2375  -0.0193   0.2360  -0.4716   0.4060  -0.5348
pca_6:  0.5908  -0.4464  -0.4173   0.0153  -0.1944   0.4211   0.1600  -0.1914
pca_7:  0.3606  -0.2479   0.5408  -0.3113  -0.3891  -0.4249   0.2502   0.1569
pca_8:  0.3822   0.3840   0.3311   0.3002   0.3097   0.3697   0.3566   0.3833
-----

```

## Matrix V3\_Normalized

```

-----
pca_1:  0.1729  -0.3297   0.3782   0.6488   0.1276  -0.1935  -0.4807  -0.1127
pca_2:  0.3917   0.1238   0.0025  -0.4990   0.5881   0.0814  -0.4658  -0.1127
pca_3: -0.1593  -0.2219  -0.0702  -0.1364  -0.1620   0.1220  -0.4294   0.8224
pca_4: -0.0060  -0.2970  -0.4351   0.1064   0.5190  -0.5414   0.3023   0.2395
pca_5: -0.5216   0.5912   0.3286  -0.0139   0.1748  -0.4535  -0.1495   0.1078
pca_6: -0.4672   0.1199  -0.4996   0.3473   0.2894   0.4695  -0.2439  -0.1831
pca_7:  0.4627   0.5120  -0.4807   0.2533  -0.3269  -0.2523  -0.2259   0.0839
pca_8:  0.2947   0.3354   0.2722   0.3394   0.3480   0.3978   0.3786   0.4343
-----

```

### 4.3 Angles between rows in the PCA transforming Matrix

The acute angles between row vectors in each PCA transforming matrix are shown below. As expected they are 90 degrees angles.

Angles between rows in V1:

```
-----
pca_1: 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_2: 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_3: 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_4: 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000
pca_5: 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000
pca_6: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000
pca_7: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000
pca_8: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
-----
```

Angles between rows in V2:

```
-----
pca_1: 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_2: 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_3: 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_4: 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000
pca_5: 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000
pca_6: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000
pca_7: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000
pca_8: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
-----
```

Angles between rows in V3:

```
-----
pca_1: 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_2: 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_3: 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000 90.0000
pca_4: 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000 90.0000
pca_5: 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000 90.0000
pca_6: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000 90.0000
pca_7: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 90.0000
pca_8: 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
-----
```

## 5 Results of ICA on returns of each Index

In this section we present results obtained after performing ICA directly on the returns of each Index. Let's start with the results of performing ICA algorithm directly on the return series shown in Fig. 2 on page 4. The corresponding ICA decomposition is shown in Fig. 31. Next we show the mixing matrix and the corresponding angles:

### Matrix W

```
-----
ica_1:  1.0597  -0.0643  -0.3151  -0.0378  -0.0151  -0.0196  -0.0617  -0.1462
ica_2:  -0.0130   1.0670  -0.0866  -0.0832  -0.0582  -0.2036  -0.0808  -0.1654
ica_3:  -0.3646  -0.0328   1.0361   0.0011  -0.0257  -0.0325  -0.0861  -0.0425
ica_4:  -0.0762  -0.1012  -0.0249   1.0231  -0.0345  -0.0630  -0.0388  -0.1179
ica_5:  -0.0334  -0.0902  -0.0117  -0.0535   0.9796  -0.0649  -0.0584  -0.0743
ica_6:  -0.0253  -0.2497  -0.0680  -0.0850  -0.0607   1.1231  -0.1007  -0.0618
ica_7:  -0.0665  -0.0970  -0.0878  -0.0355  -0.0305  -0.0569   1.0230  -0.1837
ica_8:  -0.1445  -0.1194  -0.0112  -0.0554  -0.0090  -0.0858  -0.2080   1.0394
-----
```

### Matrix W Normalized

```
-----
ica_1:  0.9465  -0.0575  -0.2815  -0.0337  -0.0134  -0.0175  -0.0551  -0.1306
ica_2:  -0.0117   0.9614  -0.0780  -0.0749  -0.0524  -0.1834  -0.0728  -0.1490
ica_3:  -0.3303  -0.0297   0.9386   0.0010  -0.0233  -0.0295  -0.0780  -0.0385
ica_4:  -0.0732  -0.0972  -0.0239   0.9827  -0.0331  -0.0605  -0.0373  -0.1133
ica_5:  -0.0336  -0.0909  -0.0118  -0.0539   0.9870  -0.0654  -0.0588  -0.0748
ica_6:  -0.0217  -0.2146  -0.0585  -0.0731  -0.0522   0.9652  -0.0866  -0.0531
ica_7:  -0.0632  -0.0922  -0.0834  -0.0338  -0.0290  -0.0540   0.9722  -0.1746
ica_8:  -0.1337  -0.1105  -0.0104  -0.0513  -0.0084  -0.0793  -0.1923   0.9612
-----
```

Angles between rows of W

```

-----
ica_1:  0.0000  89.1709  55.5987  85.8861  88.8206  89.6457  86.5974  76.7662
ica_2:  0.0000   0.0000  85.4137  82.4182  83.8937  69.1378  83.5580  77.6297
ica_3:  0.0000   0.0000   0.0000  89.1138  89.3760  86.5697  83.0174  88.9570
ica_4:  0.0000   0.0000   0.0000   0.0000  86.5933  84.5255  88.2896  82.7471
ica_5:  0.0000   0.0000   0.0000   0.0000   0.0000  85.3752  86.7944  87.3476
ica_6:  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000  84.4316  85.4353
ica_7:  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000  70.7904
ica_8:  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
-----

```

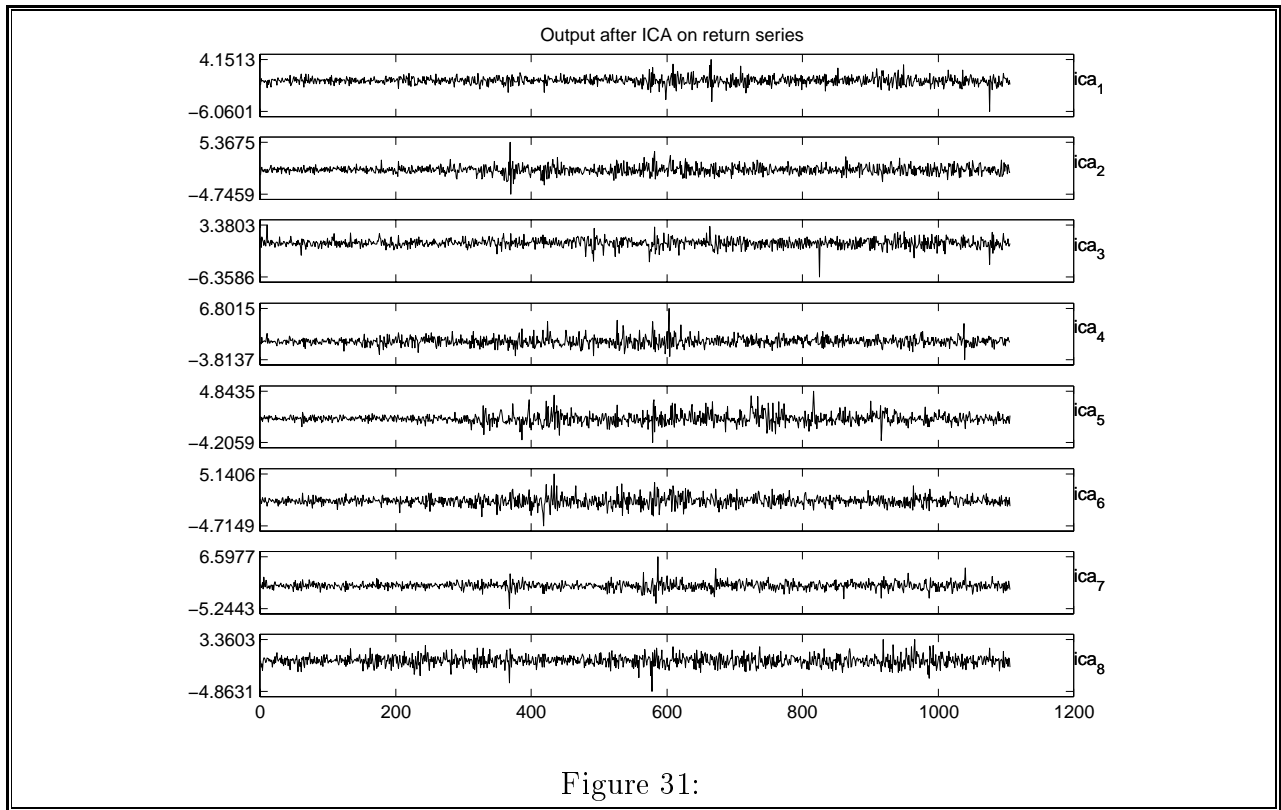
Angles between columns in the inverse of W

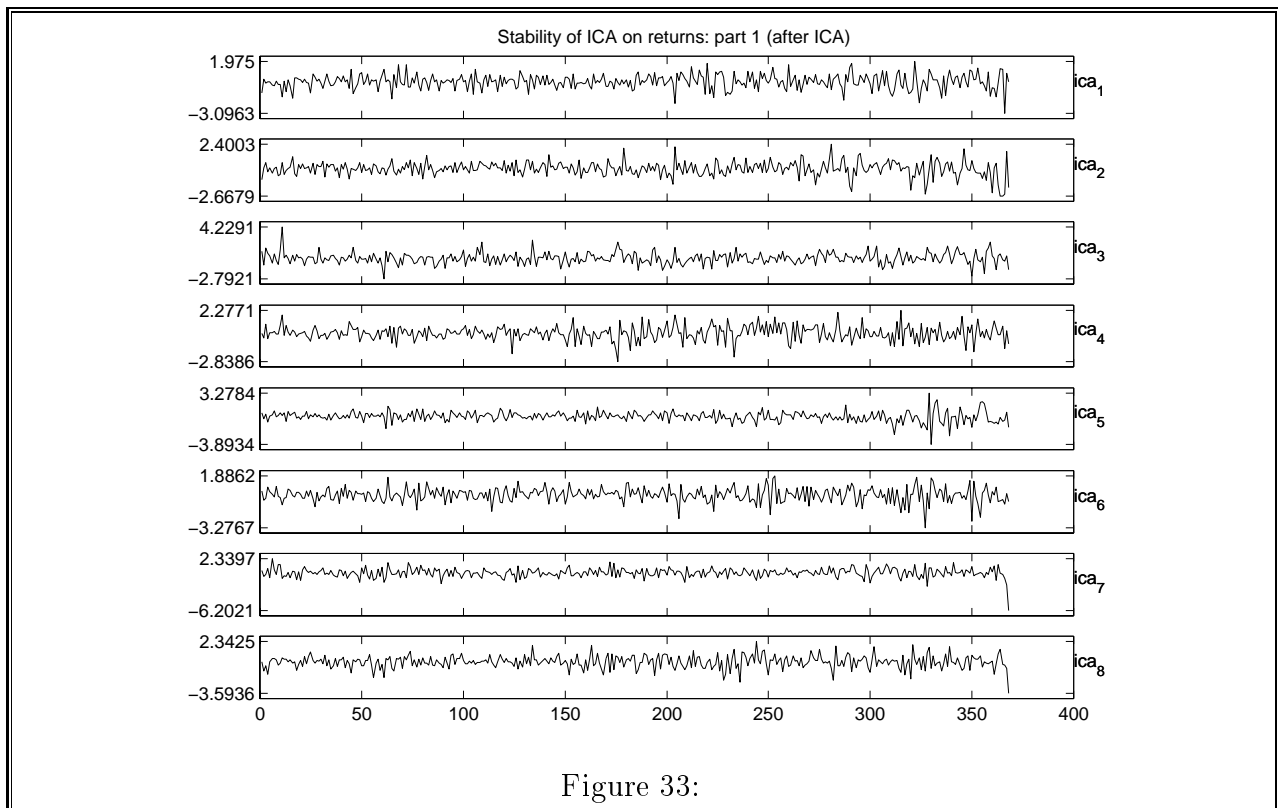
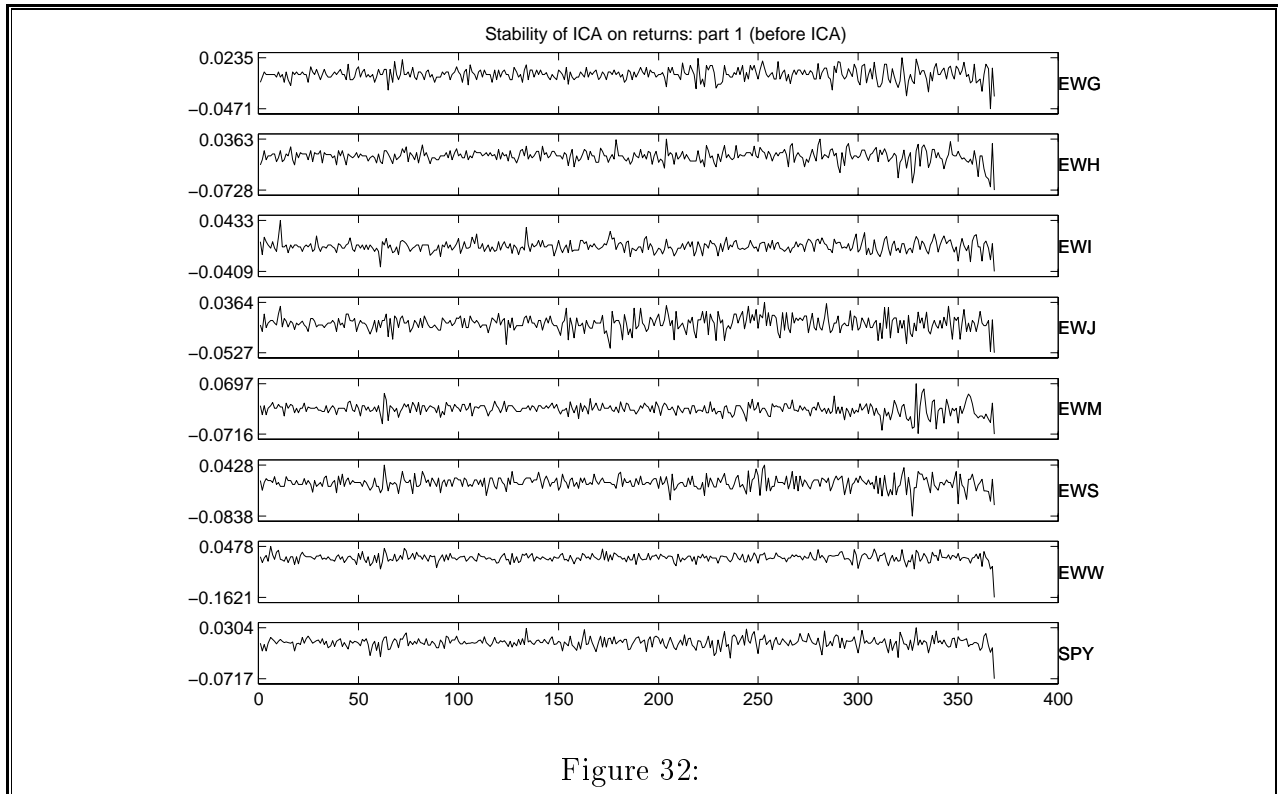
```

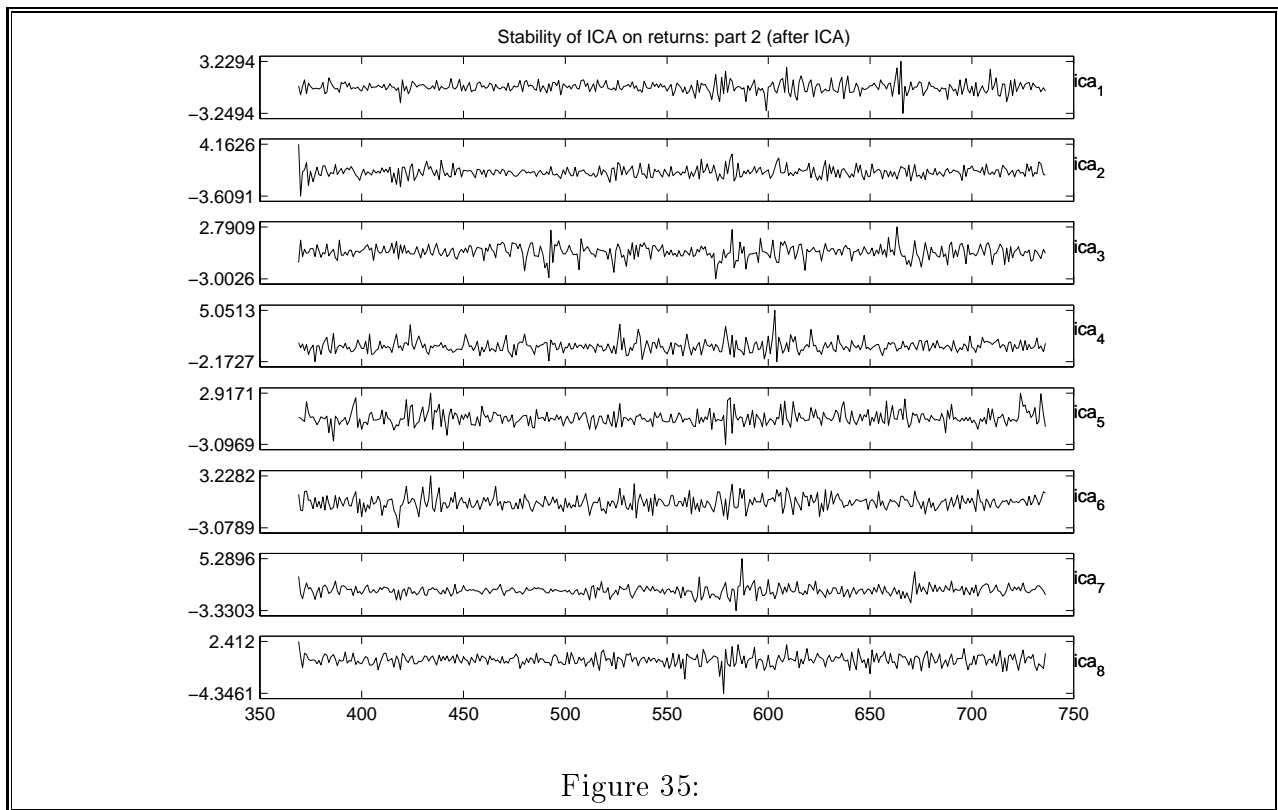
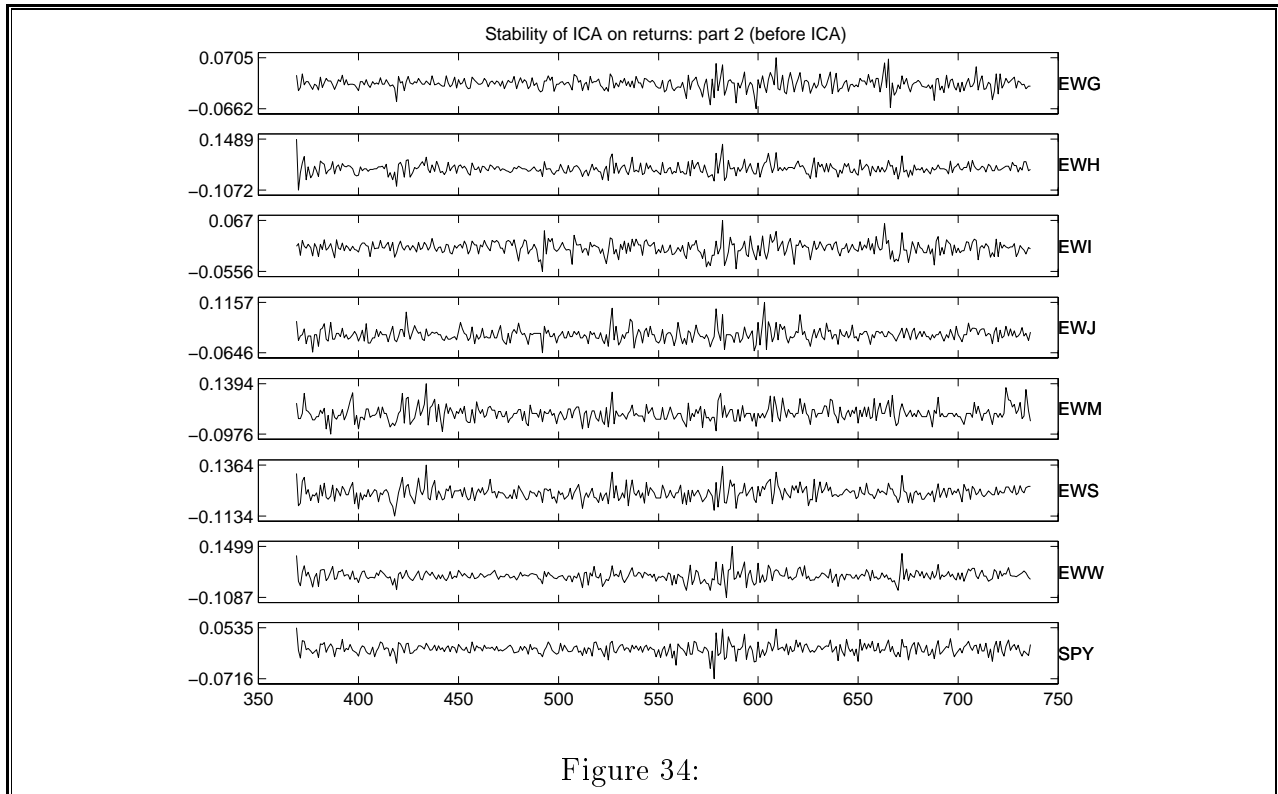
-----
ica_1:  0.0000  68.5629  48.1273  74.3154  79.1828  72.6614  65.5839  60.8292
ica_2:  0.0000   0.0000  68.9034  69.2098  73.5825  56.9729  63.6392  59.1466
ica_3:  0.0000   0.0000   0.0000  77.4983  80.0436  72.2549  66.5163  67.0386
ica_4:  0.0000   0.0000   0.0000   0.0000  79.1464  72.1907  73.9918  69.4316
ica_5:  0.0000   0.0000   0.0000   0.0000   0.0000  75.6078  76.7143  75.8261
ica_6:  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000  67.5120  65.7954
ica_7:  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000  56.2151
ica_8:  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
-----

```

Stability plots are shown from Fig. 32 to Fig. 37







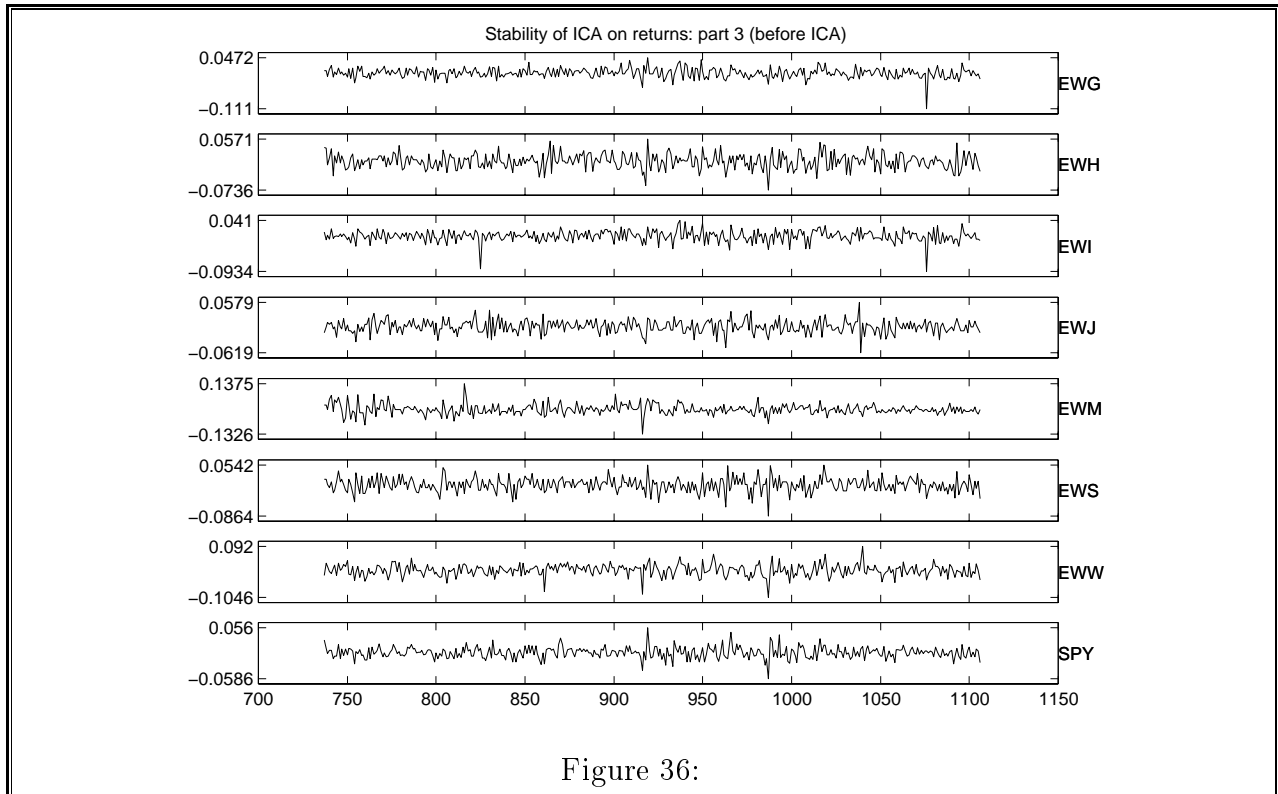


Figure 36:

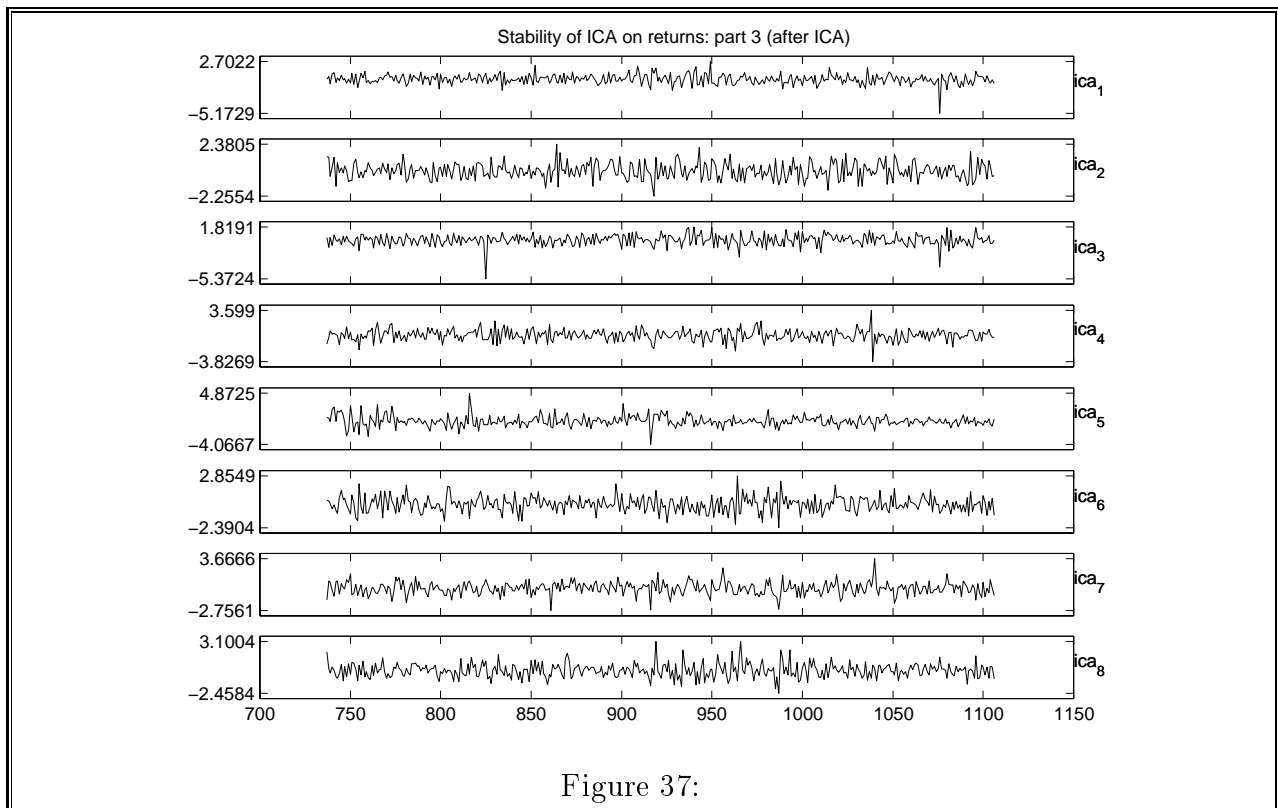


Figure 37:

## 6 Summary

A major result of this work is that care should be taken when using ICA algorithms. One need to somehow ensure that the optimal learning rate ( $\eta$ ) is used in order to have results according to the aim of ICA: finding a set of basis with minimum statistical dependency. As shown in section 2.4.2 on page 8, residual average correlation and average mutual information are highly dependent on the choice of the learning rate. In addition, when using the MSD decomposition (page 3), minimum values of the average correlation and average mutual information seems to occur at different optimal learning rates as the frequency is increased.

As expected, PCA has little influence on the average mutual information. The amount of average mutual information among the PCA output are slightly small than the amount of average mutual information among the input oscillators before performing PCA on them. Results seems to reveal similar behavior for ICA when the frequency of the oscillators increase. This result may be of interest to those performing ICA analysis on medicine (namely ICA analysis on ERP, fMRI, EEG, MEG, etc. data).

## 7 Future Work

- It is necessary to verify the statistical significance of the difference between average mutual information between the multivariate components given as input to ICA and between the corresponding output from ICA. This is a very difficult and computing intensive task. Bootstrapping could be a nice starting point. Another approach would be to use as input to ICA, data containing high values of average mutual information.
- More analysis comparing ICA and PCA is required, particularly when using both methods as a way of reducing or eliminating statistical redundancy between the components of high-dimensional data. This endeavor will be highly dependent on the criterion (basically distanced measures) adopted to sort and analyze the output. This type of analysis is very important when doing reconstruction of the of the original data from a few components of the ICA(PCA) outputs.
- It would be of profit to compare the performance of non-linear PCA algorithms in relation to linear ICA analysis.
- In this work we analyzed returns representing exponential grow, namely  $r(t) = \ln \frac{p(t)}{p(t-1)}$ , it would be interesting to perform similar study base on returns representing arithmetic grow (i.e.  $r(t) = \frac{p(t)}{p(t-1)}$ ).

## Acknowledgments

We are grateful to Dr. Howard Yang for very helpful and illuminating discussion, and also to Matthew Saffell for very helpful comments.

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## Appendix A

Here are the names of the iShares MSCI Index Funds studied in this report:

**EWG iShares MSCI Germany Index Fund**

**EWH iShares MSCI Hong Kong Index Fund**

**EWI iShares MSCI Italy Index Fund**

**EWJ iShares MSCI Japan Index Fund**

**EWM iShares MSCI Malaysia Index Fund**

**EWS iShares MSCI Singapore Index Fund**

**EWV iShares MSCI Mexico Index Fund**

**SPY Standard and Poor's Depository Receipts**

## Appendix B

The following tables summarize statistical tests that were performed on the data before ICA(PCA) to ensure none of them were normally distributed. The meaning of the “basic statistics” summary is obvious. In relation to the Lilliefors and Kolmogorov-Smirnov tests, the parameter “H” is the result of the hypothesis test: H equal to one means to reject the hypothesis that the tested data comes from a normal distribution. The p-value of the test is represented by the parameter “p”. The hypothesis of normality is rejected by both tests. See the Statistics Toolbox manual [11] for more details.

Statistical summary of  $r = \ln\left(\frac{p_t}{p_{t-1}}\right)$  for each Index.

### Descriptive Statistics

	EWG	EWH	EWI	EWJ	EWM	EWS	EWV	SPY
kurtosis	9.0004	7.9353	7.7407	7.0488	6.2979	6.6120	9.3227	6.7672
Skewness	-0.4303	0.3969	-0.6048	0.5633	0.5547	0.3362	-0.1626	-0.2653
mean	0.0002	-0.0001	0.0002	-0.0002	-0.0007	-0.0006	0.0002	0.0007
std	0.0140	0.0208	0.0136	0.0177	0.0273	0.0237	0.0224	0.0122

### Lilliefors test

	EWG	EWH	EWI	EWJ	EWM	EWS	EWV	SPY
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
p-value	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
LSTAT	0.0686	0.0665	0.0626	0.0715	0.1247	0.0871	0.0616	0.0495
CV	0.0332	0.0332	0.0332	0.0332	0.0332	0.0332	0.0332	0.0332

### Kolmogorov-Smirnov test

	EWG	EWH	EWI	EWJ	EWM	EWS	EWV	SPY
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LSTAT	0.4775	0.4705	0.4809	0.4753	0.4651	0.4667	0.4690	0.4811
CV	0.0488	0.0488	0.0488	0.0488	0.0488	0.0488	0.0488	0.0488

Statistical summary of  $r = \ln\left(\frac{p(t)}{p(t-1)}\right)$  for each Index, after transforming  $r$  to have zero mean ( $\mu = 0$ ) and unit variance ( $\sigma^2 = 1$ ).

-----  
Descriptive Statistics  
-----

	EWG	EWH	EWI	EWJ	EWM	EWS	EWV	SPY
kurtosis	9.0004	7.9353	7.7407	7.0488	6.2979	6.6120	9.3227	6.7672
Skewness	-0.4303	0.3969	-0.6048	0.5633	0.5547	0.3362	-0.1626	-0.2653
mean	-0.0000	0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0000	-0.0000
std	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

-----  
Lilliefors test  
-----

	EWG	EWH	EWI	EWJ	EWM	EWS	EWV	SPY
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
p-value	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
LSTAT	0.0686	0.0665	0.0626	0.0715	0.1247	0.0871	0.0616	0.0495
CV	0.0332	0.0332	0.0332	0.0332	0.0332	0.0332	0.0332	0.0332

-----  
Kolmogorov-Smirnov test  
-----

	EWG	EWH	EWI	EWJ	EWM	EWS	EWV	SPY
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
p-value	0.0001	0.0001	0.0003	0.0000	0.0000	0.0000	0.0004	0.0086
LSTAT	0.0686	0.0665	0.0626	0.0715	0.1247	0.0871	0.0616	0.0495
CV	0.0488	0.0488	0.0488	0.0488	0.0488	0.0488	0.0488	0.0488

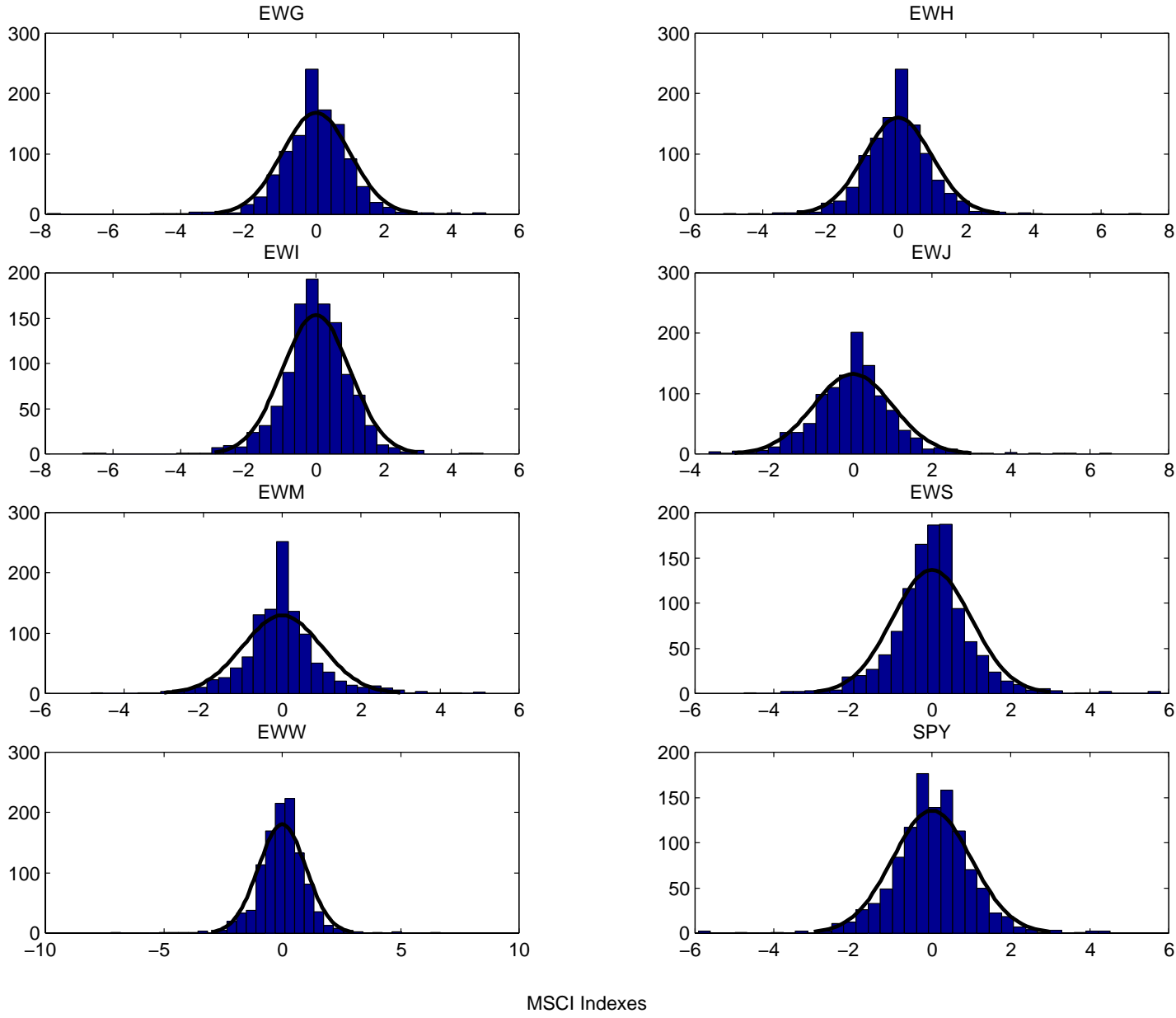


Figure 38:

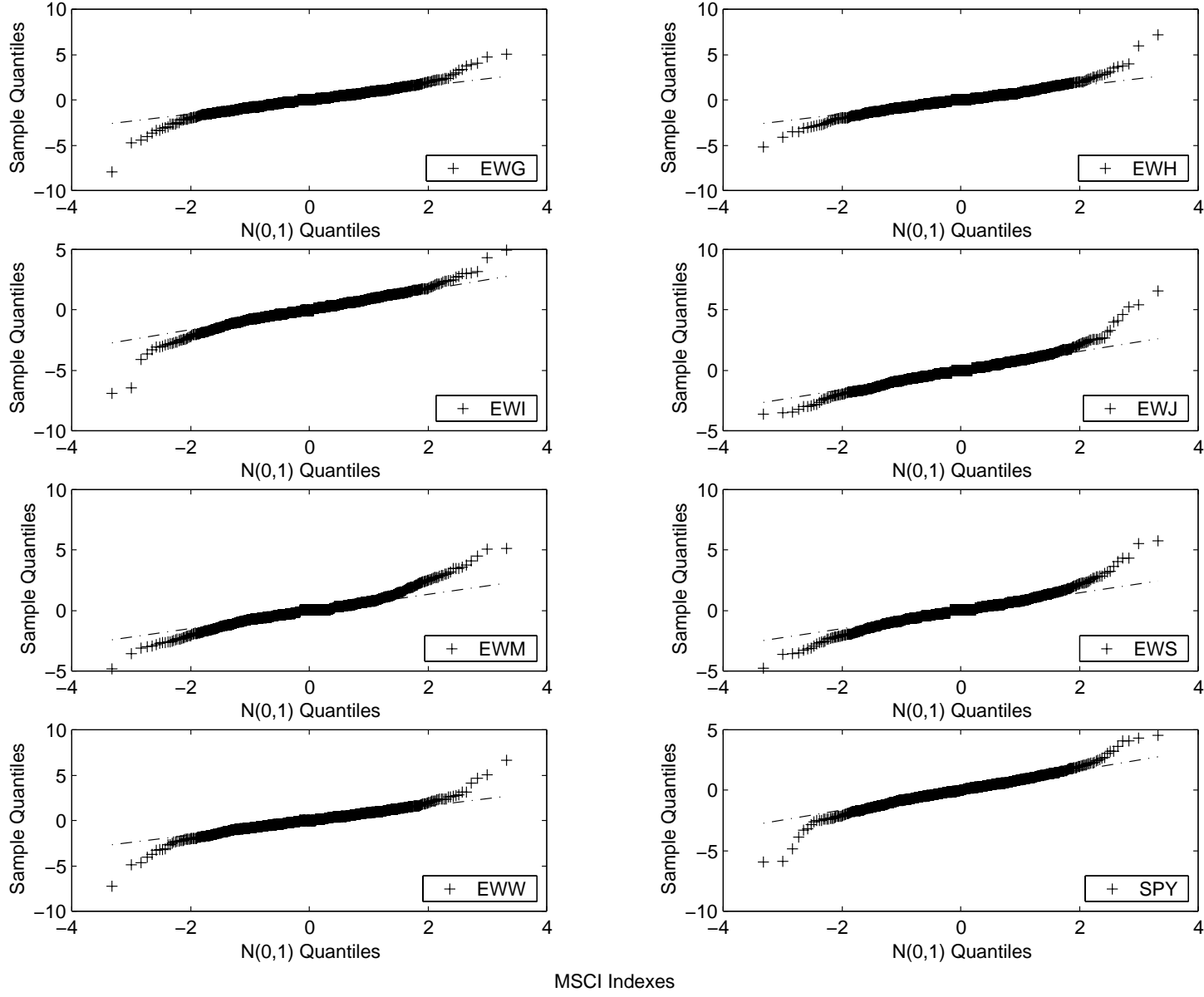


Figure 39:

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