Expected Spares Stock and Spares Expected Backorder

Fail or Demand (n)	**Fail or Demand Probability**	**Fill Rate**	**Initial Stock**	**Spares Stock (Over Stock)**	**Spares Back Order (Under Stock)**	**Expected Stock**	**Expected Back Order**
0	P(0)	P(0)	X	X		X*P(0)	
1	P(1)	P(0)+P(1)	X	X-1		**X*P(0) + + (X-1)*P(1)**	
2	P(2)	P(0)+P(1)+ +P(2)	X	X-2		**X*P(0) + + (X-1)*P(1) + + (X-2)*P(2)**	
...	...	...	...	...		...	
X-2	P(X-2)	P(0)+P(1)+ +P(2)+....+ +P(X-2)	X	2		**X*P(0) + + (X-1)*P(1) + + (X-2)*P(2) + ... + + 2*P(X-2)**	
X-1	P(X-1)	**$\sum_{n=0}^{X-1} P(n)$**	X	1		**X*P(0) + + (X-1)*P(1) + + (X-2)*P(2) + ... + + 2*P(X-2) + + 1*P(X-1)**	
X	P(X)		X	0		**$\sum_{n=0}^{X} (X-n)*P(n)$**	
X+1	P(X+1)		X		1		1*P(X+1)
X+2	P(X+2)		X		2		1*P(X+1) + + 2*P(X+2)
X+3	P(X+3)		X		3		1*P(X+1) + + 2*P(X+2) + + 3*P(X+3)
...	...		...		...		
∞	P(n=∞)	**$\sum_{n=0}^{\infty} P(n) = 1$**	X		∞		**$\sum_{n=X+1}^{\infty} (n-X)*P(n)$** **or** **$\sum_{j=1}^{\infty} j*P(X+j)$**

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EBO CALCULATION (Ref.: Diaz & Fu, Becker)

$$EBO = \sum_{k=s+1}^{\infty} (k-s) * POISSON(k, mean)$$ (definition expression)

$$E\left[B(s)\right] = E\left[B(s-1)\right] - \sum_{k=s}^{\infty} POISSON\ (k, mean)$$ or

$$E\left[B(s)\right] = E\left[B(s-1)\right] - \left[1 - \sum_{k=0}^{s-1} POISSON\,(k, mean)\right]$$ (recursive expression)

$$EBO = PL * POISSON\ (s, PL, FALSE\ ) + (PL - s) * \left[1 - POISSON\ (s, PL, TRUE)\right]$$

(general expression)

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Recursive expression deduction for Poisson distribution:

$$E\left[B(s)\right] = \sum_{k=s+1}^{\infty}(k-s)*POISSON(k,mean) =$$

$$= POISSON(k=s+1,mean) + \sum_{k=s+2}^{\infty}(k-s)*POISSON(k,mean)$$

$$E\left[B(s+1)\right] = \sum_{k=s+2}^{\infty}(k-s-1)*POISSON(k,mean) =$$

$$= \sum_{k=s+2}^{\infty}(k-s)*POISSON(k,mean) - \sum_{k=s+2}^{\infty}POISSON(k,mean)$$

$$E\left[B(s+1)\right] = E\left[B(s)\right] - POISSON(k=s+1,mean) - \sum_{k=s+2}^{\infty}POISSON(k,mean) =$$

$$= E\left[B(s)\right] - \sum_{k=s+1}^{\infty}POISSON(k,mean) =$$

$$= E\left[B(s)\right] - (1 - \sum_{k=0}^{s}POISSON(k,mean))$$

$$\boxed{E\left[B(s+1)\right] = E\left[B(s)\right] - \left(1 - POISSON\left(s;mean;TRUE\right)\right)}$$

(MS Excel notation)

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General expression deduction (of EBO with Poisson distribution)

$$EBO(s) = \sum_{k=s+1}^{\infty}(k-s)*Poisson(k,pipeline) =$$

$$= \sum_{k=s+1}^{\infty}k*Poisson(k,pipeline) - s * \sum_{k=s+1}^{\infty}Poisson(k,pipeline) =$$

$$= \sum_{k=s+1}^{\infty}k*Poisson(k,PL) - s\left[1-PoissonAc\left(s,PL\right)\right]$$

$$\sum_{k=s+1}^{\infty}k*Poisson(k,PL) = (s+1)\frac{PL^{s+1}}{(s+1)!}e^{-PL} + (s+2)\frac{PL^{s+2}}{(s+2)!}e^{-PL} + (s+3)\frac{PL^{s+3}}{(s+3)!}e^{-PL} + ... =$$

$$= PL * e^{-PL}\left[\frac{PL^{s}}{s!} + \frac{PL^{s+1}}{(s+1)!} + \frac{PL^{s+2}}{(s+2)!} + \frac{PL^{s+3}}{(s+3)!} + \frac{PL^{s+4}}{(s+4)!} + \frac{PL^{s+5}}{(s+5)!} + ...\right] =$$

$$= PL\left[1 - PoissonAc(s-1,PL)\right] = PL\left\{1 - \left[PoissonAc(s,PL) - Poisson(s,PL)\right]\right\}$$

$$EBO(s) = PL\left[1 - PoissonAc(s,PL) + Poisson(s,PL)\right] - s\left[1 - PoissonAc(s,PL)\right] = PL\left[1 - PoissonAc(s,PL)\right] + PL * Poisson(s,PL) - s\left[1 - PoissonAc(s,PL)\right]$$

$$\boxed{EBO(s) = PL * Poisson(s,PL) + (PL - s)\left[1 - PoissonAc(s,PL)\right]}$$

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Expected Stock with Poisson distribution

$$\text{ExpectedStock} = \text{ES} = \sum_{k=0}^{S} (S-k) * Poisson\ (k, pipeline\ ) = \sum_{k=0}^{S} S * Poisson(k, pipeline) - \sum_{k=0}^{S} k * Poisson(k, pipeline) =$$

$$= S * \sum_{k=0}^{S} Poisson(k, pipline) - \sum_{k=0}^{S} k * Poisson(k, pipeline) = S * PoissonAc(S, pipeline) - \sum_{k=0}^{S} k * Poisson(k; pipeline)$$

$$\sum_{k=0}^{S} k * Poisson(k, pipeline) = e^{-PL} \left[0 * \frac{PL^{0}}{0!} + 1 * \frac{PL^{1}}{1!} + 2 * \frac{PL^{2}}{2!} + \frac{PL^{3}}{3!} + \text{..........} + S * \frac{PL^{S}}{S!} \right] =$$

$$= PL * e^{-PL} * \left[1 + \frac{PL^{1}}{1!} + \frac{PL^{2}}{2!} + \frac{PL^{3}}{3!} + \text{..........} + (S-1) * \frac{PL^{S-1}}{(S-1)!} \right] = \text{PL * PoissonAc(S-1, PL)}$$

$$ES = S * PoissoAc(S, pipeline) - PL * PoissonAc(S-1, pipeline) = S * PoissoAc(S, pipeline) - PL * \left[PoissonAc(S, pipeline) - Poisson(S, pipeline) \right]$$

ExpectedStock = ES(S) = (S – PL) * PoissonAc(S, PL) +PL * Poisson (S, PL)

If EBO(S) = PL*Poisson(S, PL) + (PL-S)*[1 – PoissonAc(S, PL)] = PL*Poisson(S, PL) + (PL-S) – (PL-S)*PoissonAc(S, PL) =

= PL*Poisson(S, PL) + (PL-S) + (S - PL)*PoissonAc(S, PL) = ES(S) + PL – S

EBO(S) = ES(S) + PL - S

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