Quantum theory of charged-particle beam optics

by

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Abstract

Charged-particle beam optics, or the theory of transport of charged-particle beams through electromagnetic systems, is traditionally dealt with using classical mechanics. Though the classical treatment has been very successful, in designing and working of numerous charged-particle optical devices, it is natural to look for a deeper understanding based on the quantum theory, since any system is quantum mechanical at the fundamental level. With this motivation, the quantum theory of charged-particle beam optics is being developed currently by Jagannathan *et al.*; this formalism is specifically adapted to treat the problems of beam optics. The present thesis is an elaboration of this new formalism of the quantum theory of charged-particle beam optics with illustrations of applications to several practically important systems. The essential content of the thesis can be summarized briefly as follows.

Quantum mechanics of the optics of charged-particle beams transported through an electromagnetic lens or other such optical systems is analyzed, at the level of single-particle dynamics, treating the electromagnetic fields as classical and disregarding the radiation aspects, using essentially an algebraic approach. The formalism is based on the basic equations of quantum mechanics appropriate to the situations. For situations when either there is no spin or spin can be treated as a spectator the scalar Klein-Gordon and Schrödinger equations are used as the basic equations for relativistic and nonrelativistic cases respectively. For spin-$\frac{1}{2}$ particles, a treatment based on the Dirac equation is presented taking fully into account the spinor character of the wavefunction. The underlying powerful algebraic machinery of the formalism makes it possible to do computations to any degree of accuracy in any situation from electron microscopy to accelerator optics. The power of the formalism is demonstrated by working out the examples which include the axially symmetric magnetic round lens (of importance for electron microscopy and other micro-electron-beam device technologies) and the magnetic quadrupole lens (of importance for accelerator optics). It is found that the quantum theory at the scalar (spin-less) level gives rise to interesting small additional contributions to the classical paraxial and aberrating behaviours. These contributions are directly proportional to powers of the de Broglie wavelength. The Dirac theory further gives rise to spinor contributions which are also directly proportional to powers of the de Broglie wavelength.. Thus, it is clear that these quantum contributions are of significance only at very low energies; this explains the grand success of the classical theory so far.

It is very interesting to note that the quantum correction terms arising from the Klein-Gordon theory and the scalar approximation of the Dirac theory do not coincide and have some small differences between them. The classical, or geometrical, charged-particle optics is obtained in the classical limit of the quantum theory as should be.

The formalism based on the Dirac theory is further applied to the study of the spin-dynamics of a Dirac particle with anomalous magnetic moment being transported through a magnetic optical element. This naturally leads to a unified treatment of both the orbital (the Lorentz and the Stern-Gerlach forces) and the spin (Thomas-Bargmann-Michel-Telegdi equation) motions. This is illustrated by computing, under the paraxial approximation, the transfer maps for the phase-space and spin components in the cases of normal and skew magnetic quadrupole lenses. The quantum mechanics of the concept of spin-splitter devices, proposed recently for achieving polarized beams, is also understood using our formalism.

An alternate approach to the quantum theory of charged-particle beam optics based on the Wigner phase-space distribution function is also presented briefly, restricting to the example of magnetic round lens treated under the paraxial approximation. The possibility of extension of such an approach to the Dirac, or the spinor case, is also noted.

The concluding section lists some interesting observations and points out a few directions for future research.

Publications leading to the Thesis

Parts of the research work leading to this thesis have been published, which include:

1. S. A. Khan and R. Jagannathan
   Theory of relativistic electron beam transport based on the Dirac equation
   Proceedings of the 3rd National Seminar on Physics and Technology of Particle Accelerators and their Applications (Nov. 1993, Calcutta), Ed. S. N. Chintalapudi (IUC-DAEF, Calcutta) 102-107.

2. S. A. Khan and R. Jagannathan
   Quantum mechanics of charged-particle beam optics: An operator approach
   Presented at the *JSPS-KEK International Spring School on High Energy Ion Beams – Novel Beam Techniques and their Applications* (March 1994, Japan), `Preprint: IMSc/94/11` (The Institute of Mathematical Sciences, Madras, Mar. 1994).

3. S. A. Khan and R. Jagannathan
   On the quantum mechanics of charged particle beam transport through magnetic lenses
   Phys. Rev. E **51**, 2510-2516 (1995).

4. R. Jagannathan and S. A. Khan
   Wigner functions in charged-particle optics
   Selected Topics in Mathematical Physics – Professor R. Vasudevan Memorial Volume, Eds. R. Sridhar, K. Srinivasa Rao and V. Lakshminarayanan (Allied Publishers, New Delhi, 1995) 308-321.

5. R. Jagannathan and S. A. Khan
   Quantum theory of the optics of charged particles
   Advances in Imaging and Electron Physics Vol.**97**, Ed. P. W. Hawkes (Academic Press, San Diego, 1996) 257-358.

6. M. Conte, R. Jagannathan, S. A. Khan and M. Pusterla
   Beam optics of the Dirac particle with anomalous magnetic moment
   Particle Accelerators **56**, 99-126 (1996).

7. S. A. Khan
   Transport of Dirac-particle beams through magnetic quadrupoles
   `Preprint: IMSc/96/33` (The Institute of Mathematical Sciences, Madras, Dec. 1996).