A fuzzy logic approach to modelling the mechanical behaviour of functionally graded materials (FGM).

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Abstract

Functionally graded materials (FGM), like other composites, are designed to achieve levels of performance superior to that of homogeneous materials by combining the desirable properties of each constituent phase. To offset the adverse effects of discontinuities in layered composites, the composition of an FGM is varied over length scales that are significant in comparison to the overall dimensions of the body. The shape of this material gradient is an important parameter in determining the properties of an FGM structure. It is through the variation of effective material properties (such as modulus) that the nature of the spatial variations of the constituents affects the thermo-mechanical behavior of the FGM. The manufacturing processes for these FGM have their loopholes (like simple geometry of components, gradient limited in one dimension). Hence further forming operations are required to widen its scope of applications, which is why it is necessary to foreordain the deformation behaviour of FGM by some mathematical approach. Hence a new technique based on the mathematical rules of fuzzy logic has been approached here. This approach scores over other modeling methods in the ease of use and range of inputs over which it can be used. The simulated values have been compared with experimental data from literature.

Introduction.

The need to improve service properties adapted to the working environment of a particular application has brought about the increasing acceptance of functionally graded materials in the recent past. FGM show an improved quality potential hence can withstand most harsh environments. FGM provide a solution to many advanced applications where two or more materials with distinct properties are required to be put together. FGM accommodates gradual transition of the properties of different materials, where the mismatches in mechanical and thermal properties are reduced to a minimum. There are, at present many manufacturing processes for the production of FGM, most of them being powder or melt metallurgical processes. These processes have been successful in producing FGM with very simple shape and geometry, hence a large domain of its applications have been cut short. Further processing of these gradient materials can widen their domain of applications manifold. In order to able to evaluate the feasibility of the deformation process the mechanical behaviour of the FGM has to be predetermined. In order to achieve this, a new approach has been developed using the theory of fuzzy logic has been employed.

Fuzzy logic

Fuzzy logic is one of the more popular techniques for the simulation of real life living or non living systems. Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth i.e., truth values between "completely true" and "completely false". It was introduced by Dr. Lofti Zadeh of UC, Berkeley in the 1960's as a means to model the uncertainty of natural language. This technique emulates the human decision making process in which not every decision is discrete. There is a certain amount of indecision, fuzziness in the decision. This concept has been developed by Dr. Zadeh and is now widely in use in a wide array of applications. This fuzzy logic approach to model the mechanical behaviour of Functionally Graded Materials (FGM), has been described in this paper

Approach

FGM, as the name suggests has a locally heterogeneous composition. This results in locally different deformation behaviour hence an inhomogeneous mechanical behaviour on deformation across the materials crossection. Hence it would lead to fundamentally erroneous results if the mechanical behaviour of FGM is directly determined by the flow curve.

Hence for the simulation of the mechanical behaviour of FGM we subdivide the material into zones where homogeneity can be assumed (for Eg. fig. 1). For these homogeneous zones yield strength, k_f (an inherent property of a material calculated from its flow curve) is calculated individually and then summarized using a mathematical model. The integration of the mathematical model into a fuzzy logic model enables the predetermination of the yield strength of every homogeneous zone. Hence it is possible to determine the plastic deformation behavior of FGM. The use of such an approach allows the minimization of experimental expenditure.

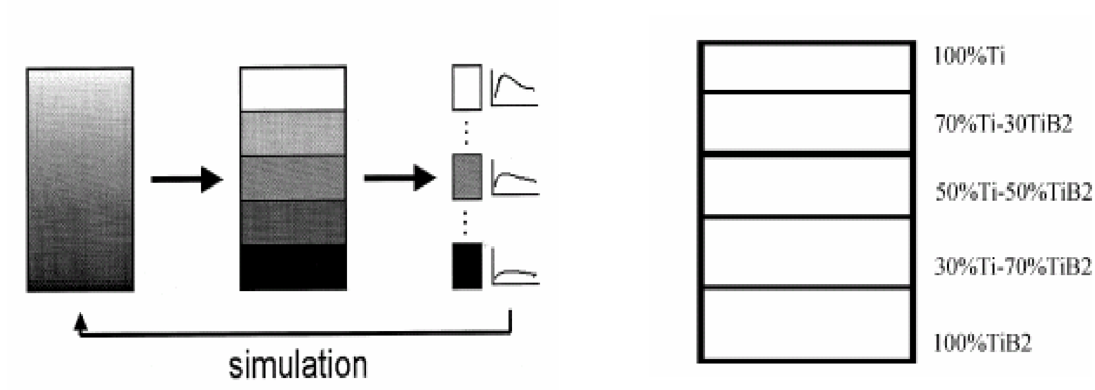


Fig. 1, Subdivision of the FGM into homogenous zones.

The main reason behind the usage of fuzzy logic for modeling the mechanical behaviour of FGM is that all the measured values represent no absolute value since they have some tolerances. Hence they can be considered as fuzzy values.

Model Theory.

Theory of the deformation behaviour of FGM.

The mechanical behaviour of a material is the response of a material when it is subjected to deformation. This deformation is primarily characterized by its yield strength which depends on both inherent properties (Eg, Microstructure, chemical composition etc.) and procedural details (Eg, Strain rate, Temperature etc.).From past and present literature it is evident that many modeling approaches have been employed, all of which have been developed from the fundamental equation for calculation of yield strength (Equation 1) k_f, during hot working process. The dependence of k_f on deformation parameter is expressed by power and exponential functions associated multiplicatively.

$$k_f = k_{f_0} \cdot k_\partial \cdot k_\Phi \cdot k_\Psi \tag{1}$$

Here k_f is the yield strength and k_{f_0} is the basic value of yield strength and the other factors are thermodynamic deformation on the material.

The material discussed in this paper comes under the class of composite materials. A composite is a multiphase material formed from a combination of materials which differ in composition or form, remain bonded together, and retain their identities and properties. Composites maintain an interface between components and act in concert to provide improved specific or synergistic characteristics not obtainable by any of the original components acting alone. Composites include:

- Fibrous (composed of fibers, and usually in a matrix).
- Laminar (layers of materials).
- Particulate (composed of particles or flakes, usually in a matrix) and
- Hybrid (combinations of any of the above).

If such a composite material has two or more ductile phases it becomes simple to estimate the yield strength of a material depending on the phase fraction using the phase rule of mixtures. In particulate composites the k_f in addition to the matrix composition, depends on the size of the particle size, shape and volume fraction (Eg, nanocomposites have better properties than micro size particle reinforced composites).

The most commercially viable and industrially used method for the manufacture of FGM is powder metallurgy. One of the main characteristics of powder metallurgical products is the porosity and hence it has to be considered. In case of such compressible materials volume is assumed to be variable. Hydrostatic stress also causes an effect and hence must be included. Hence a compaction term, Φ_i, i=1, 2, 3 must be included. In addition to all other factors Φ_p degree of compaction must also be included in the consideration (Equation 2).

$$\Phi_1 + \Phi_2 + \Phi_3 + \Phi_p = 0 \qquad (2)$$

Since the relative density of the material poses a resistance to deformation its effect must be included in the model used and it can be represented by equation 3.

$$k_f = k_{f_0} \cdot \rho_{rel}^2 \qquad (3)$$

Here k_f is the yield strength, k_{f_0} is the yield strength of the incompressible material and ρ_{rel} is the relative density of the material which is equal to 1 for the incompressible material.

Theory of approximation using fuzzy logic.

Fuzzy logic is a mathematical theory of inexact reasoning that allows modeling of the reasoning process of humans in linguistic terms. It is very suitable in defining the relationship between system inputs and desired outputs. Hence it is used in many control, production and optimization processes. The approach using fuzzy logic model involves three steps in which the approximation is done – Fuzzification, inference and defuzzification and this process has been illustrated in great detail below.

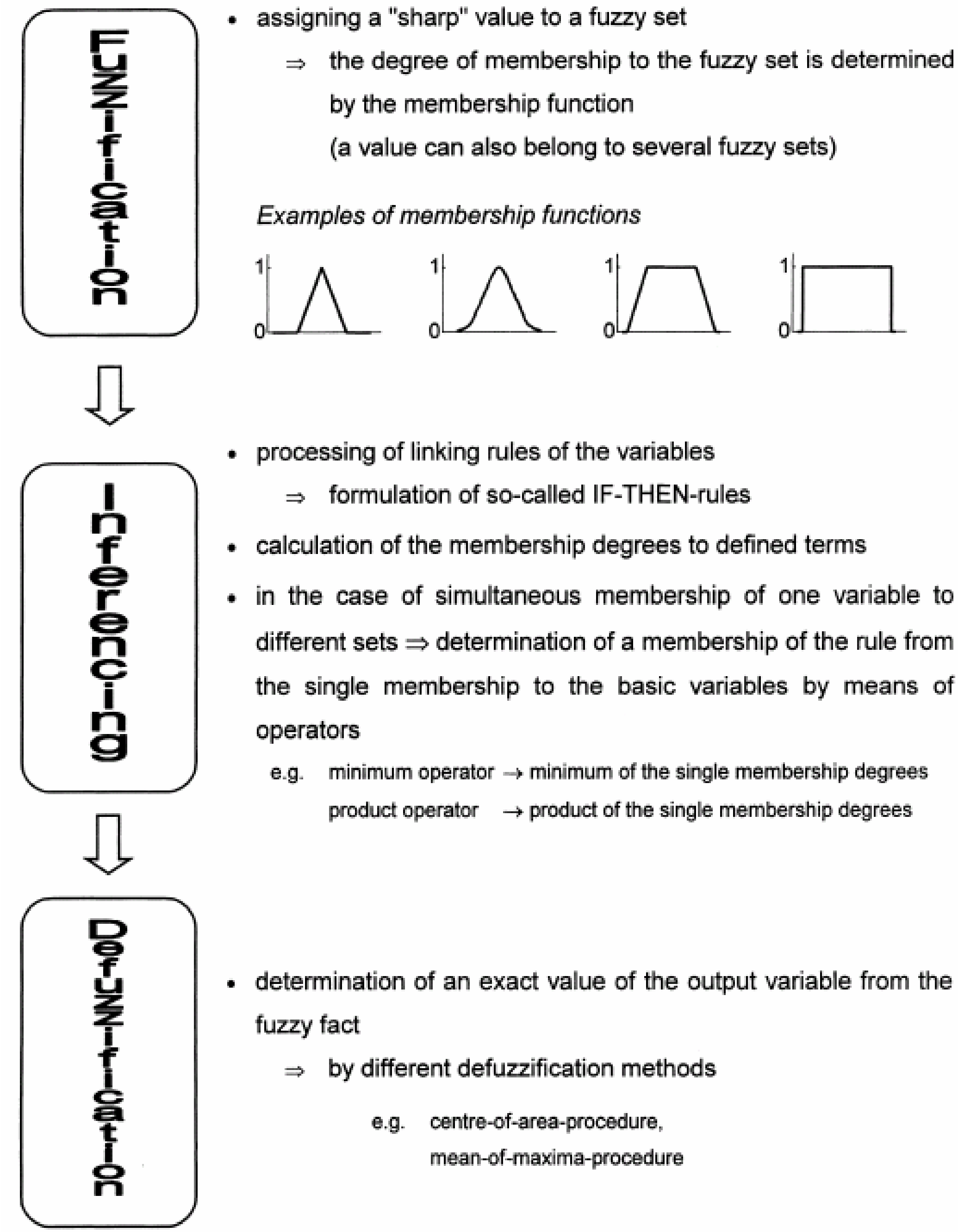
- Firstly the values are fuzzified i.e., they are associated with membership functions. And then getting the sharp value to the fuzzy set by determining the degree of membership from the membership functions. The membership function can be of many types but the triangular membership function is usually used for simplicity.

- Formulating the IF-THEN-ELSE linking rules and calculation of the degrees of membership according to the formulated rules.

- In case of simultaneous membership of one variable to different sets, membership of the rule from single member to the basic variables has to be determined.

- Defuzzification by determination of the exact value of the fuzzy variable from the fuzzy fact. This can be achieved by different defuzzification criteria; mostly center of area criteria is used owing to its simplicity.

The above steps have been schematically expressed below,



By the use of such an algorithm the calculation of k_f can be carried out more efficiently. The values of the yield strength $k_f = f(\varphi)$ along with the deformation parameter and the composition were used as inputs. Here the numeric inputs were fuzzyfied by assigning triangular, Δ membership function. The membership degree of a given value was calculated using equation 4.

$$\mu = \frac{|X_A - x|}{W} \tag{4}$$

Here μ is the membership degree and X_A is the basic variable (k_{f_0}) and x is the given value k_f and W is the width of the membership function.

The input variables were associated by IF-THEN-ELSE rules with each other. For calculating the membership of a rule both minimum and product operators were tested. The defuzzification step was done using a modified center of area method as shown in equation 5.

$$x_s = \frac{\sum_{i=1}^{n}(x_i \cdot \mu_i)}{\sum_{i=1}^{n}\mu_i} \qquad (5)$$

Here μ_i is the membership degree for the rule i.

Results

The tested material systems have shown expected theoretical dependencies of k_f on the influencing variables. k_f is proportional to the deformation rate and inversely proportional to the deformation temperature as well as composition. More amount of plastic flow occurs in systems with low yield strength and the ones with high yield strength act as rigid die and is almost undeformed.

The comparison between fuzzy logic simulated and experimental data for the composite system Al/Cu shows good qualitative agreement with each other. The variation of yield strength with the deformation rate has been shown in figure 2.

The compressive flow curve of the same composite has also been simulated and plotted and it can be clearly seen that the simulated values agree with the experimental data to a great extent as can be seen in figure 3. This eliminates the large inaccuracies found in conventional phenomenological approaches when the parameter range of the influencing variables is widened. Hence the use of fuzzy logic does not limit the amount of parametric considerations. However it has to be taken care, that the width of the membership function has to be chosen such that no definition gaps result (in case of W being small) or the computed values approximate too strongly (in case of W being large).

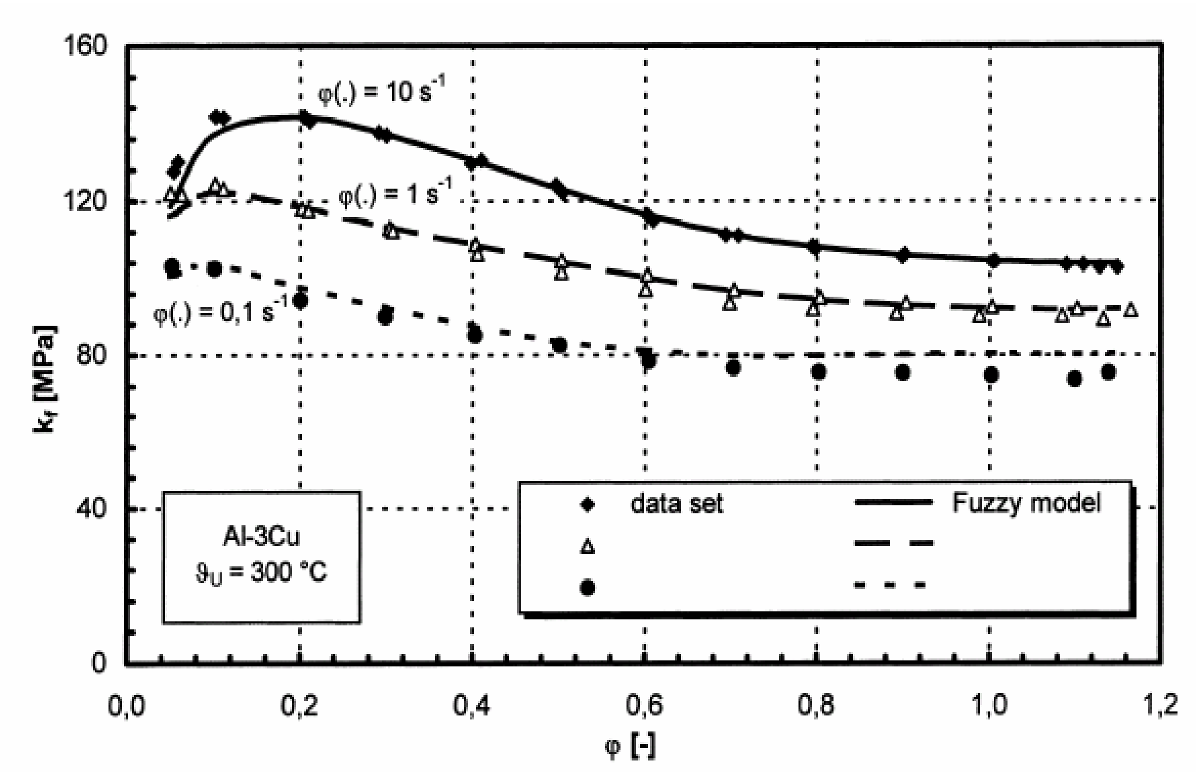


Figure 2, Comparison of simulated and experimental flow curves at constant temperature for Al/Cu composite system.

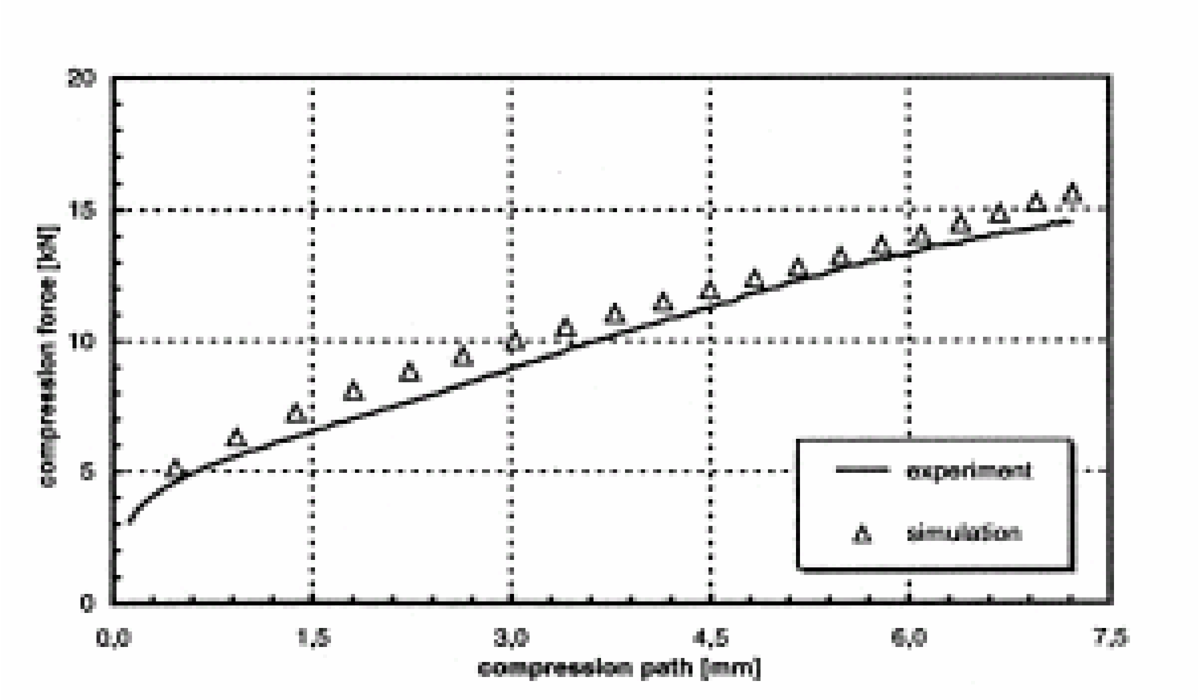


Figure 3, comparison of the fuzzy approximated and experimental compressive flow curve for the Al/Cu composite system used.

For the simulation of the deformation behaviour of FGM, fuzzy approximation such as the integration of the variation in phase fraction is necessary and this approach created no hindrances in such computations. Hence in general it can be evaluated that the simulation quality can be greatly improved by the use of fuzzy approximation.

Summary and conclusions.

An approach was made to model the mechanical deformation of Functionally Graded Material using fuzzy logic. The model was based of the fundamental equation of yield strength. The fuzzy approximated values showed the deformation behaviour of FGM, qualitatively as well as quantitatively with high accuracy. The calculation using fuzzy logic is ideally suited to this problem.

However, as has been found out in past and present the computational time required for large data sets is still on the higher side and further work must be done to optimize the time of computation. Also work can be done by the integration of Finite Element Model with the current model which might result in better simulation. Further work can also include evolutionary algorithms for multiobjective mechanisms, which is highly probable in the working environment of engineering components.

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