1. Solution: D

Let

G = event that a viewer watched gymnastics

B = event that a viewer watched baseball

S = event that a viewer watched soccer

Then we want to find

$$\Pr\left[(G\cup B\cup S)^c\right]=1-\Pr(G\cup B\cup S)$$

$$=1-\left[\Pr(G)+\Pr(B)+\Pr(S)-\Pr(G\cap B)-\Pr(G\cap S)-\Pr(B\cap S)+\Pr(G\cap B\cap S)\right]$$

$$=1-(0.28+0.29+0.19-0.14-0.10-0.12+0.08)=1-0.48=0.52$$

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2. Solution: A

Let R = event of referral to a specialist

L = event of lab work

We want to find

P[R∩L] = P[R] + P[L] – P[R∪L] = P[R] + P[L] – 1 + P[~(R∪L)]

= P[R] + P[L] – 1 + P[~R∩~L] = 0.30 + 0.40 – 1 + 0.35 = 0.05 .

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3. Solution: D

First note

$$P[A\cup B]=P[A]+P[B]-P[A\cap B]$$

$$P[A\cup B']=P[A]+P[B']-P[A\cap B']$$

Then add these two equations to get

$$P[A\cup B]+P[A\cup B']=2P[A]+(P[B]+P[B'])-(P[A\cap B]+P[A\cap B'])$$

$$0.7+0.9=2P[A]+1-P[(A\cap B)\cup(A\cap B')]$$

$$1.6=2P[A]+1-P[A]$$

$$P[A]=0.6$$

4. Solution: A

For $i=1$, 2, let

R_i = event that a red ball is drawn form urn i

B_i = event that a blue ball is drawn from urn i .

Then if x is the number of blue balls in urn 2,

$$0.44=\Pr[(R_1\cap R_2)\cup(B_1\cap B_2)]=\Pr[R_1\cap R_2]+\Pr[B_1\cap B_2]$$

$$=\Pr[R_1]\Pr[R_2]+\Pr[B_1]\Pr[B_2]$$

$$=\frac{4}{10}\left(\frac{16}{x+16}\right)+\frac{6}{10}\left(\frac{x}{x+16}\right)$$

Therefore,

$$2.2=\frac{32}{x+16}+\frac{3x}{x+16}=\frac{3x+32}{x+16}$$

$$2.2x+35.2=3x+32$$

$$0.8x=3.2$$

$$x=4$$

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5. Solution: D

Let N(C) denote the number of policyholders in classification C . Then

N(Young ∩ Female ∩ Single) = N(Young ∩ Female) – N(Young ∩ Female ∩ Married)

= N(Young) – N(Young ∩ Male) – [N(Young ∩ Married) – N(Young ∩ Married ∩ Male)] = 3000 – 1320 – (1400 – 600) = 880 .

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6. Solution: B

Let

H = event that a death is due to heart disease

F = event that at least one parent suffered from heart disease

Then based on the medical records,

$$P[H\cap F^c]=\frac{210-102}{937}=\frac{108}{937}$$

$$P[F^c]=\frac{937-312}{937}=\frac{625}{937}$$

and $P[H\mid F^c]=\dfrac{P[H\cap F^c]}{P[F^c]}=\dfrac{108}{937}\Big/\dfrac{625}{937}=\dfrac{108}{625}=0.173$

7. Solution: D

Let

A = event that a policyholder has an auto policy

H = event that a policyholder has a homeowners policy

Then based on the information given,

$$\Pr(A \cap H) = 0.15$$

$$\Pr(A \cap H^c) = \Pr(A) - \Pr(A \cap H) = 0.65 - 0.15 = 0.50$$

$$\Pr(A^c \cap H) = \Pr(H) - \Pr(A \cap H) = 0.50 - 0.15 = 0.35$$

and the portion of policyholders that will renew at least one policy is given by

$$0.4\ \Pr(A \cap H^c) + 0.6\ \Pr(A^c \cap H) + 0.8\ \Pr(A \cap H)$$

$$= (0.4)(0.5) + (0.6)(0.35) + (0.8)(0.15) = 0.53 \qquad (= 53\%)$$

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8. Solution: D

Let

C = event that patient visits a chiropractor

T = event that patient visits a physical therapist

We are given that

$$\Pr[C] = \Pr[T] + 0.14$$

$$\Pr(C \cap T) = 0.22$$

$$\Pr(C^c \cap T^c) = 0.12$$

Therefore,

$$0.88 = 1 - \Pr[C^c \cap T^c] = \Pr[C \cup T] = \Pr[C] + \Pr[T] - \Pr[C \cap T]$$

$$= \Pr[T] + 0.14 + \Pr[T] - 0.22$$

$$= 2\Pr[T] - 0.08$$

or

$$\Pr[T] = (0.88 + 0.08)/2 = 0.48$$

9. Solution: B

Let

M = event that customer insures more than one car

S = event that customer insures a sports car

Then applying DeMorgan's Law, we may compute the desired probability as follows:

$$\Pr(M^c \cap S^c) = \Pr\left[(M \cup S)^c\right] = 1 - \Pr(M \cup S) = 1 - \left[\Pr(M) + \Pr(S) - \Pr(M \cap S)\right]$$

$$= 1 - \Pr(M) - \Pr(S) + \Pr(S|M)\Pr(M) = 1 - 0.70 - 0.20 + (0.15)(0.70) = 0.205$$

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10. Solution: C

Consider the following events about a randomly selected auto insurance customer:

A = customer insures more than one car

B = customer insures a sports car

We want to find the probability of the complement of A intersecting the complement of B (exactly one car, non-sports). But P ($A^c \cap B^c$) = 1 – P ($A \cup B$)

And, by the Additive Law, P ($A \cup B$) = P (A) + P (B) – P ($A \cap B$).

By the Multiplicative Law, P ($A \cap B$) = P (B | A) P (A) = 0.15 * 0.64 = 0.096

It follows that P ($A \cup B$) = 0.64 + 0.20 – 0.096 = 0.744 and P ($A^c \cap B^c$) = 0.744 = 0.256

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11. Solution: B

Let

C = Event that a policyholder buys collision coverage

D = Event that a policyholder buys disability coverage

Then we are given that P[C] = 2P[D] and P[$C \cap D$] = 0.15 .

By the independence of C and D, it therefore follows that

0.15 = P[$C \cap D$] = P[C] P[D] = 2P[D] P[D] = $2(P[D])^2$

$(P[D])^2$ = 0.15/2 = 0.075

P[D] = $\sqrt{0.075}$ and P[C] = 2P[D] = $2\sqrt{0.075}$

Now the independence of C and D also implies the independence of C^C and D^C . As a result, we see that P[$C^C \cap D^C$] = P[C^C] P[D^C] = (1 – P[C]) (1 – P[D])

= (1 – $2\sqrt{0.075}$) (1 – $\sqrt{0.075}$) = 0.33 .

12. Solution: E

"Boxed" numbers in the table below were computed.

	High BP	Low BP	Norm BP	Total
Regular heartbeat	0.09	0.20	0.56	0.85
Irregular heartbeat	0.05	0.02	0.08	0.15
Total	0.14	0.22	0.64	1.00

From the table, we can see that 20% of patients have a regular heartbeat and low blood pressure.

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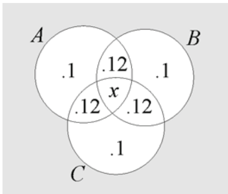
13. Solution: C

The Venn diagram below summarizes the unconditional probabilities described in the problem.



In addition, we are told that

$$\frac{1}{3} = P\left[A \cap B \cap C \mid A \cap B\right] = \frac{P\left[A \cap B \cap C\right]}{P\left[A \cap B\right]} = \frac{x}{x+0.12}$$

It follows that

$$x = \frac{1}{3}(x+0.12) = \frac{1}{3}x + 0.04$$

$$\frac{2}{3}x = 0.04$$

$$x = 0.06$$

Now we want to find

$$\begin{aligned} P\left[(A \cup B \cup C)^c \mid A^c\right] &= \frac{P\left[(A \cup B \cup C)^c\right]}{P\left[A^c\right]} \\ &= \frac{1 - P\left[A \cup B \cup C\right]}{1 - P\left[A\right]} \\ &= \frac{1 - 3(0.10) - 3(0.12) - 0.06}{1 - 0.10 - 2(0.12) - 0.06} \\ &= \frac{0.28}{0.60} = 0.467 \end{aligned}$$

14. Solution: A

$$p_k = \frac{1}{5}p_{k-1} = \frac{1}{5}\frac{1}{5}p_{k-2} = \frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}p_{k-3} = \ldots = \left(\frac{1}{5}\right)^k p_0 \quad k \geq 0$$

$$1 = \sum_{k=0}^{\infty} p_k = \sum_{k=0}^{\infty}\left(\frac{1}{5}\right)^k p_0 = \frac{p_0}{1-\frac{1}{5}} = \frac{5}{4}p_0$$

$p_0 = 4/5$.

Therefore, P[N > 1] = 1 – P[N ≤1] = 1 – (4/5 + 4/5 · 1/5) = 1 – 24/25 = 1/25 = 0.04 .

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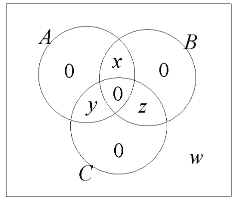
15. Solution: C

A Venn diagram for this situation looks like:



We want to find $w = 1 - (x + y + z)$

We have $x + y = \frac{1}{4}, \quad x + z = \frac{1}{3}, \quad y + z = \frac{5}{12}$

Adding these three equations gives

$$(x+y) + (x+z) + (y+z) = \frac{1}{4} + \frac{1}{3} + \frac{5}{12}$$

$$2(x+y+z) = 1$$

$$x + y + z = \frac{1}{2}$$

$$w = 1 - (x+y+z) = 1 - \frac{1}{2} = \frac{1}{2}$$

Alternatively the three equations can be solved to give x = 1/12, y = 1/6, z =1/4 again leading to $w = 1 - \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{4}\right) = \frac{1}{2}$