In Repair j = R(t)	p(r) (*)	Fill Rate R(t) < n	Available Spare s = S(t)	Waiting Spare i = W(t)	Expected Number In Repair	Expected Nr of Available Spares	P(n,s=0) PBO(n)	P[n,i=0]	Expected Number of Waited Spares E[BO(w)]	$E[BO^2(n)]$ $Var[BO(n)] =$ $= E[BO^2(n)] - E^2[BO(n)]$
0	p(0)	p(0)	**n**	0	0*p(0)	n*p(0)		p(0)		
1	p(1)	p(0)+p(1)	**n-1**	0	0*p(0)+1*p(1)	n*p(0)+(n-1)*p(1)		p(0)+p(1)		
2	p(2)	p(0)+p(1)+ +p(2)	**n-2**	0	0*p(0)+ … + +2*p(2)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)		p(0)+p(1)+ +p(2)		
3	p(3)	p(0)+p(1)+ +p(2)+p(3)	**n-3**	0	0*p(0)+ … + +3*p(3)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)+(n-3)*p(3)		p(0)+p(1)+ +p(2)+p(3)		
…..	….	…..	**…..**	0		…..		…..		
n-2	p(n-2)	p(0)+p(1)+ +p(2)+p(3)+ …+ +p(n-2)	**2**	0	0*p(0)+ … + +(n-2)*p(n-2)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)+(n- 3)*p(3)+ +….+2*p(n-2)		p(0)+p(1)+ +p(2)+p(3)+…+ +p(n-2)		
n-1	p(n-1)	$\sum_{j=0}^{n-1} p(j)$	**1**	0	0*p(0)+ … + +(n-1)*p(n-1)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)+(n- 3)*p(3)+….+ +2*p(n-2)+1*p(n-1)		p(0)+p(1)+ +p(2)+p(3)+…+ +p(n-1)		
n	p(n)		**0**	**0**	0*p(0)+ … + +(n-1)*p(n-1)+ +n*p(n)	$\sum_{j=0}^{n} (n-j) * p(j)$	p(n)	$\sum_{j=0}^{n} p(j)$		
n+1	p(n+1)		0	**1**	0*p(0)+ … + +(n+1)*p(n+1)		p(n+1)		1*p(n+1)	1^2*p(n+1)
n+2	p(n+2)		0	**2**	etc		p(n+1)+ + p(n+2)		1*p(n+1)+2*p(n+2)	1^2*p(n+1)+2^2*p(n+2)
n+3	p(n+3)		0	**3**	etc		etc		1*p(n+1)+ … +3*p(n+3)	1^2*p(n+1)+ … + +3^2*p(n+3)
…..	…..	…..	0	**…..**	……	………	….		…..	…..
∞ (t → ∞)		$\sum_{j=0}^{\infty} p(j) = 1$	0	**∞**	$\sum_{j=0}^{\infty} j * p(j)$	$\sum_{j=0}^{\infty} (n-j) * p(j)$	$\sum_{j=n}^{\infty} p(j)$		$\sum_{j=n+1}^{\infty} (j-n) * p(j)$ or $\sum_{i=1}^{\infty} i * p(n+i)$	$\sum_{j=n+1}^{\infty} (j-n)^2 * p(j)$ or $\sum_{i=1}^{\infty} i^2 * p(n+i)$

(*) **p(j)** is poisson or negative binomial distribution. **n** = initial spares.

$$FillRate(n) + PBO(n) = \sum_{j=0}^{n-1} p(j) + \sum_{j=n}^{\infty} p(j) = 1 \Rightarrow PBO(n) = 1 - FillRate(n)$$

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Fill Rate = FR = $\sum_{j=0}^{n-1} p(j)$

Expected Number of Items in Repair ER (or in pipeline) = PL = $\sum_{j=0}^{\infty} j * p(j)$

For Poisson distribution ER = PL = Failure (or Repair) rate (λ) * Time to Repair (T) = λ * T

Expected Number of Available Spares (Expected Stock) = ES = $\sum_{j=0}^{n} (n-j) * p(j)$ **where n is initial stock n=S(t=0)**

Expected Number of Waited Spares (or Expected BackOrder) = EBO = $\sum_{j=n+1}^{\infty} (j-n) * p(j)$

$$\sum_{j=0}^{\infty} (n-j) * p(j) = \sum_{j=0}^{n} (n-j) * p(j) + \sum_{j=n+1}^{\infty} (n-j) * p(j) = \sum_{j=0}^{n} (n-j) * p(j) - \sum_{j=n+1}^{\infty} (j-n) * p(j) = \textbf{ES} - \textbf{EBO}$$

$$\textbf{ES} - \textbf{EBO} = \sum_{j=0}^{\infty} (n-j) * p(j) = \sum_{j=0}^{\infty} n * p(j) - \sum_{j=0}^{\infty} j * p(j) = n * \overbrace{\sum_{j=0}^{\infty} p(j)}^{1} - \sum_{j=0}^{\infty} j * p(j) = \textbf{n} - \textbf{PL}$$

ES – EBO = n – PL or

n = PL + ES - EBO

with PL = **λ * T for Poisson distribution**

The above expression is known as "stock balance equation".

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Spares Inventory Model - Simulation with MS-Excel

Failue Rate => 0,02 **Time to Repair =>** 150

Spares In Repair N(t)	p(j) Poisson	Fill Rate P(j) Poisson	Initial Spare	Spares on Hand	Spare on Back-order	Expected Num-ber of Spares in Repair	Expected Num-ber of Spares on Hand	Expected Num-ber of Spares on Backorder	Expected Num-ber of squa-re of BO	Variance
0	0,049787068	0,049787068	**9**	9	0	0,000000000	0,4480836153	0,0000000000	0,0000000000	0,0000000000
1	0,149361205	0,199148273	9	8	0	0,149361205	1,6429732561	0,0000000000	0,0000000000	0,0000000000
2	0,224041808	0,423190081	9	7	0	0,597444820	3,2112659097	0,0000000000	0,0000000000	0,0000000000
3	0,224041808	0,647231889	9	6	0	1,269570243	4,5555167557	0,0000000000	0,0000000000	0,0000000000
4	0,168031356	0,815263245	9	5	0	1,941695666	5,3956735344	0,0000000000	0,0000000000	0,0000000000
5	0,100818813	0,916082058	9	4	0	2,445789734	5,7989487881	0,0000000000	0,0000000000	0,0000000000
6	0,050409407	0,966491465	9	3	0	2,748246174	5,9501770083	0,0000000000	0,0000000000	0,0000000000
7	0,021604031	0,988095496	9	2	0	2,899474394	5,9933850712	0,0000000000	0,0000000000	0,0000000000
8	0,008101512	**0,996197008**	9	1	0	2,964286488	6,0014865830	0,0000000000	0,0000000000	0,0000000000
9	0,002700504		9	0	0	2,988591024	**6,0014865830**	0,0000000000	0,0000000000	0,0000000000
10	0,000810151		9	0	1	2,996692536		0,0008101512	0,0008101512	0,0008094948
11	0,000220950		9	0	2	2,999122989		0,0012520518	0,0016939525	0,0016923848
12	0,000055238		9	0	3	2,999785840		0,0014177646	0,0021910907	0,0021890806
13	0,000012747		9	0	4	2,999951553		0,0014687531	0,0023950448	0,0023928876
14	0,000002732		9	0	5	2,999989794		0,0014824107	0,0024633331	0,0024611355
15	0,000000546		9	0	6	2,999997989		0,0014856886	0,0024830001	0,0024807928
16	0,000000102		9	0	7	2,999999628		0,0014864056	0,0024880192	0,0024858098
17	0,000000018		9	0	8	2,999999935		0,0014865502	0,0024891761	0,0024869663
18	0,000000003		9	0	9	2,999999989		0,0014865773	0,0024894202	0,0024872102
19	0,000000000		9	0	10	2,999999998		0,0014865821	0,0024894677	0,0024872578
20	0,000000000		9	0	11	3,000000000		0,0014865829	0,0024894764	0,0024872664
21	0,000000000		9	0	12	3,000000000		0,0014865830	0,0024894778	0,0024872679
22	0,000000000		9	0	13	3,000000000		0,0014865830	0,0024894781	0,0024872681
23	0,000000000		9	0	14	3,000000000		0,0014865830	0,0024894781	0,0024872682
24	0,000000000		9	0	15	3,000000000		0,0014865830	0,0024894781	0,0024872682
25	0,000000000		9	0	16	3,000000000		0,0014865830	0,0024894781	0,0024872682
26	0,000000000		9	0	17	3,000000000		0,0014865830	0,0024894781	0,0024872682
27	0,000000000		9	0	18	3,000000000		0,0014865830	0,0024894781	0,0024872682
28	0,000000000		9	0	19	3,000000000		0,0014865830	0,0024894781	0,0024872682
29	0,000000000		9	0	20	3,000000000		0,0014865830	0,0024894781	0,0024872682
30	0,000000000		9	0	21	**3,000000000**		**0,0014865830**	**0,0024894781**	**0,0024872682**

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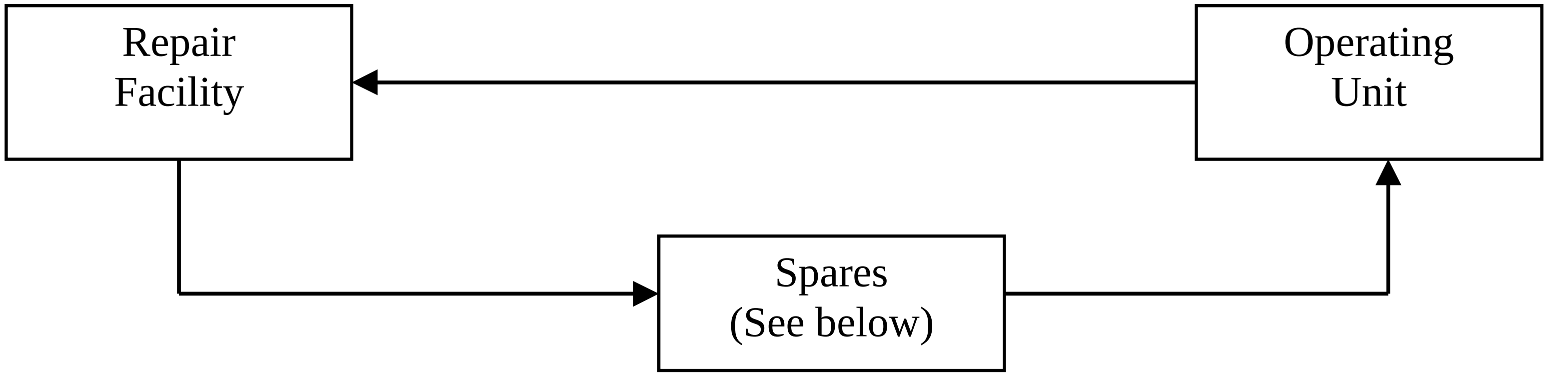

Part Type = 1, 2, 3, ... , k

Demand Rate = $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k$

Time to Repir = $\nu_1, \nu_2, \nu_3, \ldots, \nu_k$

Quantity of Parts = $n_1, n_2, n_3, \ldots, n_k$

Cost of Parts = $c_1, c_2, c_3, \ldots, c_k$

Budget = c_0

MS-Excel Formula with $n_1\text{-}1, n_2\text{-}1, n_3\text{-}1, \ldots, n_k\text{-}1 \geq 0$

Maximize

$$R = \frac{\lambda_{1*}\text{Poisson}(n_1\text{-}1, \lambda_1*\nu_1, \text{True}) + \lambda_{2*}\text{Poisson}(n_2\text{-}1, \lambda_2*\nu_2, \text{True}) + \lambda_{3*}\text{Poisson}(n_3\text{-}1, \lambda_3*\nu_3, \text{True}) + \ldots + \lambda_{k*}\text{Poisson}(n_k\text{-}1, \lambda_k*\nu_k, \text{True})}{\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_k}$$

With restriction of budget ($) $n_1*c_1 + n_2*c_2 + n_3*c_3 + \ldots + n_k*c_k \leq c_0$

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