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Economic and financial prediction using rough sets model

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Abstract

A state-of-the-art review of the literature related to economic and financial prediction using rough sets model is presented, with special emphasis on the business failure prediction, database marketing and financial investment. These three applications require the accurate prediction of the future states based on the identification of patterns in the historical data. In addition, the historical data are in the format of a multi-attribute information table. All these conditions suit the rough sets model, an effective tool for multi-attribute classification problems. The different rough sets models and issues concerning the implementation of rough sets model – indicator selection, discretization and validation test, are also discussed in this paper. This paper will demonstrate that rough sets model is applicable to a wide range of practical problems pertaining to economic and financial prediction. In addition, the results show that the rough sets model is a promising alternative to the conventional methods for economic and financial prediction.

Keywords: Rough sets model; Economic and financial prediction

1. Introduction

In the last decade, the amount of data collected and generated in all industries is growing at a fast rate (Brachman et al., 1996). From the financial sector to manufacturing operations, more and more companies are relying on the analysis of huge amount of data to compete. Although ad hoc mixtures of statistical techniques and file management tools once sufficed for analyzing mounds of data, the size of modern data warehouses, the mission-critical nature of the data and the speed with which analyses need to be made now call for a new approach.

A new generation of techniques and tools is emerging to intelligently assist humans in analyzing mountains of data, finding useful knowledge and in some cases performing analysis automatically. These techniques and tools are the subject of the growing field of data mining and knowledge discovery in database (KDD) (for details about KDD and data mining, please refer to Derry, 1997; Fayyad et al., 1996; Piatetsky-Shapiro, 1996).

The different data mining methods have different goals. In general, two types are distinguished:

- verification, in which the system is limited to verifying a user's hypothesis, and

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- discovery, in which the system finds new patterns.

Discovery includes prediction, through which the system finds patterns to help the future behavior of some entities; and description, through which the system finds patterns in order to present the patterns to users in an understandable form.

Following these two goals, many techniques have been applied to a variety of domains in business data mining, including marketing, finance, banking, manufacturing and telecommunications. Some results obtained were translated directly into business plans, considerably improving the quality of companies' decisions.

Among these methods, the Rough Sets Theory of Pawlak (1982) – as a new knowledge discovery tool with many advantages – inspired many scholars to do research in adapting this theory to the business domain. So far they have achieved many promising results (Polkowski and Skowron, 1998; Shen et al., 2000; Slowinski, 1992; Tsumoto et al., 1996; Weiss, 1996, 1997; Ziarko, 1993).

The rough sets model has the following advantages (Dimitras et al., 1999; Greco et al., 1998):

- It is based on the original data only and does not need any external information, unlike probability in statistics or grade of membership in the fuzzy set theory (Krusinska et al., 1992; Dubois and Prade, 1992; Skowron and Grzymala-Busse, 1993).
- The rough sets model is a tool suitable for analyzing not only quantitative attributes but also qualitative ones.
- It discovers important facts hidden in data and expresses them in the natural language of decision rules.
- The set of decision rules derived by the rough sets model gives a generalized description of the knowledge contained in the financial information tables, eliminating any redundancy typical of the original data.
- The decision rules obtained from the rough sets model are based on facts, because each decision rule is supported by a set of real examples.
- The results of the rough sets model are easy to understand, while the results from other methods (credit scoring, utility function, outranking relation) need an interpretation of the technical parameters, with which the user may not be familiar.

In the following sections, a state-of-the-art review of the literature relating to economic and financial prediction using the rough sets model is presented. Three major application areas, business failure prediction, database marketing and financial investment, are focused because of their similar prediction characteristics, i.e., the existing patterns are discovered from the historical database to predict the future outcomes. In addition, the historical database is in the format of a multi-attribute information table. All these conditions made the rough sets model well suited for the application areas. Different rough sets models which were used in these applications and some pre-processing and post-processing issues for the implementation of some rough sets models are also discussed.

2. Basic Rough Sets Theory

Rough Sets Theory is a powerful mathematical tool to handle vagueness and uncertainty inherent in making decisions. The concept of Rough Sets Theory is founded on the assumption that every object of the universe of discourse is associated with some information. For example, if objects are assets listed on a market, the information about the assets are composed of their price behavior and economic characteristics. Objects characterized by the same information are indiscernible (similar) in view of their available information. The indiscernibility relation generated in this way is the mathematical basis of the Rough Sets Theory. The most important problems that can be solved by Rough Sets Theory are: finding description of sets of objects in terms of attribute values, checking dependencies (full or partial) between attributes, reducing attributes, analyzing the significance of attributes, and generating decision rules (Pawlak, 1997).

Since Pawlak proposed the Rough Sets Theory in 1982 (Pawlak, 1982), this theory has been well studied by many researchers and has made great progress. For the reader's convenience, some concepts in the theory are presented in this section. For a detailed review on the Rough Sets Theory, readers can refer to Komorowski et al. (1999).

2.1. Information systems

In the Rough Sets Theory, *information systems* are used to represent knowledge. The notion of an information system presented here is described in Pawlak (1991) and Hampton (1997, 1998a,b).

An information system $S = (U, \Omega, V_q, f_q)$ consists of:

U – a nonempty, finite set called the *universe*;
Ω – a nonempty, finite set of *attributes*;
$\Omega = C \cup D$, in which C is a finite set of *condition attributes* and D is a finite set of *decision attributes*;

For each $q \in \Omega, V_q$ is called the *domain of q*;
f_q – an *information function* $f_q: \ U \to V_q$.

Objects can be interpreted as cases, states, processes, patients and observations. *Attributes* can be interpreted as features, variables and characteristic conditions. A special case of information systems called *decision table* or *attribute-value table* is applied in the following analysis. In a decision table, the row and column correspond to objects and attributes, respectively.

2.2. Lower and upper approximations

Due to imprecision which existed in the real world data, there are always conflicting objects contained in a decision table. Here conflicting objects refer to the two or more objects that are indiscernible by employing any set of condition attributes, but they belong to different decision classes. Such objects are called *inconsistent*. This decision table is called *inconsistent decision table*. In the Rough Sets Theory, the approximations of sets are introduced to deal with inconsistency. If $S = (U, \Omega, V_q, f_q)$ is a decision table, $B \subseteq \Omega$ and $X \subseteq U$, then the *B-lower* and *B-upper approximations* of X are defined, respectively, as follows:

$$\underline{B}X = \bigcup\{Y \in U/\mathrm{IND}(B) : Y \subseteq X\}, \tag{1}$$

$$\overline{B}X = \bigcup\{Y \in U/\mathrm{IND}(B) : Y \cap X \neq \emptyset\}, \tag{2}$$

where $U/\mathrm{IND}(B)$ denotes the family of all equivalence classes of B (classification of U); $\mathrm{IND}(B)$, called the B-indiscernibility relation, is defined as follows:

$$\mathrm{IND}(B) = \{(x, y) \in U^2 : \text{ for every } a \in B, \ a(x) = a(y)\}. \tag{3}$$

The set $BN_B(X) = \overline{B}X - \underline{B}X$ is called the *B-boundary* of X.

$\underline{B}X$ is the set of all elements of U which can be certainly classified as elements of X, employing the set of attributes B. $\overline{B}X$ is the set of elements of U which can be possibly classified as elements of X using the set of attributes B.

2.3. Quality of approximation

One measure to describe the inexactness of approximation classifications is called the *quality of approximation* of Ω by B. It expresses the percentage of objects, which can be correctly classified into class Ω employing the attribute B:

$$\gamma_B(\Omega) = \frac{\sum \mathrm{card}(\underline{B}X_i)}{\mathrm{card}(U)}. \tag{4}$$

If $\gamma_B(\Omega) = 1$, then the decision table is consistent, otherwise, it is inconsistent.

2.4. Reducts and core

An important issue in the Rough Sets Theory is about attribute reduction, in such a way that the reduced set of attributes provides the same *quality of approximation* as the original set of attributes. There are two fundamental concepts in connection with this attribute reduction. The B-reduct of Ω, denoted by $\mathrm{RED}(B)$, is the minimal subset of Ω, which provides the same *quality of approximation* of objects into elementary classes of B as the whole attributes Ω. The B-core of Ω, $\mathrm{CORE}(B)$, is the essential part of Ω, which cannot be eliminated without disturbing the ability to classify objects into elementary classes of B. It is the intersection of all reducts.

$$\mathrm{CORE}(B) = \bigcap_{R_i \in \mathrm{RED}(B)} R_i, \quad i = 1, 2, \ldots \tag{5}$$

Computing reducts is a non-trivial task that cannot be solved by a simple-minded increase of computational resources (Komorowski et al., 1999). This is one of the bottlenecks of the Rough Sets Theory. Recently, Wroblewski (1995, 1998) applied the genetic algorithm to the reduct generation problem and obtained sufficient reducts in an acceptable time, except for the case that the number of attributes is very high.

2.5. Decision rules

The problems of inducing decision rules have been extensively investigated in many fields, in particular in the machine learning domain (Michalski, 1983; Shavlik and Dietterich, 1990; Weiss and Kulikowski, 1990). The Rough Sets Theory can also be applied to different stages of rule induction and data processing. However, one aspect that distinguishes the Rough Sets Theory from typical machine learning systems is that the Rough Sets Theory does not correct or aggregate the inconsistency in the input data (Stefanowski, 1998b). The lower and upper approximations are applied to describe the inconsistency and consequently, deterministic and non-deterministic rules are induced.

Procedures for the derivation of decision rules from decision table were presented by Grzymala-Busse (1992), Skowron (1993), Slowinski and Stefanowski (1992), Stefanowski and Vanderpooten (1994) and Ziarko et al. (1993). More advanced rule induction methods have been studied in Bazan (1998) for comparing the dynamic and non-dynamic methods of induction rules from decision tables, Grzymala-Busse and Zou (1998) and Stefanowski (1998b) with focus on the induction rules from inconsistent decision table.

A decision rule can be expressed as a logical statement:

IF conjunction of elementary conditions;

THEN disjunction of elementary decisions.

Decision rules induced from a decision table can be applied to classify new objects. Specifically, the classification of a new object can be supported by matching its description to one of the decision rules. The matching may lead to one of the four situations:

(i) The new object matches exactly one of the deterministic decision rules.

(ii) The new object matches exactly one of the non-deterministic decision rules.

(iii) The new object does not match any of the decision rules.

(iv) The new object matches more than one rule.

In (i), the sorting suggestion is obvious. In (ii), however, the suggestion is not direct since the matched rule is ambiguous. In this case, the decision maker (DM) is informed of the number of sorting examples that support each possible category. The number is called the *strength*. If the *strength* of one category is greater than the *strength* of other categories occurring in the non-deterministic rule, one can conclude that according to this rule, the considered object most likely belongs to the strongest category.

Situation (iii) is more difficult to solve. In this case, one can help the DM by presenting him with a set of rules 'nearest' to the description of the new object. The notion of the nearest involves the use of the distance measure. Slowinski and Stefanowski (1994) have proposed a distance measure based on a *valued closeness relation* R having some good properties.

Situation (iv) may also be ambiguous if the matched rules (deterministic or not) lead to different classes. Here, the suggestion can be based either on the *strength* of possible classes or on an analysis of the sorting examples that support each possible class. In the latter case, the suggested class is that which is supported by a sorting problem closest to the new object based on the relation R.

In the solutions to situations (ii) and (iv) mentioned above, one situation is missing, that is the situation whereby the *strengths* of both categories are the same in which case there is no clear indication of how to classify the new object. Tay and Shen (2001) did some research on this problem. They applied the self-organizing map (SOM) to extract the inner relationship inherent in data sets. This inner relationship is helpful in distinguishing the 'strong' objects from the 'weak' objects. By 'strong object', one means that the inner category for the object as determined by SOM is the same as the original value of decision attribute. Otherwise, the object is termed a 'weak object'. The information inherent in the data sets helps to remove the uncertainty from the system and increase the classification accuracy on the new objects. This is especially efficient for inconsistent systems.

3. Application of rough sets model in economic and financial prediction

The applications of rough sets model in economic and financial prediction can be divided into three main areas: business failure prediction, database marketing and financial investment. The corresponding references are given in Table 1, together with the rough sets models applied. (The details of the different models formulated based on the Rough Sets Theory are described in Section 4.) The applications are described here to show the diversity of the problems that rough sets models can handle. In the following, the details of applying rough sets model to financial domains are introduced.

3.1. Business failure prediction

Business failure prediction is a scientific field in which many academic and professional people are interested. Financial organizations, such as banks, credit institutes, clients, etc. need these predictions for evaluating firms in which they have an interest. A large number of methods such as discriminant analysis, logit analysis, probit analysis and recursive partitioning algorithm have been applied to model this problem. Most of these methods have already been investigated in the course of comparative studies. Dimitras et al. (1996) gave a complete review of the methods used for the prediction of business failure and new trends in this area. Although some of these methods led to models with a satisfactory ability to discriminate between healthy and bankrupt firms, they suffered from some limitations, often due to the unrealistic assumption of statistical hypotheses or due to a confusing language of communication with the DMs (experts in this domain). Compared with

Table 1
The main application areas and their corresponding rough sets model

Rough sets models	Business failure prediction	Database marketing	Financial investment
RSES			Bazan et al. (1994)
			Baltzersen (1996)
LERS		Poel (1998)	
DataLogic	Szladow and Mills (1993)	Mills (1993)	Ziarko et al. (1993)
		Mrozek and Skabek (1998)	Golan (1995)
			Golan and Edwards (1993)
			Ruggiero (1994a,b,c)
			Skalko (1996)
			Lin and Tremba (2000)
		Eiben et al. (1998)	
TRANCE		Kowalczyk and Slisser (1997)	
		Kowalczyk and Piasta (1998)	
		Kowalczyk (1998a)	
ProbRough		Poel (1998)	
		Poel and Piasta (1998)	
Dominance relation	Greco et al. (1998)		
RoughDas and ProFit	Slowinski and Zopounidis (1994, 1995)		Susmaga et al. (1997)
	Dimitras et al. (1999)		
	Slowinski et al. (1997)		
	Slowinski et al. (1999)		
Hybrid model	Ahn et al. (2000)		
	Hashemi et al. (1998)		

these methods, the rough sets model appeared to be an effective tool for the analysis of financial information tables describing a set of objects (firms) by a set of multi-valued attributes (financial ratios) (Dimitras et al., 1996).

The rough sets model has also been used for the analysis and explanation of financing decisions in a Greek industrial development bank called ETEVA (Slowinski and Zopounidis, 1994, 1995). The ETEVA bank was interested in investing its capital in the better firms so the risk involved in investing was the primary element in the assessment of a firm. A sample of 39 companies was chosen. With the help of a DM (financial manager of ETEVA), the information table was built with 12 attributes (financial ratios) and 1 decision attribute with three categories, which indicated whether the company was 'acceptable', 'unacceptable' or 'uncertain'. The rules generated from the information table, on one hand, can be used to reveal the financial policy applied in the selection of viable firms. On the other hand, they can be used to evaluate another sample of firms which seek financing from ETEVA bank for the first time, although there was no validation test provided in the paper. Slowinski and Zopounidis (1995) illustrated how the rough sets model was a useful tool for the discovery of a preferential attitude of the DM in multi-attribute sorting problems, especially for the bankruptcy risk evaluation of firms.

The application of the rough sets approach in business failure prediction was investigated by Slowinski et al. (1999) and Dimitras et al. (1999). In their works, the rough set approach was tested for its prediction ability and was compared with three other methods: C4.5 inductive algorithm, discriminant analysis and logit analysis. In this case, 40 failed firms and 40 matching healthy firms from 13 industries were selected, meeting the criteria of (a) having been in business for more than 5 years and (b) data availability. Altogether 12 financial ratios were selected by a DM (the credit manager of a large Greek bank) to compose the information table with 1 decision attribute (0 – fail; 1 – healthy). Slowinski et al. (1999) and Dimitras et al. (1999) used a distance measure based on a valued closeness relation (Slowinski, 1993), VCR, to determine which category a test object belongs to in the case of no rules matching this object. The decision rules generated from reducts selected by the DM were applied to the validation data sets, which were comprised of the previous three years' data (year -1, -2 and -3) of the same firms. By comparing the predictive accuracy, the rough sets model was found to be more accurate than the classical discriminant analysis by an average of 6.1% per case using a minimal set of reduced rules. It also outperformed the C4.5 inductive algorithm as far as the classification accuracy is concerned. The superiority over logit analysis was not as distinct as that over discriminant analysis. Szladow and Mills (1993) presented a comparative study of the rough sets model versus multi-variable discriminant analysis (MDA) for prediction of corporate bankruptcy from five financial ratios: working capital, retained earnings, earnings before interest and taxes, market value of equities and sales to total asset volumes. By applying the rough sets model, correct predictions for bankrupt firms were increased from 96.0% for MDA to 97.0% for a rough sets model. The above comparison results showed that the prediction model based on the rough sets approach has more advantages over classical statistical models, such as discriminant analysis, logit analysis and probit analysis.

The study of the financial characteristics of the acquired firms aims at discriminating the acquired firms from the non-acquired ones is another application area of business failure prediction problems. In Slowinski et al. (1997), 30 acquired firms and a sample of equivalent non-acquired firms were chosen to be studied, using the rough sets model, to create patterns which would be able to distinguish between the two classes of firms. The rough sets model used here was the same as in Dimitras et al. (1999). The comparison of predictive accuracy with the discriminant analysis also showed that the rough sets approach was a strong alternative for the prediction of company acquisition.

3.2. Database marketing

Database marketing is a capacious term related to the way of thinking and acting which contains the application of tools and methods in studies and formation of the companies surroundings, their structure and internal organization in order that they could achieve success on a fluctuating and difficult to predict consumer market. In simplicity, database marketing can be defined as a method of analyzing customer data to look for patterns among existing preferences and to use these patterns for a more targeted selection of the customers (Fayyad et al., 1996). Database marketing is characterized by enormous amounts of data at the level of the individual consumer. However, these data have to be turned into information in order to be useful. To this end, several different problem specifications can be investigated. These include market segmentation, cross-sell prediction, response modelling, customer valuation and market basket analysis (Ananyan, 2000). Building successful solutions for these tasks requires applying advanced data mining and machine learning techniques to find relationships and patterns in historical data and using this knowledge to predict each prospect's reaction to future situations. The rough sets model has also been applied in this domain.

Poel (1998) gave a rough overview of the database marketing with the focus on customers' response modelling. The problem was about how to use the past transaction data of customers, personal characteristics and their response behavior to determine whether these clients were good mailing prospects during the next period. Real-world data were collected from one of the largest European mail-order companies. Poel (1998) applied statistical techniques (discriminant analysis, logistic regression, CART and CHAID), machine learning methods (C4.5), mathematical programming (linear programming classification), rough sets models (LERS and ProbRough) and neural networks to model the customer response problems. The performance of each method was evaluated on the basis of two criteria: the percentage classified correctly in the validation samples and gains chart analysis. (Here the gains chart analysis is widely used in comparing alternative techniques as shown in Furness, 1994. It is in fact an application of the Lorenz curve of incremental expenditure to the database marketing setting (Thompson, 1994).) The result of onefold cross-validation test showed that the rough sets model scored second (with a classification accuracy of 74.35%), following the CHAID (with a classification accuracy of 74.62%), and was characterized by only a small drop-off between learning and estimation samples. In the tenfold cross-validation test, the rough sets model offered a similar solution to CART, with a higher mean accuracy and lower standard deviation, in terms of the number of rules generated and in terms of the variables in the rules. From the analysis of the gains chart, two rough sets models – LERS and ProbRough had distinctly different gains charts compared with other methods. The results generated by LERS offered good predictive capability at the end of the chart, whereas ProbRough only showed top performance at the lower side of the chart. The comparison among these methods revealed that the classical statistical parametric techniques, discriminant analysis and logistic regression, performed very well for the relevant range of gains charts. The machine learning method, rough sets model and neural network, which were new to database marketing, can also be used successfully as techniques for response modelling.

Poel and Piasta (1998) further discussed the efficacy of the rough sets model in response modelling for the mailing-order prediction problem. By the analysis of the classification ability of the rough sets model, they drew the conclusion that the results obtained from the application of rough sets model rediscovered the variables – recency, frequency and monetary (RFM) value, to be the most significant predictors for mail-order buying behavior. Further, a significant finding on the importance of non-RFM variables in predicting purchasing behavior as the ratio of misclassification costs became larger, and was agreeable with prior beliefs of marketing managers. These results had not been revealed by other data mining efforts before. From this point of view, rough sets model can excavate the inherent important factors contained in the system.

Using the rough sets approach to model customer retention is another application area in the database marketing (Eiben et al., 1998; Kowalczyk, 1998a; Kowalczyk and Piasta, 1998; Kowalczyk and Slisser, 1997). The retention of its customers is very important to a commercial entity, in particular, a bank. When a client decides to move to another bank, it usually implies some financial losses for this bank. Therefore, banks are very interested in identifying some mechanisms behind such decisions and determining which clients are about to leave the bank. One way to find such potential customers is to analyze the historical data which describe customer behavior in the past. Kowalczyk and Slisser (1997) gave a simple description on how to model the customer retention problem using the rough sets model. It included the following steps: the conceptual analysis of problem, initial analysis of available data, identification of the more important attributes, construction of models and extraction of rules from the database. By applying these steps in modelling customer retention, it showed that the clients who were investors for a long time, invested money in funds with very small risk, and acquired small profits – had stopped their relationship with the company. This result was similar to the experience of an expert. In addition, compared with genetic programming, logistic regression and CHAID, the model based on the rough sets was a more efficient and simpler approach and it identifies more factors influencing the customers' behavior.

Besides the above two main applications, the rough sets approach has been applied to evaluate the qualifications of credit cards applicants (Mrozek and Skabek, 1998; Piasta and Lenarcik, 1996), draft an advertising budget for a company (Mrozek and Skabek, 1998) and predict the likely buyers for a company (Mills, 1993). In their works, the rough sets model was used to analyze the data on current customers, patterns were created to describe typical customers. Then any new prospect who fits these patterns would be identified as a potential customer. The rough sets model was a easy tool to help a manager organize the data and look at it in a different way.

3.3. Financial investment

Many financial analysis applications employ predictive modelling techniques, for example, statistical regression and neural network, to create and optimize portfolios and to build trading systems. These problems belong to the domain of financial investment.

Building trading systems using the rough sets model was studied by several researchers. Ziarko et al. (1993), Golan and Edwards (1993) and Golan (1995) applied the rough sets model to discover strong trading rules, which reflect highly repetitive patterns in data, from historical database of the Toronto stock exchange (TSE). By using the 1980 stock and economic data, they extracted the trading rules of five major TSE companies' stocks. These five companies were the Bank of Montreal, the Bell Canada Enterprises, the Imperial Oil, the Loblaws and the Northern Telecom. As indicated by the experts, these rules described the stock behavior of these companies and their sensitivity to market or economic indices, although not all the rules discovered by the rough sets model were of high quality. The 'generalized rules' (with a low roughness parameter, this concept will be introduced in Section 4) extracted by the rough sets model were all recognized rules or relationships in the investment industry, while the "exact rules" (with a high roughness parameter up to 1) made less sense to them. Their works proposed a methodology which established the rough sets model as a good candidate for knowledge discovery in stock market data.

Bazan et al. (1994) discussed the same trading system building problem using a rough sets model. In his work, 15 market indicators were collected and the problem was focused on how to deduce the rules that map the financial indicators at the end of a given month into the stock price changes a month later. Aimed at reducing the reduct set computation time, new solutions – conceptual clustering, extracting new attributes from decision table and joining groups of attributes – were discussed to meet the hardware requirement. The preliminary results using a rough sets model seemed to be only satisfactory with a classification accuracy of 44%. Further, there were still some problems, such as data filtration, incomplete data and evolution learning problem, to be studied. Baltzersen (1996) also did some research on the Total Index of the Oslo stock exchange (OSE) using rough sets model. His studies included the data collection and selection, conversion time-series to rough sets objects, reduct analysis and rules construction, using rough sets model to forecast the development of the index. Although the classification accuracy was not satisfactory (from 25% to 45% for different discretization methods), he extracted several clear indications for some of the factors whose change had more effect than their level, such as the slope of the interest rate, currency rate and consumption rate of gasoline for motor vehicles were more important than themselves.

Building trading systems on the S&P index was another major application in financial investment. Skalko (1996), following the KDD procedures, extracted a set of rules from the S&P 100 index, put/call data, New York stock exchange (NYSE) trading statistics, and US Treasury bond yield data from October 1987 to December 1994. These extracted rules were applied to trade using data from January 1995 to December 1995. There were nine trades initiated by the trading rules, among which seven were profitable. By analysis of the results, Skalkos proposed that the interest rates and market sentiment played important roles in a short-term trading system. His study was noteworthy for its use of technical analysis as well as financial data.

Ruggiero (1994a) has also done a lot of research on the building trading systems on S&P 500. He developed a set of rules to predict short and long positions in the S&P 500 while recognizing different market price cycles (Ruggiero, 1994a). It was claimed that excellent performance was achieved by using strong rules while discarding weak rules in trading. Over the whole trading period, this system achieved correct calling of over 70% of the positive moves in the next 5 weeks and average transaction represented over $25,000 profit per S&P 500 contract. In Ruggiero (1994b), rules to predict strong rallies in the S&P 500 of 2% or more were developed. The trading system extracted the strong generalization rules for predicting S&P 500 'buy' and 'sell' signals. Ruggiero (1994c) also developed a hybrid trading system incorporating neural networks, rough sets and a spreadsheet. Neural network models were supervised by rules generated by rough sets to correct for possible errors in predictions. The system allowed the user to address and exploit inefficiencies that exist in the market. This trading system obtained the reduction of drawdown by 25–50% and increased the average winner/loser ratio by 50–100%. The average trade length was reduced by 50–80%.

The rough sets model was also applied to the domains of portfolio management. Greco et al. (1996) did some research on the stock selection problem of Italian Stock Market based on the rough sets approach. Twenty-two Italian 'Blue Chips' listed for a long period were chosen to compose an equally weighted portfolio. Seven factors being proxies of risk exposures were selected to compose the decision table. This portfolio was studied using the rough sets approach and two conventional approaches of multi-factor models (MFM). The results show that the rough sets approach outperforms the MFM in the case of small samples and during turbulent periods. In addition, the rough sets approach provides a specific integration of classical market description for the investment process. It is a promising tool in portfolio risk management with focus on the behavioral finance.

Portfolio tilting was another application. The notion of 'Tilting' in portfolio management was to define a systematic approach to construct a portfolio which had a higher (or lower) value of a particular attribute than the value which can be found in a benchmark portfolio (such as mean–variance framework). For example, an investor preferred a low dividend paying stock over high dividend paying stock (all else being equal). This implied that the portfolio was tilted in such a way that either it would include a higher number of low dividend paying stocks or would have a higher investment in the few low dividend paying stocks. Susmaga et al. (1997) studied the portfolio tilting problem according to some well-established price-related stock attributes. The data set used included all the publicly traded companies listed on the TSE. By using the *n*-fold cross-validation test, it was concluded that for potential investors the price-related attributes were more significant in designing successful investment strategies. The predictive quality on stock of the top performer groups in the next year reached up to 70%, using the rules generated only from 'Common Reduct'.

This indicated that the rough sets analysis was useful in determining the contributions of attributes to identify top stock performers, allowing also for a construction of the decision rules which might be applied to the evaluation of new stocks.

It can be seen that the rough sets model is a promising alternative to the conventional methods. In the following sections, different rough sets models and some problems related to the applications of the rough sets model are introduced and discussed.

4. Different rough sets models and corresponding software used in economic and financial prediction

Since the rough sets theory was proposed in 1982, this theory has been studied and modified by many researchers. Consequently, different rough sets models were advanced to widen the applications of the rough sets theory in economic and financial prediction. These models and their corresponding applications are given in Table 1.

Learning from examples using rough sets (LERS) is a rule induction system developed by Grzymala-Busse (1992, 1998). There are two different approaches for rule induction in this system, which are computing sufficient rule sets using a machine learning approach and computing all rules by a knowledge acquisition approach. In both approaches the user has an additional choice between local and global algorithms. Among these options, learning from examples module, version 2 (LEM2), which is a local algorithm using the machine learning approach, is most widely used in practice. Poel (1998) applied LERS into database marketing with the focus on customers' response modelling and presented promising results on this new alternative approach. Stefanowski (1998a) proposed a modified LEM2 algorithm to handle directly continuous attributes and discretize them inside the learning algorithm while creating elementary conditions. This algorithm extracts better sets of decision rules than LEM2 so as to enhance the predictive accuracy of rough set-based rule induction system.

Bazan et al. (1994) and Baltzersen (1996) engaged in market data analysis with the aid of the rough set expert system (RSES) (Bazan and Szczuka, 2000). The rule induction system of this software is based on the Boolean Reasoning and dynamic reducts (Bazan, 1998). Besides all basic operations of the rough sets theory, this software also provides the discretization algorithms and template generation algorithms.

The Variable Precision Rough Sets Model was proposed by Ziarko (1993) as a derivative of the basic rough sets model. This model broadened thedeterministic data dependencies, which was the foundation of a basic rough sets model, to non-deterministic relationships. Given an acceptable rule probability β, the strong non-deterministic rules were extracted. These rules were likely to be correct or almost correct but their usefulness depends on β. In Ziarko et al. (1993), the Variable Precision Rough Sets Model (Ziarko, 1993), which has been developed into a commercial software – DataLogic, was applied to extract the trading rules. β is set to 0.55 in their works. The same rough sets model was applied to build the trading system in Lin and Tremba (2000), Skalko (1996), Golan (1995), Golan and Edwards (1993) and Ruggiero (1994a,b).

Rough Classifier, developed by Lenarcik and Piasta (1992) and Rough Data Models introduced by Kowalczyk (1998b) were two approaches that avoided the use of reducts. Both approaches were focused on finding a relatively simple partition of the attribute space and then drawing some conclusions from the structure of this partition. Two systems that were representative for these approaches were ProbRough (Piasta and Lenarcik, 1996, 1998; Lenarcik and Piasta, 1998) and TRANCE (Kowalczyk, 1996, 1998b). These two systems were mainly used in the database marketing, such as customer retention modelling (Eiben et al., 1998; Kowalczyk and Slisser, 1997; Kowalczyk and Piasta, 1998; Kowalczyk, 1998a), purchase prediction (Poel and Piasta, 1998), respond modelling (Poel, 1998) and bankruptcy prediction (Piasta and Lenarcik, 1996).

The rough sets theory was founded on the assumption that every object of the universe was associated with some information. Objects characterized by the same description were indiscernible in view of the available information about them, which has been defined in Section 2 of this paper as indiscernibility relation. Greco et al. (1998), taking into account the ordinal properties of considered evaluation criteria, proposed the dominance relation instead of the discernibility relation to reconstruct the rough sets model. This new model not only maintained the best properties of the basic rough sets model but also presented more understandable rules to the user. In addition, the rules based on dominance relation were also better adapted to sort new actions than the rules on indiscernibility. This new model was applied to bankruptcy risk evaluation, which has been discussed in Slowinski and Zopounidis (1995). Compared with previous results, the new model presented a smaller number of reducts (only 4 for dominance relation against 26 for indiscernibility relation) and a larger core ({Attribute7, Attribute 9} against {Attribute 7}). These two features were generally recognized as desirable properties of a good approximation (Pawlak, 1991; Slowinski and Stefanowski, 1996). Moreover, the decision rules from dominance relation generally perform better when applied to new objects.

The valued closeness relation (VCR) (Slowinski, 1993; Slowinski and Stefanowski, 1994) was proposed to solve the problem of no rules matching the new object during the prediction phase. It involved indifference, strict difference and veto thresholds on particular attributes, used in concordance and discordance tests. The goals of these tests were to:

(i) characterize a group of attributes considered to be in accordance with the affirmation 'object x is close to rule y', and assess the relative importance of this group;

(ii) characterize among the attributes, which are not in concordance with the above affirmation, the ones whose opposition is strong enough to reduce the credibility of the closeness, which would result from taking into account just concordance, and to calculate the possible reduction that would thereby result.

Dimitras et al. (1999) compared the classification results before and after the applications of VCR. They indicated that on the average, 60% of objects not classified by exactly matching rules were classified correctly by the VCR-nearest rules, which was a better result than random classification of these objects. In addition, using VCR also supported that two objects were marginal which had not been detected before the application of VCR. This distance measure has also been used in portfolio tilting (Susmaga et al., 1997). The software including this technique is RoughDas and ProFit (Mienko et al., 1996). Now a new extension of this software called ROSE (Predki et al., 1998; Slowinski and Stefanowski, 1998) is available which provides different techniques on rule induction. In addition, besides all basic operations of the rough sets theory, ROSE provides several approximation techniques to avoid data discretization such as similarity relation and dominance relation. These techniques can be easily controlled by users.

In recent research works (Yasdi, 1995; Hashemi et al., 1998; Ahn et al., 2000), the rough sets approach combined with neural network has been used in economic and financial prediction. In these hybrid models, the rough sets approach took the role of a preprocessor for the neural network – reducing the information table. This had a great significance for neural network in that reduction of attributes prevents the overfitting problem and saves training time. Furthermore, removing conflicting objects and training neural network with consistent cases can improve the performance as well as reduce the training time. Ahn et al. (2000) applied this hybrid model to predict the business failure for over 1200 healthy firms and 1200 failed firms in Korea. The results showed that this hybrid model outperformed the discriminant analysis model and neural network model.

5. Discussion on some problems related to the implementation of rough sets model

In the aforementioned economic and financial applications using the rough sets model, the same procedures are used. The first step is data collection and data selection. In this step, the data availability and data reliability should be considered. Then these data are pre-processed to construct the information table, which is the knowledge representation in a rough sets model. Here pre-processing includes the calculation of indicators for time series, discretization of indices or indicators for continuous values, rescaling the attributes for nominal values and removing the outliers and missing values. After the pre-processing step, the rough sets model is applied to extract rules from decision table. In this step, various methods are applied to solve different problems by their characteristics, which have been discussed in Section 4. The extracted rules are first tested for their validation, then they are used to build a trading system for financial investment or prediction of business failure. This procedure is illustrated in Fig. 1.

In this procedure, it can be seen at every step that there are several choices to be made. Different selections give different outcomes. In the following, some of them will be discussed.

5.1. Indicator/attribute selection

The quality of indicators affects the final generated rules because the rough sets model can only extract the inherent principle existing in the information table, which means that if the indicators do not represent the related information about system, then the generated rules would be dispersed and become insignificant. There are many methods for estimating the importance of a single attribute. For example, one can calculate various statistical measures of dependency, or, more often, information gain (Quinlan, 1986). But these methods fail if one wants to build a model based on a combination of several attributes. Kowalczyk (1996) and Kowalczyk and Piasta (1998) did some research on this problem. First, they generated numerous plots which were routinely used in statistical data analysis: frequency histograms, means,

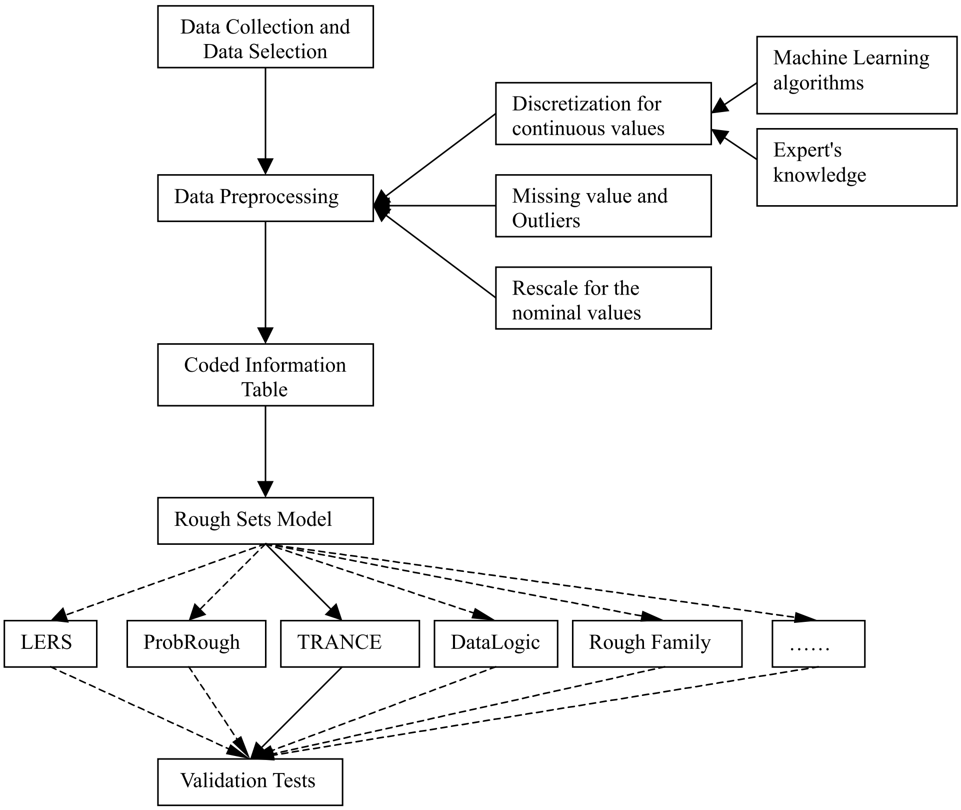

Fig. 1. The procedure of economic and financial prediction using rough sets model.

density estimates. Through the visual inspection of these plots, the whole data set was divided into two subsets. In order to identify the most important attributes among the calculated ones, three measures were taken – correlation coefficients, coefficients of concordance and information gain. By the analysis of these three measures each attribute was given the significance value. This method seemed effective in the modelling of customer's retention problem (Kowalczyk and Slisser, 1997) and gave satisfactory results when combined with rough sets model. Due to its sparse applications in other areas, this method was still under validation considerations. Among the rest of researcher's works, the indicators' collection and selection were all supervised by the experts. This was also a reliable method to ensure the correlation and correction of attributes.

5.2. Discretization

Before the data can be sent into some rough sets models, they must first be discretized. As a result of discretization, the precision of the original data will be decreased but its generality will be increased. When the subintervals for the discretization were specified by a domain expert following his judgement or using norms established in the subject domain, then they were called expert discretization. On the other hand, when they were defined automatically, they were called automatic discretizations (Susmaga et al., 1997). There are a lot of supervised discretization and unsupervised automatic discretization methods (Dougherty et al., 1995; Fayyad and Irani, 1993; Quinlan, 1986) studied in machine learning community. But these methods are seldom applied to economic and financial prediction based on the rough sets model, except that Slowinski et al. (1997) used the Minimal Class Entropy method (Fayyad and Irani, 1992) to discretize the financial ratios in cooperation with financial experts. The majority of researchers gave the coded information table (after discretization) based on the knowledge of domain experts. Definitely the experience of experts can give more reasonable cut points than the automatic discretization method. But sometimes due to a lack of experts' supervisions, or more new indicators were involved in this model, we had to look for the help of automatic discretization methods.

The rough sets community has been also committed to constructing effective algorithms for new feature extraction, in particular the efforts have been focused on discretization and symbolic attribute value grouping (Chmielewski and Grzymala-Busse, 1996; Lenarcik and Piasta, 1992, 1993a,b, 1997; Nguyen, 1997; Nguyen and Skowron, 1995 and 1997). The most successful among these methods are:

- discretization techniques (Nguyen, 1997, 1998a,b; Nguyen and Skowron, 1995, 1997);
- methods of partitioning (grouping) of nominal attribute value sets (Nguyen, 1997; Nguyen and Nguyen, 1998; Nguyen and Skowron, 1997);
- combinations of the above methods (Nguyen and Nguyen, 1998).

Another way to avoid the discretization problem is to apply the rough sets models directly to the continuous decision table. The indiscernibility relation is the mathematical basis of the Rough Sets Theory. All the following concepts and operations are based on this generalization. Some researchers extend this relation to a more general concept which admits the ambiguity which existed, such as the similarity relation proposed by Slowinski and Vanderpooten (1995, 2000) (see a further discussion in Greco et al., 1999a, 2000a,b) and tolerance relation proposed by Skowron and Stepaniuk (1996, 1998). These generalizations provide more possibilities for approximation definition, approximation tuning and primitive definable sets choosing. Based on these general relations, the Rough Sets Theory can be applied to continuous data sets directly.

5.3. Validation test of results

The major validation tests for classification problem (Weiss and Kulikowski, 1990) can be categorized as:

- *Random tests*. The testing sample is selected and classified only once, with its size being set arbitrarily to a constant percentage of all objects or generated randomly. Generally, random validation tests should be used with large data sets.

- *N-fold cross-validation tests*. The set of all available objects is initially divided into N disjoint subsets, called folds. The learning and classifying phases are then conducted N times, with each of the folds acting successively as the testing sample and all the remaining folds as the learning sample. The main idea behind this is that every object is classified exactly once and it serves to generate the decision rules $N-1$ times. The final results of the N-fold cross-validation test are given as averages of the N individual test. This type of test should be used with medium-size data sets.
- *Leaving-1-out test*. It is a variation of the N-fold cross-validation test in which N is set to the number of all objects. This test should be applied to small data sets (<100).

The 10-fold cross-validation method was used in Susmaga et al. (1997) to assess the quality of the rules as classifiers. The results reported in his work were very promising.

In the case of the financial investment, the validation tests were processed using the trading performance on the testing period, i.e., the profit and loss, the Sharpe Ratio and winning trades. Skalko (1996) and Ruggiero (1994a,b,c) used these measures to evaluate the effectiveness of rough sets model on building trading systems.

6. Concluding remarks

In the above review of rough sets model for economic and financial prediction, it has been demonstrated that rough sets model is a promising alternative method to conventional methods. However, thus far the use of Rough Sets Theory has been restricted to the classification problem because classification objects can be directly put into the decision table (Pawlak, 1994). This usage can be generalized from the applications described in previous sections. Greco et al. (1997a,b) extended the usage of Rough Sets Theory to the choice and ranking problems, which are often encountered in economic and financial decision making problems. In their approaches, the conventional decision table is replaced by a pairwise comparison table, i.e., a decision table whose objects and entries are pairs of actions and binary relation instead of single action and attribute values, respectively. In this way, the built preferential model is much closer to the natural reasoning of the decision making problem. This approach will broaden the application of Rough Sets Theory in the economic and financial problems.

Moreover, Greco et al. (1999b, 2000c), Kryszkiewicz (1998), Piasta and Lenarcik (1998), Stefanowski and Tsoukias (1999) have also undertaken some research on the application of Rough Sets Theory in case that there is missing information in the decision table. Sakai and Okuma (1999, 2000) proposed a definition for dependencies of attributes on non-deterministic information and presented an algorithm to check it. All these researches will enhance the capability of the Rough Sets Theory to solve practical problems. Recently, Kumar (1998) presented efficient exact and approximate techniques for identifying reducts from a database system using simple structured query language (SQL) commands. His method is closest to Rough Sets Theory and extends it to an approach which can be easily implemented on a relational database system.

We are confident that by taking advantage of its own unique characteristics in data mining techniques, rough sets models can be applied to a broader application area.

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