On Reliability Analysis of Forward Loop Forward Hop Networks

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Abstract. A common technique to improve the reliability of loop (or ring) networks is by introducing link redundancy; that is, by providing several alternative paths for communication between pairs of nodes. With alternative paths between nodes, the network can now sustain several node and link failures by bypassing the faulty components. However, faults occurring at strategic locations in a ring can prevent the computation by disrupting I/O operations, blocking the flow of information, or even segmenting the structure into pieces which can no longer be suitable for any practical purpose.

An extensive characterization of fault-tolerance in FLFH networks is given in this paper. The characterization has revealed several properties which describe the problem of constructing subrings and linear arrays in the presence of node failures in the FLFH network for a specific link configuration. Also in this paper, bounds are established on the degree of fault tolerance achievable in a redundant FLFH network when performing a computation that requires a fixed number of operational nodes. Also the bounds on the size of the problems guaranteed to be solved in the presence of a given number of faults in the network are derived.

1 Introduction

Loop (or ring) networks are commonly used for the interconnection of computers that are in close proximity. Reliability is a severe problem in single-loop networks; any single node or link failure can disrupt communication. A way to provide higher reliability in loop networks is to introduce redundancy. The idea here is to design regular multi-connected loop network so that several alternative paths for communication exists between nodes. In other words, each node is connected to two or more nodes in the network. With alternate paths between nodes, the network can sustain several node and link failures. Several ring networks, suggested in [1,4,12,14], are based on this principle. Although reliability can be improved in these networks, it is still possible that faults occurring in strategic locations in the network can logically disconnect the ring structure.

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Disconnection of a ring does not mean that the entire structure is unusable. In fact, it still might be possible to find one or more subrings or linear arrays in the original ring in spite of the failure of some nodes.

In this paper, the inherent limits to reconfigurability of FLFH networks are studied by using (*i*) the properties of catastrophic fault patterns, and (*ii*) the necessary and sufficient conditions for construction of subrings and linear arrays in presence of faults. In particular, bounds on the size of the problems one can be guaranteed to solve in the presence of a given number of faults in the network are derived.

The organization of this paper is as follows. In Section 2, basic concepts are introduced and basic conditions on FLFH network are discussed. In Section 3, properties of catastrophic fault patterns are discussed followed by the bounds on the sizes of connected components, such as subrings and linear arrays. An algorithm to get the optimal linear array in the generalized FLFH network is given is Section 4. Finally, limits to reconfiguration of FLFH topologies are studied in Section 5.

2 Preliminaries

The basic components of such a ring network are the processing elements (PEs) namely P_0 to P_{n-1}. We sometimes refer to a processor P_i as processor i. There are two kinds of links : *regular* and *bypass*. A regular link connects P_i to $P_{(i+1)\ mod\ n}$, its direction is from P_i to $P_{(i+1)\ mod\ n}$, while the bypass links connect non-neighbors. The bypass links are used strictly for reconfiguration schemes, otherwise they are redundant. From now on we use $\aleph(n, G)$ to denote that the ring network $\aleph$ has n PEs and has a redundancy set G. We now formally introduce the following definitions:

Definition 1. A FLFH network $\aleph(n, G)$ consists of a set $V = \{P_0, P_1, \ldots, P_{n-1}\}$ of n PEs and a set $G = \{1, 2, \ldots, g\}$ of g unidirectional links. We say $\aleph(n, G)$ has link redundancy or link configuration G if, the directed links are from p_i to $p_{i+t\ (\text{mod}\ n)}$ for $1 \le i \le n$ and $1 \le t \le g$.

We now define a redundant linear array and show its relationship with the FLFH network.

Let $S = \{LCU, A, RCU\}$ represent a redundant linear array in which LCU and RCU denote Left and Right Control Unit respectively and $A = \{P_0, P_1, \ldots, P_{n-1}\}$ denotes a redundant linear array of PEs. The LCU and RCU which interface with the linear array A are responsible for all the I/O functions. All the links are unidirectional. There exists a regular link from P_i to P_{i+1}, $0 \le i < n-1$. Any link connecting P_i to P_j where $j > i$ is a bypass link. The length of a bypass link, connecting P_i and P_j, is the distance in the array between P_i and P_j; i.e., $|j - i|$. Note that for each $P_i \in A$ with $i > N - g$ there exists a link from P_i to RCU. Similarly, LCU is connected to $P_1, P_2, \ldots, P_g$. Every P_i has in-degree (also out-degree) g, LCU has out-degree g, and RCU has in-degree g.

There is very close relationship between a FLFH network and a redundant linear array: a ring is essentially a redundant linear array in which end points of the array are connected to each other in a wrap-around manner rather than to LCU or RCU. We will denote the array A on n PEs by simply $A(n)$.

Definition 2. Given a ring network $\aleph(n, G)$, a fault pattern for $\aleph$ is a set of integers $F = \{ f_1, f_2, \ldots, f_m\}$ where $m \leq n$, $f_j \leq f_{j+1}$ and $f_j \in [0, n-1]$. The set F represents the set of faulty PEs in $\aleph$.

There are patterns of faults whose occurrence in $\aleph$ can block the flow of information from one side of the pattern to the other through the faulty region in any fixed direction. In the case of a linear array, the occurrence of one such fault pattern can cause the disconnection of the I/O ports. Such patterns are considered to be catastrophic for a linear array, but we cannot say it is really catastrophic for a ring.

Definition 3. A fault pattern F is *catastrophic* for an array $A(n)$ with link redundancy G if I and O are not connected in the presence of such an assignment of faults.

Definition 4. A fault pattern F is blocking for $\aleph(n, G)$ if it is catastrophic for a redundant linear array A on $n' \geq n$ nodes with redundancy G and vice versa.

Many properties and results of catastrophic fault patterns for linear arrays apply to FLFH networks. In the following we will summarize some known properties of catastrophic fault pattern for linear array and also characterize catastrophic fault pattern for FLFH networks.

3 Fault-Tolerance Analysis of FLFH Networks

In this section properties of catastrophic fault patterns in FLFH networks are discussed. This section also deals with the degree of fault tolerance that can be achieved in a ring in the presence of node failures. An extensive characterization of catastrophic fault patterns for linear array has been given in [6,7,8]. As discussed in [7,8], the only pattern that can be blocking for $A(n)$, where $G = \{1, 2, \ldots, g\}$, is a cluster of at least g faulty nodes.

Definition 5. The *decomposition* of a fault pattern F is a set $B = \{F_1, F_2, \ldots, F_p\}$, where

1. Each F_i is a set of contiguous faulty PEs,
2. $\forall\ i \neq j$, $F_i \bigcap F_j = \emptyset$,
3. $\bigcup_i F_i = F$
4. The cardinality p of the set B should be as minimum as possible.

Definition 6. For a fault pattern F, the *blocking dimension*, given by $D(F)$, is the number of elements in the decomposition of F of cardinality $\geq g$. Each of the F_i those count into blocking dimension is called a *blocking pattern.*

Definition 7. A fault pattern F is said to be *catastrophic* for $FLFH$ ring $\aleph(n, G)$ if $\aleph - F$ is not connected, that is, the network becomes disconnected once the faulty processors, and their incident links are removed.

3.1 Properties of Catastrophic Fault Patterns

Theorem 1. *F is catastrophic for a $FLFH$ ring $\aleph(n, G)$ if and only if $D(F) \geq 2$.*

Proof: Obviously, for $D(F) = 0$ the ring is not catastrophic. It will be proved that if $D(F) = 1$, then $\aleph - F$ is connected and, thus F is not catastrophic. $D(F) = 1$ means that there cannot be two disjoint blocking fault patterns. Consider two arbitrary non-faulty nodes x and y in $\aleph - F$. We need to show that one of the paths x to y or y to x, should be present. Now a path from x to y does not exit only if there is a blocking fault pattern among x, $x+1 \pmod n$, $x+2 \pmod n), \ldots, y$. Similarly no path exists from y to x, only if there is a blocking fault pattern among y, $y+1 \pmod n$, $y+2 \pmod n \ldots, x$. But, as there is only one blocking fault pattern, one of the paths x to y or y to x exists.

It suffices to show that if $D(F) = 2$ then F is catastrophic. Let $B = \{F_1, F_2, \ldots\}$ be a decomposition of F such that $|F_1| \geq g$ and $|F_2| \geq g$. Let S_1 (S_2) be the set of non-faulty nodes between F_1 and F_2 (F_2 and F_1) in the clockwise direction. Notice that S_1 and S_2 are disjoint and neither of the two are empty, otherwise $D(F) = 1$. Now take $x \in S_1$ and $y \in S_2$. Since F_1 and F_2 are blocking, there is neither a path from x to y nor from y to x. □

The occurrence of more than one disjoint blocking patterns thus logically disconnects a ring structure. However, disconnection of a ring does not mean that the entire structure is unusable. In fact, it still might be possible to find one or more subrings or linear arrays in the original ring in spite of the failure of some nodes. These subrings and linear arrays can be utilized in computations requiring fewer processing elements. We now deal with the degree of fault tolerance that can be achieved in a ring in the presence of node failures. The degree of fault tolerance is characterized in terms of bounds on the sizes of the subrings and linear arrays it might still be possible to obtain in the presence of faults.

3.2 Subring Results

Theorem 2. *For a $FLFH$ ring $\aleph(n, G)$ and a fault pattern F, $D(F) = 0$ if and only if there exists a subring $\aleph(n - |F|, G)$.*

Proof: $D(F) = 0$ implies that there is no fault cluster of size g or larger. That is, in the decomposition of F, $|F_i| = \ell_i < g$, $\forall i$. Since $G = \{1, 2, \ \ldots, \ g\}$, there exists a link of length $l_i + 1 \in G$ to bypass the fault pattern F_i. It is clear that a ring can be formed on the remaining $n - |F|$ nodes by using regular link of length 1 and links of length $l_i + 1$. □

Obviously, if there exists a ring on $n - |F|$ nodes, there exists a linear array on $n - |F|$ nodes.

3.3 Linear Array Results

Tyszer [13] had studied the problem of constructing a linar array in the ring when $|F| < 2g$. Tyszer's result is formally described by the following lemma.

Lemma 1. *If $|F| < 2g - 2$, it is possible to create a linear array $A(n - |F|)$.*

We now generalize this result for FLFH network $\aleph(n, G)$. Tyszer's result then follows as a simple corollary of this generalized result. The generalized version is as follows:

Theorem 3. *For a FLFH network $\aleph(n, G)$ and a fault pattern F, there exists a linear array $A(v)$ regardless of the number of faults $|F|$, where*

$$v = \frac{n - |F|}{Max\ \{1, D(F)\}}$$

Proof: If $D(F) = 0$, then the theorem follows from Theorem 2. Let $D(F) > 0$. Let $F_{i1}, F_{i2}, \ldots, F_{iD(F)}$ be the blocking fault patterns in the decomposition of F. Removal from $\aleph$ of these blocking patterns will disconnect the ring into $D(F)$ connected components, $C_1, C_2, \ldots, C_{D(F)}$. Let n_i denote the number of non-faulty elements in C_i. Without loss of generality, let $n_1 \geq n_i$, $\forall i$, $1 < i \leq D(F)$, that is, C_1 is the connected component containing the largest number of non-faulty elements. Then, $n_1 \geq \frac{N-|F|}{D(F)}$, by pigeon hole principle. Without loss of generality, let C_1 be

$$S_1, F_2, S_2, F_3, \ldots, S_{q-1}, F_q, S_q.$$

where F_i's are non-blocking fault patterns in the decomposition of F and all nodes in F_i are to the left of the nodes in S_i (which consists solely of non-faulty nodes), and all nodes in S_i are to the left of the nodes in F_{i+1}; all operations are modulo n. An array on these S_i's can be constructed as follows:

1. Connect every node in S_i using regular link.
2. Connect S_i to S_{i+1} using a bypass link of length $l_{i+1} + 1$, this is clearly possible since $l_{i+1} = |F_{i+1}| < g$.

□

Note that $|F| < 2g - 2$ implies that $D(F) \leq 1$. Thus, by Theorem 3, there exists a linear array of $n - |F|$ working PEs. In other words, the result by Tyszer, follows as a simple corollary of Theorem 3.

4 Optimal Linear Array Formation Strategy for Generalized FLFH Networks

In this section, we consider generalized FLFH networks with an arbitrary link structure, and study the problem of optimal linear array construction in the presence of a set of blocking fault patterns. We consider optimality with respect to the size of the reconfigured array.

A generalized FLFH network is similar to the FLFH network defined before with the only difference being that the redundancy set is not constrained to have all the elements from 1 to g. So any proper subset of $G = \{1, 2, \ldots, g\}$ will be a redundancy set for a generalized FLFH network. We now introduce the following definitions:

Definition 8. A generalized FLFH network $\aleph(n, G)$ consists of a set $V = \{P_0, P_1, \ldots, P_{n-1}\}$ of n PEs and a set $G = \{g_1 = 1, g_2, \ldots, g_k\}$, $1 < g_2 < g_3 \ldots < g_k$, of k unidirectional links. We say $\aleph(n, G)$ has link redundancy or link configuration G if, the directed links are from p_i to $p_{i+g_t \pmod n}$ for $1 \leq i \leq n$ and $g_t \in G$, $1 \leq t \leq k$.

In generalized FLFH networks, a cluster of atleast g faulty nodes is not the only blocking fault pattern. For example, consider the fault pattern $F = \{0, 2, 8\}$ in the generalized FLFH network $\aleph(10, G)$ with $G = \{1, 3\}$ as shown in Figure 1. It can be noted that F is a blocking pattern but it is not a cluster of 3 faulty processors. We know that a fault pattern F is blocking for $\aleph(n, G)$ if it is catastrophic for a redundant linear array A on $n' \geq n$ nodes with redundancy G and vice versa. The fault pattern in Figure 1 is a catastrophic fault pattern for a linear array with unidirectional link redundancy $G = \{1, 3\}$, occurring as a blocking pattern in generalized FLFH network with $G = \{1, 3\}$. Thus the number of blocking fault patterns in FLFH network $\aleph(n, G)$ is the same as the number of catastrophic fault pattern for linear array with link redundancy G. A count on the number of catastrophic fault patterns in linear array with link redundancy $G = \{1, 2, g\}$, $g > 2$, is given in [7].

Definition 9. Given a generalized FLFH network $\aleph(n, G)$ and a fault pattern F, we construct a graph $H = (V, E)$ as follows: The set V of vertices is the set of working processors, and the set E of edges are the directed links between two working processors. We preserve the direction of the link, in the edge representing it. We call the graph H the *Auxiliary graph* of $\aleph$ for the fault pattern F.

By definition of Auxiliary graph it follows that a fault pattern F is a blocking fault pattern for for generalized FLFH $\aleph$, if and only if the corresponding auxiliary graph H is directed acyclic graph (DAG)[3]. We use the concept of derived graph to solve our problem.

We say that a linear array formed is *optimal* if it contains maximum possible working PEs. Obviously, the linear array should be a single connected component. If the fault pattern F cuts the network into disconnected components, such a linear array will be entirely within a component. Now we use the directed acyclic graph property of the auxiliary graph to show that we can find the optimal linear array in polynomial time. We may have several connected components of the auxiliary graph H. Then in each of the connected components we can find the all pairs longest path to get the optimal linear array. The longest path problem is solvable in linear time for directed acyclic graph, as given in [3].

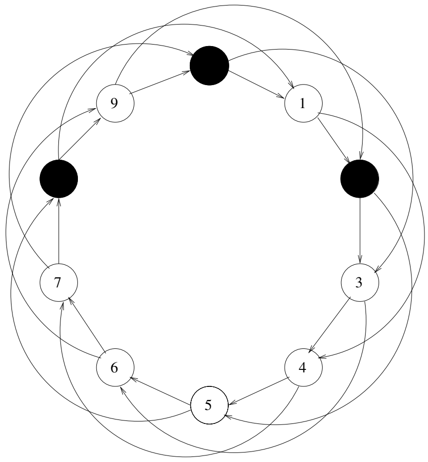

Fig. 1. A blocking fault pattern

5 Limits to Reconfigurability of Generalized FLFH Network

In this section, we are concerned about the guaranteed fault tolerance in a redundant ring network in the presence of PE failures. It is assumed that some form of reconfiguration mechanism is built into the network and is triggered on the occurrence of one or more PE failures. The guaranteed level of fault tolerance is related to the limits to reconfigurability. In order to quantify the limits to reconfigurability of redundant rings, we focus on performing computations that require a fixed number of connected non-faulty PEs. As seen earlier, it still might be possible to find one or more subrings or linear arrays in the original ring in spite of the failure of some nodes. These subrings and linear arrays can be utilized in computations requiring fewer processing nodes. In this section, we derive bounds on the size of the problems one can be guaranteed to solve in the presence of a given number of faults in the network.

Definition 10. A ring problem X of size $|X| = m$ is a problem whose solution requires a ring of size at least m.

Definition 11. An array problem X of size $|X| = m$ is a problem whose solution requires a linear array of size at least m.

Definition 12. A generalized FLFH ring network $\aleph(n, G)$ is "(k, m)-Ring Intolerant" if in the presence of k faults, it is not possible to solve all ring-problems of size m or greater.

Definition 13. A generalized FLFH ring network $\aleph(n, G)$ is "(k, m)-Array Intolerant" if in the presence of k faults, it is not possible to solve all array-problems of size m or greater.

An obvious but useful property linking ring intolerance and array intolerance is the following.

Lemma 2. *If a generalized FLFH ring network $\aleph(n, G)$ is (α, β)-Array Intolerant then it is also (α, β)-Ring Intolerant.*

Theorem 4. *A generalized FLFH ring network $\aleph(n, G)$ is "$(\alpha g_k, \beta)$-Array Intolerant", where $\alpha \geq 1$ and $\beta = \lceil \frac{n}{\alpha} \rceil - g_k + 1$.*

Proof: If we can show that αg_k faults can be arranged in such a way that none of the linear arrays is of size β, then we are done. Let $F_1, F_2, \ldots, F_\alpha$ be α identical blocking fault patterns, each consisting of exactly g_k faults. There is a total of αg_k faults in $\aleph$. Now place the $F_1, F_2, \ldots, F_\alpha$ in $\aleph$ in such a way that F_i's are equally spaced. The number of non-faulty PEs between F_i and F_{i+1}, is $\leq \lceil \frac{n}{\alpha} \rceil - g_k$. This implies that in occurrence of αg_k faults in $\aleph$, an array may be formed on $\lceil \frac{\aleph}{\alpha} \rceil - g_k$ non-faulty PEs, but one cannot guarantee $\lceil \frac{n}{\alpha} \rceil - g_k + 1$ sized linear array. Therefore, it follows that $\aleph$ is $(\alpha g_k, \beta)$-Array Intolerant. □

In other words, it is always impossible to solve an array-problem of size $|X| = m$ in presence of αg_k faults if $m > \lceil \frac{n}{\alpha} \rceil - g_k$. The intolerance of a generalized FLFH ring network can therefore be shown and rephrased in terms of the largest subring and largest linear array possible in it, in presence of faults.

6 Conclusions

A characterization of fault-tolerance of FLFH networks is given in this paper. It is shown that some of the existing results follow as simple corollaries to the ones established in this paper. The characterization has revealed several properties which describe the problem of constructing subrings and linear arrays in the presence of node failures in the FLFH network for a specified link configuration. Also in this paper, bounds on the degree of fault tolerance that can be achieved in a FLFH network when performing a computation (on a linear array as well as on ring) that requires a fixed number of operational nodes are established. Bounds on the size of the problems one can guarantee to solve in the presence of a given number of faults in the network are also derived. Efficient reconfiguration strategy for generalized FLFH network is also shown. Currently, work is underway for similar analysis of FLBH (Forward Loop Backward Hop) networks.

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