SPARE PARTS STOCK LEVEL CALCULATION

1.0 Objective

The purpose of this paper is to describe a simple technique to calculate the aircraft spare parts quantity (for fleet size and inventory) taking into account item reliability that is modeled by Poisson process.

2.0 Calculation Factors

The factors considered in this technique are:

1. The reliability of item to be spared (expressed as fail, removal, replacement or inversely as mean time between failures, removals **MTBR** in flight hours, replacements).
2. The number of items installed per aircraft (indicated as **A**).
3. Required probability that a spare will available when needed, that is the chance of having a spare part in inventory when required (90% =< **P** =< 95%), also called Fill Rate or confidence level.
4. Number of aircraft (fleet size) to be supported **N**.
5. Period to be supported as operational time or between initial and subsequent order (time **T** in months).
6. Average aircraft utilization (**M** in flight hour per month or day per aircraft). It may be in %, 7.5FH/day =7.5FH/24h=31.25%.

Spares are divided, for application of this method, in repairable and non-repairable. For repairable parts, a stock level of spares is calculated to compensate items undergoing the process of repair.

7. For repairable items the average period that it takes to repair (indicated as time between repairs **RT**) is considered instead of time of support T of point 5 above.

It is necessary an additional stock level of spares to compensate for the condemnation or scrapage of repairable items.

8. The condemnation for repairable is expressed as scrap rate (e.g R ≅ 0.02 or 2%).

3.0 Poisson Distribution

This distribution is used quite frequently in spares quantity determination as a forecasting technique with use of reliability analysis.
The demand for spares covering replacement or failed items occurring as a result of maintenance action are events described as Poisson distributed when they occur at a constant average rate and the number of events occurring in any interval are independent of the number of events occurring in any other time interval.
For the purpose of reliability analysis

$$f(x;\lambda,t) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

where λ = failure rate
t = time considered (or total operation period of all flight item)
x = number of failure (or number of spares required)
λt = mean value (number of failures in time t)

For the case in terms of the probability of n or fewer failures in time t

$$R(t) = \sum_{x=0}^{n} \frac{(\lambda t)^x e^{-\lambda t}}{x!} = e^{-\lambda t}\left[1 + \lambda t + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{(\lambda t)^n}{n!}\right]$$

4.0 Calculation Method

4.1 Non-reparable items

For these items the number of failures is equal to number of spares. The quantity of spares is the minimum value of **n** that satisfies the following condition

$$P =< \sum_{x=0}^{n} \frac{(\lambda t)^x e^{-\lambda t}}{x!} = e^{-\lambda t}\left[1 + \lambda t + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{(\lambda t)^n}{n!}\right] \text{ (as close as possible)}$$

where t = total operating time of all flight items or

$$\lambda t = \frac{1}{MTBR} t = \frac{AxNxMxT}{MTBR} = \frac{AxNxM}{MTBR} T$$

Note: the above equation can also directly be used as a deterministic method for computing spares quantity.
T may be mean resupply time, order or production lead time.

4.2 Example

Calculate the spare quantity of a non-repairable item which is installed 4 units per a/c (A=4EA) and having a mean time between removal of 7.500 flight hours (MTBR=7.500FH) for a fleet of 2 aircraft (N=2a/c) operating each one 225 flight hours per month (M=225FH/month/ac) and an initial period of 2 years (T=24months) to achieve a confidence level of 90% (P=0.90)?

$$\frac{AxNxMxT}{MTBR} = \frac{(4EA)x(2ac)x(225FH/ac/month)x(24month)}{7.500FH} = 5.76EA$$

So recursively
for 0 spare, P = exp(-5.76) = 0.003 = 0.3% < 90%
for 1 spare, P = 0.003(1+5.76) = 0.02 = 2% < 90%
for 2 spares, P = 0.003(6.76+16.6) = 0.07 = 7% < 90%
for 3 spares, P = 0.003(23.36+31.85) = 0.17 = 17% < 90%
for 4 spares, P = 0.003(55.21+45.9) = 0.303 = 30.3% < 90%
for 5 spares, P = 0.003(101.1+52.8) = 0.462 = 46.2% < 90%
for 6 spares, P = 0.003(153.91+50.7) = 0.614 = 61.4% < 90%
for 7 spares, P = 0.003(204.6+41.7) = 0.74 = 74% < 90%
for 8 spares, P = 0.003(246.3+30.1) = 0.83 = 83% < 90%
for 9 spares, P = 0.003(276.4+34.2) = 0.932 = 93.2% **> 90%**

and the recommended quantity is **9 spares**.

4.3 Repairable Items

For these items the number of failure is different from number of spares. It is a direct application of Palm's theorem.
The stock level of spare is to compensate repairable items undergoing the process of repair and expressed by

$$\lambda t = \frac{1}{MTBR}t = \frac{AxNxMxRT}{MTBR} = \frac{AxNxM}{MTBR}RT$$

To replace first failed item it is necessary an initial single spare. Substituting n by (n-1) in previous equation the probability method becomes

$$P =< \sum_{x=0}^{n-1} \frac{(\lambda t)^x e^{-\lambda t}}{x!} = e^{-\lambda t}\left[1 + \lambda t + ... + \frac{(\lambda t)^{n-1}}{(n-1)!}\right]$$

4.4 **Example**

Considering previous example, calculate spares for a repairable item with time to repair of 3 months (RT = 3 months).

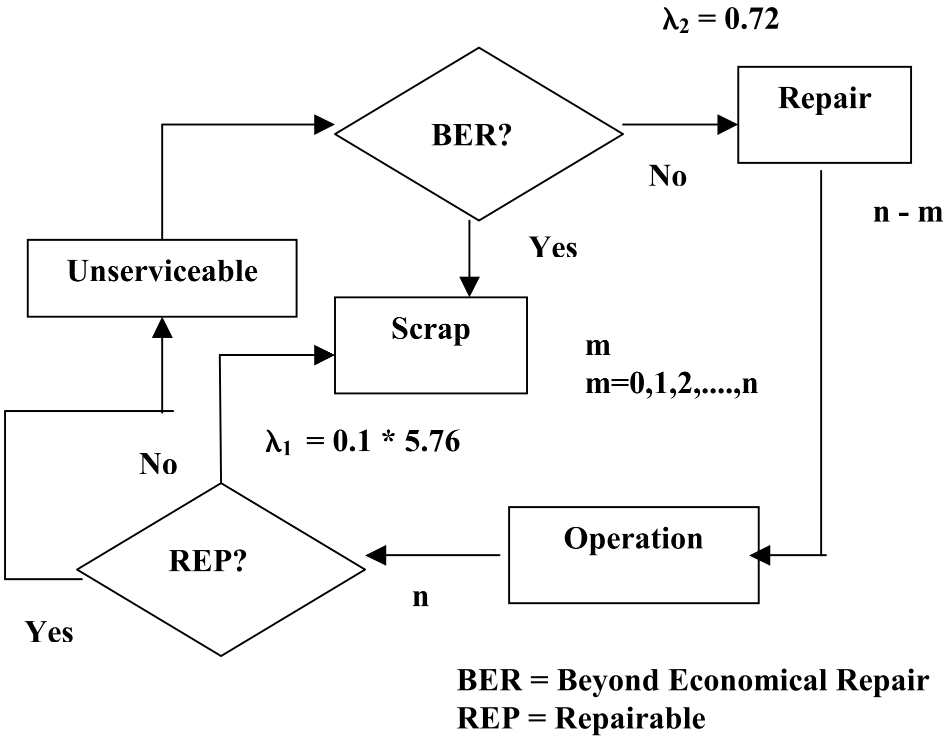
$$\frac{AxNxMxRT}{MTBR} = \frac{(4EA)x(2ac)x(225FH/ac/month)x(3month)}{7.500FH} = 0.72EA$$

For 0 spare, P = exp(-0.72) = 0.487 = 48.7% < 90%
for 1 spare, P = 0.487(1+0.72) = 0.837 = 83.7% < 90%
for 2 spares, P = 0.487(1.72+0.259) = 0.963 = 96.3% **> 90%**

The recommended quantity is (n-1) = 2 or **n = 3 spares**.

4.5 **Repairable Items with Scrap Rate**

Repairable items returned to shop are sometimes condemned, i.e., not repaired because through inspection, it is decided that some items were not economically feasible to repair. The condemnation generally results in an increase of spare parts requirement. For this case, the probability method is result of a combination of repairable and non-repairable calculation.
The description of process is as below and figures are from above examples.



4.6 Example

If it is considered a repairable, item with scrap rate of 10% (R=0.10) in the above problem how many spares are necessary?

1st approach

9 failures (from 4.2) in the period requiring maintenance and 0.1 x 9 ≅ 1 EA becomes scrap. The total number to support the operation is 3 EA (from 4.4) plus 1 EA discarded summing **4 spares**.

2nd approach (neglecting scrap during repair)

The total mean is given by

$\lambda_1 + \lambda_2 = 0.72 + 0.1 * 5.76 = 1.296$

For 0 spare, P = exp(-1.296) = 0.2736 = 27.36% < 90%
for 1 spare, P = 0.2736(1+1.296) = 0.6282 = 62.82% < 90%
for 2 spares, P = 0.2736(2.296+0.840) = 0.8580 = 85.80% < 90%
for 3 spares, P = 0.2736(3.136+0.363) = 0.9573 = 95.73% > **90%**
for 4 spares, P = 0.2736(3.499+0.118) = 0.9895 = 98.95% > **90%**
for 5 spares, P = 0.2736(3.617+0.030) = 0.9978 = 99.78% > **90%**
The recommended quantity is (n-1) = 3 or **n = 4 spares**.

3rd approach (neglecting scrap during repair)

The probability for number of spares to cover items scrapped plus in repair are given seeing following table.
Numbers of items to scrap m and in repair (n-m) may have any distribution called P_1 and P_2.

Number of spares P	To scrap P_1	In repair P_2
n	m; mean=λ_1	(n – m); mean=λ_2
0	0	0
1	0	0; 1
	1	0
2	0	0; 1; 2
	1	0; 1
	2	0
….	….	….
n	0	0; 1; 2; ….; n
	1	0; 1; 2; ….; (n-1)
	2	0; 1; 2; ….; (n-2)
	….	
	m	0; 1; 2; ….; (n-m)
	….	….
	n-1	0; 1
	n	0

$P(n=0)=P(m=0;\lambda_1)*P((m-n)=0;\lambda_2)$ if P_1, P_2 are Poisson.

$$P(n=1)=P(m=0;\lambda_1)*(P((m-n)=0;\lambda_2)+P((m-n)=1;\lambda_2))+$$
$$+P(m=1;\lambda_1)*P((m-n)=0;\lambda_2)$$

$$P(n=2)=P(m=0;\lambda_1)*(P((m-n)=0;\lambda_2)+P((m-n)=1;\lambda_2)+P((m-n)=2;\lambda_2)+$$
$$+P(m=1;\lambda_1)*(P((m-n)=0;\lambda_2)+P((m-n)=1;\lambda_2))+$$
$$+P(m=2;\lambda_1)*P((m-n)=0;\lambda_2)$$

$$P(n) = P(n;\lambda_1)*P(0;\lambda_2)+$$
$$+P(n-1;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+$$
$$+P(n-2;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+P(2;\lambda_2))+$$
$$+ ....+$$
$$+P(m;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+...+P(n-m;\lambda_2))+$$
$$+..........+$$
$$+P(2;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+...+P(n-2;\lambda_2))+$$
$$+P(1;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+...+P(n-1;\lambda_2))+$$
$$+P(0;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+...+P(n;\lambda_2))$$

Or recursively

$P(n) = P(n-1) + P(n; \lambda_1)* P(0; \lambda_2)+ P(n-1; \lambda_1)* P(1; \lambda_2)+....+ P(1; \lambda_1)* P(n-1; \lambda_2) + P(0; \lambda_1)* P(n; \lambda_2)$

$P(n)=P(n-1)+Conv(P_1(n); P_2(n))$ n=1, 2, 3, …. and $P(0)=P_1(0)*P_2(0)$

where $Conv(P_1(n); P_2(n))$ means convolution of $P_1(n)$ and $P_2(n)$ for any distribution $P_1(n)$ and $P_2(n)$

For the numerical example:

i	$P(i; \lambda_1=0.576)$	$P(i; \lambda_2=0.720)$
0	0.5621	0.4868
1	0.3238	0.3505
2	0.0933	0.1262
3	0.0179	0.0303
….	….	….

If n = 0 then
P(0) = P(0; 0.576) * P(0; 0.72)
P = exp(-0.576) * exp(-0.72) = 0.5621*0.4868 = 0.2736 < 90%

If n = 1 then
P(1) = P(0; 0.576) *(P(0; 0.72)+ P(1; 0.72)) +
+ P(1; 0.576) * P(0; 0.72)=
= 0.5621 (0.4868+0.3505) +0.3238*0.4866= 0.6282 < 90%

If n = 2 then
P(2)=P(0; 0.576)*(P(0; 0.72)+P(1; 0.72)+P(2; 0.72))+
+P(1; 0.576)*(P(0; 0.72)+P(1; 0.72))+P(2; 0.576)*P(0; 0.72)=
=0.5621(0.4868+0.3505+0.1262)+0.3238(0.4868+0.3505)+
0.0933*0.4868=0.5416+0.2711+0.0454=0.8581 < 90%

If n = 3 and using recursive formula then
P(3)=P(2)+P(3;0.576)*P(0;0.72)+P(2;0.576)*P(1;0.72)+P(1;0.576)*
P(2;0.72)+P(0;0.576)*P(3;0.72)=
=0.8581+0.5621*0.0303+0.3238*0.1262+0.09333*0.3505+
+0.0179*0.4868=0.8581+0.0170+0.0409+0.0327+0.009=
=0.8581+0.0996=0.9577 **>90%**

The recommended quantity is (n-1) = 3 or **n = 4 spares**

Note 1 - It is observed that 2[st] and 3[rd] approaches lead to same results.

By 3[rd] approach and P(1) the equation gives for Poisson proccess:

$P(1)= P(0;\lambda_1)*(P(0;\lambda_2)+P(1;\lambda_2))+P(1;\lambda_1)*P(0;\lambda_2)=$

$$= \exp(-\lambda_1)[\exp(-\lambda_2) + \lambda_2 \exp(-\lambda_2)] + \lambda_1 \exp(-\lambda_1)\exp(-\lambda_2) =$$
$$= \exp(-(\lambda_1 + \lambda_2)) + \lambda_2 \exp(-(\lambda_1 + \lambda_2)) + \lambda_1 \exp(-(\lambda_1 + \lambda_2)) =$$
$$= \exp(-(\lambda_1 + \lambda_2)) + (\lambda_1 + \lambda_2)\exp(-(\lambda_1 + \lambda_2)) =$$

$= P(0; \lambda_1+\lambda_2) + P(1; \lambda_1+\lambda_2)$ and so on.

Note 2 - If one wants to consider scrap during repair process then the scrap rate is applied to λ_2 and added to mean of Poisson expression per 2[nd] approach.