Sikarin Yoo-Kong
syk500@york.ac.uk

Towards an entanglement measure based on the partial transpose

Stefan Weigert
slow500@york.ac.uk

Introduction

Entanglement corresponds to inseparable states of composite quantum systems. The question of how to distinguish **separable** from **entangled** states plays an important role in the theory of quantum information since entanglement is a resource to perform certain tasks.

Furthermore, it is important to quantify the amount of entanglement for a given state. For **pure** states, such a measure of entanglement is essentially unique, given by the von Neumann entropy [1]. For **mixed** states, many proposals to measure entanglement exist [2]. Many of them are difficult to actually calculate.

Conditions on measures of entanglement [3]

E1. The entanglement measure $E(\rho)=0$ iff ρ is a **separable** state.

E2. The entanglement measure $E(\rho)$ is **invariant** under **l**ocal unitary **o**perations, $U_{LO}=U_A\otimes U_B$:

$$E\left(U_{LO}\rho U_{LO}^{\dagger}\right)=E(\rho)$$

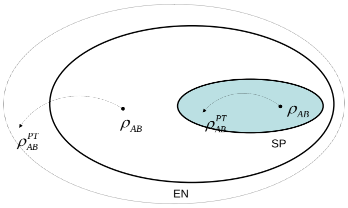
E3. The measure **does not increase** under LOCC operations:

$$E(\rho)\geq E\left(\Phi_{LOCC}\rho\right)$$

where Φ_{LOCC} is defined a LOCC map.

A continuous version of PT

In this contribution, we would like to establish an alternative approach to define the amount of entanglement of a given state. Our method is based on the **PPT-criterion** [4] which distinguishes successfully between entangled and separable states of low-dimensional systems.



The action of the **partial transposition** on separable / entangled states, respectively.

Consider composite systems of dimensions 2x2 or 2x3. We form a **convex combination** of the given density matrix ρ_{AB} with its partial transpose ρ_{AB}^{PT},

$$\rho_{AB}(\lambda)=(1-\lambda)\rho_{AB}+\lambda\rho_{AB}^{PT};\quad 0\leq\lambda\leq 1$$

which interpolates between a density matrix and a matrix with one **negative** eigenvalue if ρ_{AB} is entangled.

A candidate for a measure

Based on the convex combination of the density matrix and its partial transpose, we propose a candidate of a measure of entanglement given by

$$E_{PT}(\rho_{AB})=1-\lambda_c$$

where

$$\lambda_c=\max_{\lambda\in[0,1]}\left\{E_k\left((1-\lambda)\rho_{AB}+\lambda\rho_{AB}^{PT}\right)\geq 0,\quad k=1.....4(6)\right\}$$

is the **critical value** of λ : for $\lambda_c+\varepsilon$ with any positive ε at least one of the eigenvalues $E_k\left(\rho_{AB}(\lambda)\right)$ becomes negative.

Pure states

In this case, we can show that

E1. This measure can distinguish between entangled and separable pure states but it does not differentiate between degrees of entanglement of pure states.

$$E_{PT}\left(|\Psi\rangle\langle\Psi|_{sep}\right)=0 \text{ and } E_{PT}\left(|\Psi\rangle\langle\Psi|_{en}\right)=1$$

E2. We can also show that

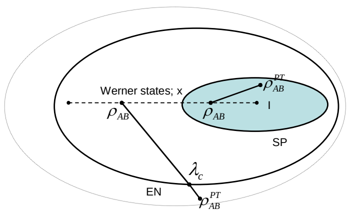
$$E_{PT}\left(U_A\otimes U_B|\Psi\rangle\langle\Psi|U_A^{\dagger}\otimes U_B^{\dagger}\right)=E_{PT}\left(|\Psi\rangle\langle\Psi|\right)$$

with $|\Psi\rangle$ either a product or an entangled state.

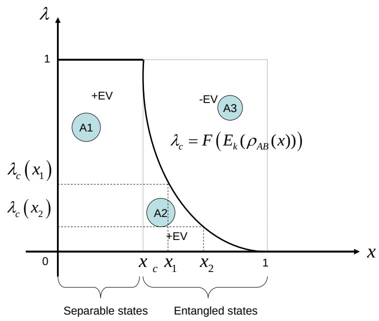
E3. We do not know whether $E_{PT}(\rho_{AB})$ decreases under LOCC:

$$E_{PT}\left(\Phi_{LOCC}|\Psi\rangle\langle\Psi|\right)\overset{?}{\leq}E_{PT}\left(|\Psi\rangle\langle\Psi|\right)$$

Mixed states: Werner states



$$\rho_{AB}(x)=x\rho_{ent}+(1-x)\rho_{sep}$$



$$\rho_{AB}(\lambda)=(1-\lambda)\rho_{AB}(x)+\lambda\rho_{AB}^{PT}(x)$$

E1. We can show that $E_{PT}\left(\rho_{AB}(0\leq x\leq x_c)\right)=.0$

For $x_c<x\leq 1$, we find that $E_{PT}\left(\rho_{AB}(x_c<x\leq 1)\right)=1-F\left(E_k\left(\rho_{AB}(x)\right)\right)$

E2. We are optimistic that we can show invariance under LO:

$$E_{PT}\left(U_{LO}\rho_{AB}(x)U_{LO}^{\dagger}\right)\overset{?}{=}E_{PT}\left(\rho_{AB}(x)\right)$$

E3. The decrease of $E_{PT}(\rho_{AB})$ has been shown for **some** LOCC operations but not for all LOCC:

$$E_{PT}\left(\Phi_{LOCC}\rho_{AB}(x)\right)\overset{?}{\leq}E_{PT}\left(\rho_{AB}(x)\right)$$

Conclusions and Discussion

The PT-entanglement measure can be shown to possess some fundamental properties attributed to a measure of entanglement. But various properties remain open:

1. Invariance under local unitary operations has been shown for pure states and is likely to hold for mixed states as well.

2. It remains to show that the PT-entanglement measure does not increase under LOCC operations for both pure or mixed states.

The proposed measure of entanglement is restricted to 2x2 and 2x3 dimensional quantum systems. For higher dimensions, one might suggest to use a similar approach using the reshuffling operation instead of the partial transpose.

References

1. C.H. Bennett, H.J. Bernstein, S. Popescu and B. Schumacher, Phys. Rev. A 53 (1996) 2046

2. M.B. Plenio and S. Virmani, quant-ph/0504163

3. V. Vedral et al., Phys. Rev. Lett. 78 (1997) 2275

4. A. Peres, Phys. Rev. Lett. 77 (1996) 1413