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A NOTE ON GENERALIZED SUPERPOSITION OF COHERENT STATES

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In this comment we present an improvement of our recent proposal [*Int. J. Mod. Phys.* **B12**, 1495 (1998)] to generate a generalized superposition of two coherent states. The basic idea here is to use a dispersive atom-field interaction together with a single atom quantum state engineering.

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Experiments done recently have achieved superposition states in different ways, presenting several schemes to prepare Schrödinger-cats and study their decoherence. In the work done by Monroe *et al.*[1] a "Schrödinger-cat"-like state of matter was generated using a single trapped ion and the work presented by Brune *et al.*[2] have produced a mesoscopic superposition of quantum states involving radiation fields with classically distinct phases.

In view of those important experiments a comment improving our previous published paper[3] seems to be very appropriated. In that work we have studied a generalized superposition of two coherent states denoted by ICCS (Intermediate Coherent–Coherent State), as well as their generation and nonclassical properties. With respect to the proposal presented for its generation we have based our arguments in what is currently named in the literature as "quantum state engineering" (QSE).[4] In Ref. 3 we have employed the QSE in the *resonant* case, which assumes the atom-field interaction Hamiltonian

$$\widehat{\mathcal{H}}_{\text{int}} = \hbar\lambda(\hat{\sigma} + \hat{a} + \hat{\sigma}^- \hat{a}^+) \tag{1}$$

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where λ is the atom-field coupling constant, which is proportional to the atomic dipole and to the field strength; $\hat{\sigma}^+(\hat{\sigma}^-)$ is the raising (lowering) Pauli operator and $\hat{a}^+(\hat{a})$ is the creation (annihilation) field operator.

We have shown in Ref. 3 how to generate the ICCS, by controlling the passage of N successive atoms, one at a time, through a high-Q cavity. However, in our previous proposal two difficulties emerge: (i) when the average excitation of the desired field to be created inside the cavity increases, then the number N of atoms necessary to create this field also increases. This characteristic of the method implies a bad "side effect": when N increases decoherence effects (due to unavoidable interaction of the cavity mode with its environment) proceed at faster rate destroying the "integrity" of the field state[5]; (ii) the ICCS defined in Ref. 3 as

$$|\psi(\xi,\phi)\rangle = \eta\left[\sqrt{\xi}|\alpha_1\rangle + e^{i\phi}\sqrt{1-\xi}|\alpha_2\rangle\right], \tag{2}$$

was represented in the Fock's basis as

$$|\psi(\xi,\phi)\rangle = \sum_n C_n|n\rangle \tag{3}$$

where

$$C_n = \eta(\xi,\phi)e^{-r^2/2}\left(\frac{r^n}{\sqrt{n!}}\right)\left(\sqrt{\xi}\,e^{in\theta_1} + \sqrt{1-\xi}\,e^{i(n\theta_2+\phi)}\right) \tag{4}$$

with $\alpha_j = re^{i\theta_j}$, $j = 1, 2$. In Ref. 3 we have shown how to generate a field state like in Eq. (2), obeying the condition (3) [plus (4)]. The point, however, is that there is not a unique set of parameters $\xi, \phi, \alpha_1, \alpha_2$ defining the coefficient C_n in Eq. (4). So, as it seems, the method of QSE employed in Ref. 3 would be free of such ambiguity only when concerning with the generation of states being a one-parameter dependent.

In this brief report, to circumvent these two difficulties, we have modified our previous strategy by alternating using a **dispersive** interaction, also employed extensively in the literature.[6] In this case, instead of using the Eq. (1), we will employ a atom-field interaction given by

$$\widehat{\mathcal{H}}_{\text{int}} = \hbar\lambda\hat{\sigma}_z\hat{a}^+\hat{a} \tag{5}$$

where $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the atomic inversion operator, $|e\rangle$ ($|g\rangle$) standing for the excited (ground) state of the (two-level) atom interacting with the field mode. As a result of this interaction the effective action of the Hamiltonian (5) upon the initial state of the whole system, atom plus field, is given by

$$|\Psi_{\text{AF}}(\tau)\rangle = (e^{-i\phi\hat{n}}|e\rangle\langle e| - |g\rangle\langle g|)|\Psi_{\text{AF}}(0)\rangle \tag{6}$$

where $\phi = \Omega^2\tau/\delta$, with Ω the Rabi frequency, τ is the interaction time between atom field inside the cavity, $\delta = \omega - \omega_0$ is the detuning parameter, $\omega(\omega_0)$ is the field (atomic) frequency.

Assuming that the atom, initially prepared in a first Ramsey zone R_1, in a superposition state (with $\xi \in [0, 1]$ and η being a normalization constant),

$$|\Psi_{\text{A}}(0)\rangle = \eta\left(\sqrt{\xi}\,|g\rangle + e^{i\varphi}\sqrt{1-\xi}\,|e\rangle\right) \tag{7}$$

enters a cavity where there is a field, initially prepared in a coherent state, described by

$$|\Psi_{\mathrm{F}}(0)\rangle = |\alpha\rangle \,. \tag{8}$$

We then find that, by substituting Eqs. (7) and (8) in Eq. (6),

$$\Psi_{\mathrm{AF}}(\tau)\rangle = \eta\left(\sqrt{\xi}\,|g\rangle\,|\alpha\rangle + e^{i\varphi}\sqrt{1-\xi}\,|e\rangle\,|\alpha'\rangle\right) \tag{9}$$

with $\alpha' - e^{-i\phi}\alpha$.

Next, we let the atom traverse a second Ramsey zone R_2, in which it suffers a $\pi/2$ state rotation, leading its component $|e\rangle$ ($|g\rangle$) to $|e\rangle + |g\rangle$ ($|g\rangle - |e\rangle$), such that

$$\begin{aligned}|\Psi_{\mathrm{AF}}(\tau)\rangle = \eta\Big\{&|g\rangle\left(\sqrt{\xi}\,|\alpha\rangle + e^{i\varphi}\sqrt{1-\xi}\,|\alpha'\rangle\right)\\ &- |e\rangle\left(\sqrt{\xi}\,|\alpha\rangle - e^{i\varphi}\sqrt{1-\xi}\,|\alpha'\rangle\right)\Big\}\end{aligned} \tag{10}$$

So, e.g., by detecting the atom in the state $|g\rangle$, the field is projected in the state

$$|\Psi_{\mathrm{F}}(\tau)\rangle = \eta\left(\sqrt{\xi}\,|\alpha\rangle + e^{i\varphi}\sqrt{1-\xi}\,|\alpha'\rangle\right). \tag{11}$$

The state in Eq. (11) coincides with the ICCS in Eq. (2) for the identification $\alpha = \alpha_1$ and $\alpha' = \alpha_2$.

The advantage of the present alternative upon that proposed in Ref. 3 has two aspects: first, the important fact that only a single atom is used in the present QSE. So decoherence effects are no longer dependent on the field excitations, due to a large number N of atoms, as in Ref. 3. Second, the parameters defining our ICCS $(\xi, \phi, \alpha_1, \alpha_2)$ are all explicitly carried through the present QSE. For our present case, the experimental setup is the same as shown in Fig. 15 of Ref. 3, with the special changes in the atom-field interaction from *resonant* [cf. Eq. (1)] to *dispersive* [cf. Eqs. (5)], and in the initial state of the whole system [cf. Eqs. (7), 8].

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