Quality control for round workpieces

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1 Introduction

An important part of the development in science and technology has, in some way or another, something to do with the car industry. Many people have worked and are working actually in problems occurring during the design and production of cars. This situation is particularly noticeable in Germany, where car companies and small industries working for them, are very common.

Here we report our advances in the direction of the quality control of some special round workpieces. In our case these workpieces are called "Flywheels" and these are part of every clutch system in the car's motor. The flywheels are no exception. Interesting problems appear in the investigation of this peculiar pieces, the reason is that flywheels are used in many other devices and situations as a mean of storing energy, including the space industry.

But our problem is rather different. We are concerned with the quality of the flywheel and specifically with the use of a "coordinate measure machine". These machines are used to take several measurements of a worked piece in a short time. The company that originally propose the problem, used to check the quality of their round worked pieces manually. They come doing this for a while, but apparently they decided to make this procedure automatic. They wanted to do that using a coordinate measuring machine.

Using lasers the machine is able to take measurements related with the worked piece and to get some finite number of data points describing the boundary. The problem, as firstly proposed to us, was to study the quality of the piece using those measurements. Unfortunately (or fortunately) the decision of changing the method was abandoned and they will continue doing all the control manually. Therefore our project stayed in the air. Having no answer to our questions about the precise definition of the goals, we have defined our own objectives.

This work is divided in two main parts. One related with the measurement of roundness (or more precisely out of roundness) and the other concerned with our idea of quality for the flywheel. For most of the methods programs have been written but just (poorly) simulated data has been used to test them. Finally, inspired in our task, some new interesting problems are proposed.

2 The Workpiece

What we got to do was to test the quality of roundness of a clutch's part. A clutch is a device that makes it possible to cut the connection between motor and wheels to change gears.

In our case the clutch is made for V.A.G., which means that it should be used in a VW or AUDI, and therefore it should be a "one-disc-friction-clutch with membrane spring". These kinds of clutches consisted of some different parts whose in general are a clutch-disc, a flywheel, a clutch-cap, a pressure-disc and a release-lever.

In simplified words a clutch works as follows. A one-disc-friction-clutch is a flywheel fixed by a shaft to the motor, so it rotates with the speed the motor does. Fixed on it there is a device called clutch-cap, which is a cap of metal, with a hole in the middle and membrane springs fixed on its inner side. These springs push the pressure-disc in the flywheel's direction. Leading through the hole in the clutch-cap there is another shaft connected to the car's gearing and to the clutch-disc, which is placed between the flywheel and the pressure-disc. Therefore it is fixed there by pressure and the rotation of the motor is transmitted to the gearing. To change gears you push the pedal connected to the release-lever, which then pushes onto the membrane-springs what reduces the pressure on the pressure-disc. This takes away the pressure from the clutch-disc and is therefore no more connected to the motor. Now is able to run free. That makes the changing of gears possible. After changing gears you release the pedal, and the clutch-disc is fixed again, so the motor's rotation is transmitted to the gearing.

Figure 1: The Flywheel

Our objective is to test with a given measuring machine whether the flywheel is produced well or not. We know that our workpiece (flywheel) is a metal plate with a whole in the middle and it has a circular boundary and should rotate quietly.

3 Coordinate Measuring Machines

The quality of round workpieces is measured automatically using a Coordinate Measuring Machine. Those machines are able to take several measurements and perform the calculations using a computer that usually comes together with the machine.

The main idea, how the roundness of a circular workpiece could be measured is to check with the help of two lasers, if the inner and the outer radius of the workpiece are in given tolerance limits. For this it is useful to put the workpiece centered on a rotating plate.

The problem that appears here is that we do not know where is the center of the workpiece until now. So we get another task, namely to estimate the center of the workpiece by measuring the coordinates of the two circles.

3.1 Getting the Data

The Measuring Machine rotates the workpiece and using some laser device, provides us with data measurements.

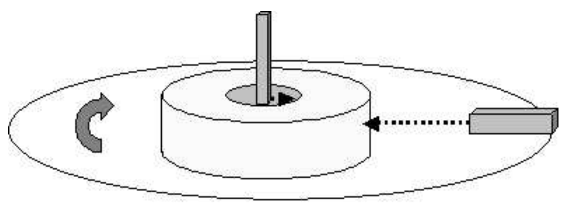

Figure 2: Worked Piece over the Rotating Plate and two Lasers

- Rotating plate is rotating in 1.5 seconds per round.
- The two lasers are taking 2^{14} measurements per round. We assume that the two lasers are measuring at the same time and that they are measuring on one line with the center of the rotating plate.
- The distances between the lasers and the center of the rotating plate is known. It is nearly exact, because it is measured by a glassmasstab.
- Laser 1 measures the distance to the outer boundary.
- Laser 2 measures the distance to the inner boundary.

Next we want to have a look at the data we get out of the measuring process. It is enough to look at the data from the second laser (the first measures analogously). Therefore we use a coordinate system placed in the known center of the rotating plate. Furthermore we know the position of the second laser on the x-axis from the measurement with the glassmasstab.

- $C = (x_c, y_c)\ldots$ center of the inner circle

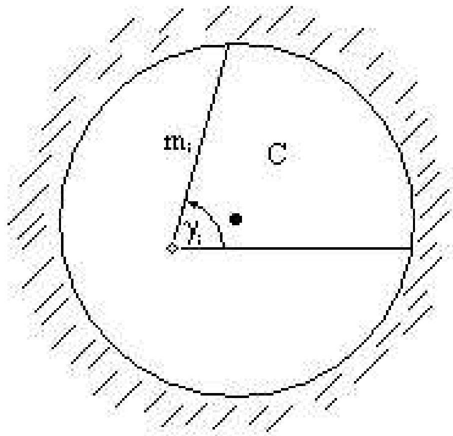

Figure 3: Data Coming from Inner-Boundary Measurements

- r ... radius of the inner circle
- d ... distance between center of the plate and second laser (known)
- m_i ... mesasured distance

The data we received can be interpreted in a natural way using polar coordinates in the form (m_i, γ_i) from the second laser, where γ_i are the angles depending on the rotation, i.e.

$$\gamma_i = i \cdot \frac{360^\circ}{\text{number of measurements}}$$

i=1, ...,number of measurements, here $i = 2^{14}$.

We can transform this data pairs to rectangular coordinates by:

$$(x_i, y_i) := (m_i \cos(\gamma_i), m_i \sin(\gamma_i))$$

As we have seen, it is possible to use polar or rectangular coordinates to represent the data. We will use both of them according to the situation.

The next step is to consider what to do with the data in order to say something about the quality of the workpiece.

4 The Roundness

Our goal is to model a process of a machine that should measure the “roundness ” of circular workpieces.

Already in this short formulation two main questions of our problem appear:

- What is roundness ?
- When can we say that our workpiece is ”round” enough ?

Because this is a very common problem, many methods have been developed. One of them is the use of reference circles, but other popular approach is the use of the Limacon curves.

4.1 Reference Circles

The measurement of roundness is a common problem of quality control and inspection of a circular workpiece. The tolerance of the out-of-roundness is the annular space between two concentric circles. A workpiece is within tolerance if its profile, is enclosed by these two circles. Four reference circles are internationally accepted for roundness measurements. These are:

- Least Squares Circles
- Minimal Zone Circles
- Maximal Inscribed Circles
- Minimal Circumscribed Circles

4.1.1 Least Square Reference Circle (LSC)

The Least Squares reference circle is a circle where, the sum of areas inside this circle, are equal to the sum of the areas outside the circle and kept to a minimum separation.

The out of roundness value is the difference between the maximum and minimum radial departure from the reference circle center. This is a very convenient reference circle to derive, as it is mathematically precise.

To calculate the LSC we have to find the radial distance between the unknown center (x_c, y_c) and a point (x_i, y_i) on the circle.

Using Pytagoras we get

$$d_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}$$

So we can calculate the discrete form errors of the circle by

$$\epsilon_i := r - d_i$$

Our goal is to minimize the form error of the circle using the Least Square Minimization, i.e.

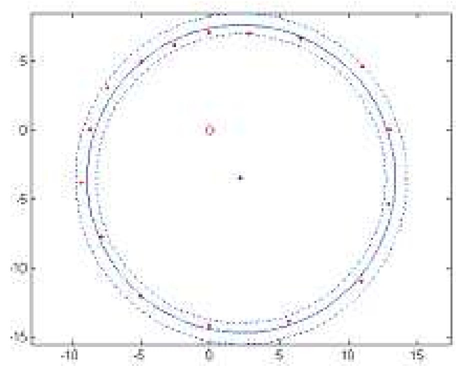

Figure 4: Least Squares Circle

$$\min \mathrm{E}(x_c, y_c, r) := \mathrm{E} = \sum_{i=1}^{n} \epsilon_i^2$$

To get this minimum we have to find z, such that

$$\nabla \mathrm{E} = g = 0$$

One method to calculate the zeroes of ∇ E is the Newton - Iteration, which is determined through:

$$z_{i+1} = z_i - J(x_i)^{-1} g(x_i)$$

Where J is the Hessian matrix of second derivatives. It is important to choose a starting vector $z^{(0)}$, which is not too far away from the solution, because the convergence of the Newton - Iteration depends on it.

There are some possibilities, how to find such a starting vector:

1. Take the mean value of the x_i s , y_i s and the desired radius, i.e.

$$x^{(0)} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$y^{(0)} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

2. Fix three measurement points and take them as corners of a triangle. Then calculate the center of the triangle and use this as $x^{(0)}$ and $y^{(0)}$. For r take again the desired radius.

This method is unreliable, it will if J is singular. There are some safeguards but in our case we can avoid this method and use a simpler one that simplifies the equations to get a linear system.

Given the data points (X_i, Y_i) which lie near a circle of unknown center and radius, we want to compute the center(X_c, Y_c), and a radius r. This is done, by minimizing a cost function. The general equation of a circle centered at (X_c, Y_c) with radius r is:

$$(X_i - X_c)^2 + (Y_i - Y_c)^2 = r^2$$

The radial distance d_i, from the (unknown) center (X_c, Y_c) to a data point (X_i, Y_i) is:

$$d_i = \sqrt{(X_i - X_c)^2 + (Y_i - Y_c)^2}$$

We want to minimize the error in the radial distances between d_i and r. The preferred method, as it eliminates computations involving the square root in the definition of d_i, is to minimize the square of the difference of the square of the distances, i.e,

$$\sum_{i=1}^{N} \epsilon_i^2 = \sum_{i=1}^{N} (d_i^2 - r_i^2)^2$$

This function has partial derivatives that are continuous. Therefore, we minimize the following function:

$$L(X_c, Y_c, r) = \sum_{i=1}^{N} ((X_i - X_c)^2 + (Y_i - Y_c)^2 - r^2)^2$$

which can be transformed into

$$L_2(X_c, Y_c, r) = \sum_{i=1}^{N} (X_i^2 - 2X_iX_c + Y_i^2 - 2Y_iY_c + \alpha)^2$$

where $\alpha = X_c^2 + Y_c^2 - r^2$ is defined to be a new unknown.

Now, roughly speaking, functions L and L_2 are "like" and have the same values with a countious change of coordinates. Finding the minimum of some of them should be equivalent to find the minimum of the other. The rigorous justification of this assumption will be skipped. Let's now calculate the minimum of L_2.

Setting each partial derivative to zero gives:

$$\frac{\partial L_2}{\partial X_c} = \sum_{i=1}^{N} 2(X_i^2 - 2X_iX_c + Y_i^2 - 2Y_iY_c + \alpha)(-2X_i) = 0$$

$$\frac{\partial L_2}{\partial Y_c} = \sum_{i=1}^{N} 2(X_i^2 - 2X_iX_c + Y_i^2 - 2Y_iY_c + \alpha)(-2Y_i) = 0$$

$$\frac{\partial L_2}{\partial \alpha} = \sum_{i=1}^{N} 2(X_i^2 - 2X_iX_c + Y_i^2 - 2Y_iY_c + \alpha) = 0$$

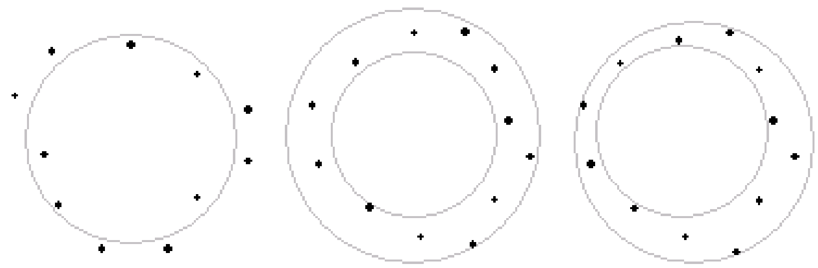

Figure 5: a)Least Squares b)Minimal Zone c)Maximal Inscribed and Minimal Circumscribed

This yields three simultaneous equations which can be simplified to be written in matrix form $Ax = b$ as,

$$\begin{pmatrix} -2\sum_{i=1}^N X_i^2 & -2\sum_{i=1}^N X_iY_i & \sum_{i=1}^N X_i \\ -2\sum_{i=1}^N X_iY_i & -2\sum_{i=1}^N Y_i^2 & \sum_{i=1}^N Y_i \\ -2\sum_{i=1}^N X_i & -2\sum_{i=1}^N Y_i & N \end{pmatrix} \begin{pmatrix} X_c \\ Y_c \\ \alpha \end{pmatrix} = \begin{pmatrix} -\sum_{i=1}^N X_i^3 - \sum_{i=1}^N X_iY_i^2 \\ \sum_{i=1}^N X_i^2Y_i - \sum_{i=1}^N Y_i^3 \\ -\sum_{i=1}^N X_i^2 - \sum_{i=1}^N Y_i^2 \end{pmatrix}.$$

Remember that X_i and Y_i are the data coordinates, then the matrix A and vector b are given. Solving the simple system give us the solution, and using $\alpha = X_c^2 + Y_c^2 - r^2$ we can determine the value for the raduis r. In that way is possible to get r, X_c and y_c using least squares.

4.1.2 Minimum Zone Circle (MZC)

The MZC is defined as two concentric circles positioned to just enclose the measured profile such that their radial departure is a minimum. The roundness value is then given as their radial separation. This circle can be estimated using the limacon approximation. We will define the equations in the limacon section.

4.1.3 Minimum Circumscribed Circle (MCC)

This is also known as the ring gauge reference circle and is the smallest circle that totally encloses the profile. Out of roundness is quantified as the largest deviation from this circle. The MCC can be also estimated using the limacon approximation.

4.1.4 Maximum Inscribed Circle (MIC)

The maximum inscribed circle, sometimes referred to as the plug gauge circle, is the largest circle that is totally enclosed by the profile. Errors are quantified as the maximum radial deviation away from this reference circle.

There is a straight way to calculate a Maximal Inscribed Circle. We can construct the MIC for a set of points, starting from an arbitrary inscribed circle

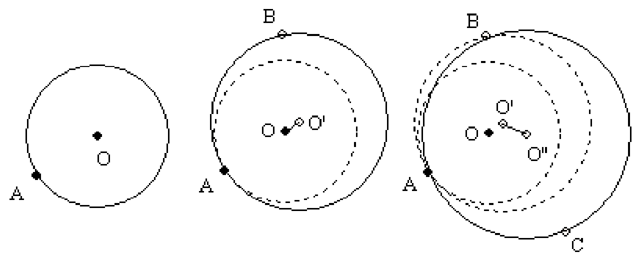

Figure 6: Construction of the Maximal Inscribed Circle

and increasing the radius keeping the data points out of the circle. At the end we should have a circle defined at least by three points and containing no other data points.

A simple geometric algorithm can be used to calculate a maximal inscribed circle.

Calculate the Least Squares Circle. Name O the center of that circle. Now take the data point A, which is closer to O. The circle with center O and radius OA contain no other data points.

If these circle pass trough no other point then we can find a larger inscribed circle, moving the point O away from A in the same line to a new point O' until the circle meets a second data point B. In the program we can find such a second point, simply by comparing the radius of the circles defined passing trough each data point and orthogonal to the line AO. The smallest circle containing O will define the point B.

Now we are looking for a third point C, such that the circle defined by ABC. Increasing the radius of the circle, but keeping the contact with A and B, eventually we will meet a third point. Again we can calculate this circle comparing the circles defined by A, B and each of the other data points. The smaller circle containing O' is an inscribed circle for the data points. We call the center of such a circle O".

We can increase the radius of that circle easily if the triangle ABC is non-acute triangle, in the same way we have found the point C.

It is easy to realize that the MIC is not always unique, but still we will get some information from it. Namely, if the MIC for the data is larger than the circle for the axis then we can be very sure that the work piece will not fit in the axis.

4.2 The Limacon Approximation

There exist an alternative approach to transform the problem in a linear one. It is very used in the practice. The idea comes from the fact that we really don't measure the "roundness" but the deviation from roundness.

Using polar coordinates, consider a circle of radius R_0 and centered in the point (E, ϕ). The variable E represents the deviation from the center and it is usually very small compared with R_0, i.e. E/R_0 is small. Let's calculate the function $k(\theta)$ representing the circle.

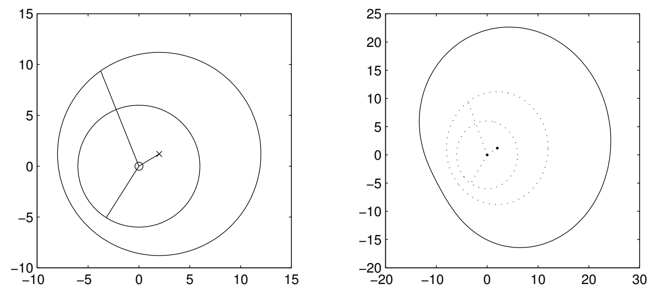

Figure 7: The Limacon Curve

From Law of cosines we have:

$$R_0^2 = E^2 + (k(\theta))^2 - 2Ek(\theta)\cos(\theta - \phi)$$

Solving for $k(\theta)$ we can represent the circle in polar coordinates,

$$k(\theta) = E\cos(\theta - \phi) + (R_0^2 - (E\sin(\theta - \phi))^2)^{1/2}$$

Now comes the interesting part. Because we want to analyze the deviation from roundness, we will intentionally increase the effect of this "out of roundness". To do that, imagine a second circle of radius L centered in the origin. If we "suppress" this circle, we keep just the deviations. Were "suppress" means to consider a curve which function is $k(\theta) - L$. Next we apply a magnification by a factor M and finally we aggregate a new radius S to recover, more or less, the original shape, but with the deviations increased. The final curve has a function of the form,

$$r(\theta) = M(k(\theta) - L) + S$$

Which is not a circle (unless $E = 0$). The curve is called Limacon. (Discovered and Named after Etienne Pascal, father of Blaise Pascal.) Substituting $k(\theta)$ and using the binomial expansion we get,

$$r(\theta) = M(R_0 - L) + S + ME\cos(\theta - \phi) - \frac{ME^2}{2R_0}(\sin(\theta - \phi))^2 + ...$$

Assuming that E/R_0 is small we can consider only the first terms of the series, having then,

$$r(\theta) = R + a\cos(\theta) + b\sin(\theta)$$

where,

$$R = M(R_0 - L) + S, a = ME\cos(\phi), b = ME\sin(\phi)$$

This assumption is valid as long as the center of the workpiece is close to the center of the rotating plate.

Now, as we can see, we need to find R, a and b such that

$$r(\theta_i) = R + a\cos(\theta_i) + b\sin(\theta_i), i = 1...N$$

where $(\theta_i, r(\theta_i))$ represents the N data measurements in polar coordinates. We get again a system of N equations i.e. a Least Squares problem, and we have seen before how to manage such a problem.

5 Modelling the Quality

Until now we have described some known methods used to describe the roundness of a curve. The original problem was to model the quality of the workpiece. To do this we have to consider the real use of the workpiece and the desired properties of it. We should remember that our workpiece is a flywheel, and it is part of the clutch system in the motor of a car.

We consider two aspects of the quality of the flywheel, one for each of the boundaries of the piece, the inner-boundary and the outer-boundary. The problem of, what to do, with the two curves at the same time is not yet considered.

The hole represented by the inner boundary of the workpiece is designed to fit in a rotating axis of the motor. A quality workpiece should fit in the axis properly. This proper fitting is an ambiguous matter. We make some simplifications in order to get some answers.

After having given some simple models for this "fitting process" a second characteristic is desired in the flywheel. The rotation has to be smooth, without vibrations or oscillations. This depends strongly on the shape of the outer boundary. We make some assumptions in this direction too, so we can get a simple model of the forces acting in the axis.

5.1 Fitting in a Rotating Axis

As we have seen, the proper fitting of the work piece in an axis is one of the desired characteristics. We want to answer the question: Will the piece fit properly in the axis? More properly: Given the data points coming from the measuring process, is it possible to predict if the piece will fit in a given axis? The assumptions we made are:

- The axis has a perfect circular shape and a known radius R.
- The shape of the axis is not deformable.
- The material of the work piece is homogeneous.

From the data points we can get some information about how large is the hole in the work piece. There are three possibilities.

- The hole fit in the axis as desired.
- The hole is too large and it will not fit properly.
- The hole is too small and it will not fit at all, or if we manage to fit it (via some expansion contraction process) some high stresses will appear in the work piece.

The first idea, to realize if the hole is too large, is to calculate the Maximal Inscribed Circle (MIC). As we have seen, this circle is in general not unique, but still we can get some information. Namely, if the MIC for the data is larger than the circle for the axis then we can be very sure that the work piece will not fit in the axis.

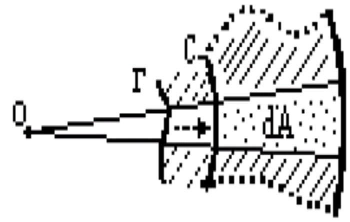

Figure 8: Deformation of the Inner Boundary

Anyhow, the use of the MIC as an estimator for the center of rotation is not a very realistic approach.

The work piece should be considered as a deformable material. If we want to consider a better model, we have to take into account this fact.

5.1.1 Simple Elastic Model

As we mentioned before, a more realistic model can be obtained if we assume that the axis will create a deformation in the workpiece. Let us make some assumptions on the elastic properties of the workpiece.

- The axis is not deformed.
- The deformation is elastic, i.e. $force \sim deformation$.
- There exist a complete contact between the workpiece and the axis.

For this analysis we will assume that what we have is a curve representing the inner boundary. This can be achieved via an interpolation procedure on the data points. Lets call Γ in C^1 the curve representing the inner boundary of the workpiece. Let C be the circle representing the axis. The curve Γ will be deformed to C. The deformation will create forces directed to the center of the curves, depending on the position of C relative to Γ.

Call x the center of C, and say that the radius is R. Take a small angle $d\theta$. The force required to deform Γ in C considering just this section will be proportional to the deformed area dA.

Let $u(\theta)$ be the unitary circle with parameter $0 < \theta < 2\pi$. We can use the same parameter for $\Gamma(\theta)$. Now is possible to calculate the area $dA = Ru(\theta) - (\gamma(\theta) - x)d\theta$.

Then the total force is proportional to the integral.

$$F = k \int_0^{2\pi} Ru(\theta) - (\gamma(\theta) - x)d\theta$$

Consider that

$$\int_0^{2\pi} u(\theta)d\theta = 0$$

We obtain

$$\frac{1}{2\pi}\int_0^{2\pi}\vec{\gamma}(\theta)d\theta = \vec{x}$$

From here we can see that x is in fact the average of Γ in the plane.
Lets estimate the integral using

$$\frac{1}{2\pi}\sum_{k=1}^{N}\vec{\gamma}(2\pi k)\frac{2\pi}{N} \approx \vec{x} \Rightarrow \frac{1}{N}\sum_{k=1}^{N}\vec{\gamma}(2\pi k) \approx \vec{x}$$

Then we can see that x is in fact an average, and that we can estimate it very easily.

5.2 Modelling the Rotation

If the center of rotation of the workpiece do not coincide with its center of mass, an oscillating force is applied on the axis. This variable rotating force depends on,

- the mass of the work piece m
- angular speed of the rotating axis w
- the distance between the center of mass and the center of rotation d

$$a_r = dw^2$$

$$F = mdw^2$$

where a_r is radial acceleration and F is a centrifugal force. In fact, this force will be applied all the time in the same direction, if we fix our frame of reference on the axis.

We have some estimators for the center of rotation, but not for the center of mass of the rotating piece. The estimation of the center of mass will require the assumption of constant density on the material. And we will use finally the outer boundary.

With this assumption there are many ways to calculate the area and center of mass of a polygon. By assuming that the polygon is not self-intersecting we can use the following formula to calculate the rectangular coordinates for the center of mass.

$$A = \frac{1}{2}\sum_{i=0}^{N-1}(X_iY_{i+1} - X_{i+1}Y_i)$$

$$C_x = \frac{1}{6A}\sum_{i=0}^{N-1}(X_i + X_{i+1})(X_iY_{i+1} - X_{i+1}Y_i)$$

$$C_y = \frac{1}{6A}\sum_{i=0}^{N-1}(Y_i + Y_{i+1})(X_iY_{i+1} - X_{i+1}Y_i)$$

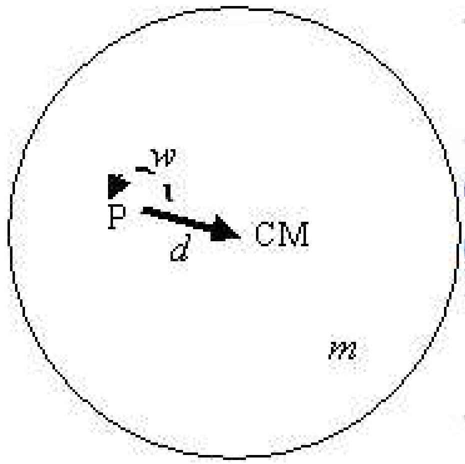

Figure 9: Force Generated by the Rotation of the Flywheel

The deviations of the center from the center of mass will result in an increment in the moment of inertia of the workpiece. We can make use of the Parallel axis theorem, which enables us to calculate the moment of inertia through any axis parallel to the axis through the center of mass of the body. If the moment of inertia increases, we need more energy to rotate the workpiece as the next formula, the parallel axis theorem, shows,

$$I_P = I_{CM} + md^2, \tag{1}$$

The kinetic energy of the rotating flywheel is given by the formula,

$$K_P = \frac{1}{2} I_P \omega^2 \tag{2}$$

here

- I_P is the moment of Inertia at the rotating center P.
- I_{CM} is the moment of Inertia throgh the axis at the center of mass.
- K_P is the Rotational Kinetic Energy

This means that workpieces with large deviations d will require high energy to move and will produce larger forces in the axis. This can be dangerous for the parts.

6 Conclusions

After having studied several methods used to measure the out of roundness for a given curve, and assuming the existence of clear tolerance intervals, we are in a good position to estimate if the dimensions of a worked piece are in its tolerance level. We have several possibilities to realize this analysis, the reference circles or the limacon approximation.

We have considered also a second possibility of not having clear tolerance intervals. And we have tried to say something related with the quality of the workpiece using only the data coming from the coordinate measuring machine.

Two characteristics defining the quality were worked. First we have assumed that the workpiece should fit properly in some rotating axis. We can use the maximal inscribed circle to give a bound for the maximal radius for the inner boundary, and we have another approach using a simple elastic model of the workpiece. In this way we have a new estimator for the center of rotation of the piece.

We can use then this rotation center and the information on the outer boundary to understand the behavior of the flywheel when it rotates. Forces and energy considerations have been mentioned.

Those two aspects can be also used to measure the quality of the workpiece independently of the existence of tolerance levels.

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