3. RELATIVISTIC QUANTUM MECHANICS OF THE PARTICLE MODEL

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The simplest particle model is made up of a single photon with its two components traveling in opposite directions. For the model at rest with respect of the observer, each of its components has one half of the total energy of the model. This is consistent with the fact that the annihilation of an electron pair (or positronium atom) normally gives out two photons traveling in opposite directions with energy of just a half of that of the electron pair mass-energy.

Since the particle model has no net electric charges[1], then, according to the Explicit Equivalence Principle, the properties of uncharged particles must come from the properties of quanta of radiation in stationary states.

3.1 Nonlocal Mass-Energy Conservation

From above it inferred that, the most demental "object-like" in nature is a quantum of radiation whose most important parameter with respect to a NL observer is its *NL* frequency. This one is equal to the virtual number of quantum waves (or quantum cycles) that are crossing some plane static with respect to the observer. According to the *wave continuity* and to the EEP

1. The quantum waves or quantum cycles (QCs) confined in the model cannot

[1] However, inside of the model, some temporal internal field gradients would come out as a consequences of phase differences.

suddenly disappear leaving nothing, or appear from nothing else.

2. *The net number (n) of quantum waves (or QCs) per unit of local time reflected in its mirrors is constant.*

- *Thus the sum of the model NL quantum frequencies, with respect to an invariable clock, must also be a constant. The same holds for many particle systems.*

Then, in general,

The sum of the NL frequencies of the quantum's confined in isolated systems, with respect to a NL observer in a fixed G potential, is constant[2]. (*NL mass-energy and frequency conservation with respect to an observer in a fixed G potential*).

3.2 Quantum Vector conservation

The minimum particle model at rest with respect to the observer is made up of two halves of a single quantum[3] traveling in opposite directions. Each one has energy equal to $\frac{1}{2}h\boldsymbol{n}_{r*}(0,r)$. The net model NL quantum vector, with respect to an observer at *r**, is:

$$\vec{Q}_{r*}(0,r)=\sum_{j=1}^{2}\vec{q}^{\,j}_{r*}(0,r)=\sum_{j=1}^{2}\frac{h}{2}\vec{n}^{\,j}_{r*}(0,r)=0 \quad . \qquad \text{E3.1}$$

If the system is strictly isolated in an isotropic space, this vector remains constantly null, *i. e.*, the system remains, indefinitely, at rest relative to the observer.

On the other hand the net *NL mass-energy* confined in the model is, by definition:

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[2] Such constant can have different values for observers in different potentials because their clocks (time units) are different with respect to each other.

[3] Individually, they may be thought as “leptons”

$$m_{r*}(0,r) = \sum_{j=1}^{2} \frac{h}{2} \boldsymbol{n}^{i}_{r*}(0,r) = h\boldsymbol{n}_{r*}(0,r) \quad . \qquad \text{E3.2}$$

$\boldsymbol{n}_{r*}(0,r)$ is the model NL frequency, at rest at r, with respect to the observer at $r*$.

In the case of the model moving relative to the observer, the model vectors are no longer opposed to each other and, therefore, the net model frequency vector is no longer zero. This one turns out to be equal to *the net rate of propagation of quantum cycles in some well-defined direction,* with respect to the observer's clock.

$$\vec{Q}_{r*}(b,r) = \sum_{j=1}^{2} \vec{q}^{\,j}_{r*}(b,r) = \sum_{j=1}^{2} \frac{h}{2} \vec{n}^{\,j}_{r*}(b,r) \neq 0 \quad . \qquad \text{E3.3}$$

When the *isolated* model is moving with respect to the observer, in a space of isotropic properties, each of its internal components recovers its original modulus and orientation after a pair of internal reflections. This means that the net quantum-vector of the model remain constant with the time. Then it may be concluded that:

> *The NL quantum vector of an isolated model moving with respect to a fixed observer, in a space free of gradients of the NL refraction index, remains constant. (NL quantum vector conservation law)*

This law accounts for momentum conservation and for the body's inertial properties.

This property can be easily generalized for a more complex *system* made up of *two or more particle models*, with some bonds between them.

Effectively, since *any bond* between two models must be a stationary wave, this one is equivalent to *third particle model*. Then *the above conservative properties also hold for the sum of the quantum vectors of the three particle models*[4]. In a

[4] Notice that the sum of these frequencies is equal to *the virtual number of quantum cycles* that are reflected by the particle model mirrors in just one standard second. Such number accounts for *the mass-energy* confined in the model.

similar way these properties can be easily generalized for any isolated system made up of many particle-models moving with respect to the observer.

For a system with z particle models, moving in a space of isotropic properties, its net NL quantum vector is also constant with respect to a fixed observer:

$$\vec{Q}_{r*}(\boldsymbol{b},r) = \sum_{i=1}^{2z} \vec{q}^{\,i}_{r*}(\boldsymbol{b},r) = \frac{h}{2}\sum_{i=1}^{2z} \vec{\boldsymbol{n}}^{\,i}_{r*}(\boldsymbol{b},r) = \text{constant} \quad . \qquad \text{E3.4}$$

On the other hand, the net NL mass-energy confined in the moving model is obviously equal to *the sum of the energies of its components.*

$$m_{r*}(\boldsymbol{b},r) = \sum_{i=1}^{z} h\boldsymbol{n}^{i}_{r*}(\boldsymbol{b},r) = \text{Constant} \qquad \text{E3.5}$$

Notice that the concept of *mass*, here, is associated to the net energy confined in the system, in rather stationary states[5]. This relation also comes out below, explicitly and naturally, during the interaction of the model with other bodies, in E3.7 and E3.8.

Equations E3.5 and E3.4 have been called here *NL mass-energy and quantum-vector conservation laws*, with respect to some fixed observer. Notice that hese laws are consequences of wavelet continuity. According to it:

> *The number of quantum waves confined in stationary radiation cannot change. Such number is also equal to the sum of the virtual number of quantum cycles reflected within a free particle model, in the observer (fixed) time unit.*

Then, from the EEP and wave continuity it may be concluded:

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[5]The inertial and gravitational properties of the model are derived below, at posterior, from the model radiation properties. Then it is meaningless to talk about inertial or gravitational mass because this value does not depend on the methods (inertial or gravitational ones) used for measuring it.

For a system of many particle models traveling in an isotropic space, the sum of their NL quantum vectors and the sum of their absolute values is constant[6].

This is obviously consistent with the conventional *momentum and mass-energy conservation, respectively.*

For NL cases in G fields, the corresponding conventional laws are not well defined because the unit systems of observers in different G potentials are not strictly the same with respect to each other. *These laws can get a well-defined meaning only if the masses or frequencies are referred to the unit system of some single observer in some well-defined state of velocity and G potential.*

3.2.1 Electromagnetic interactions with the particle model

For simplicity, let us study the local case in which it is not necessary to use subscript.

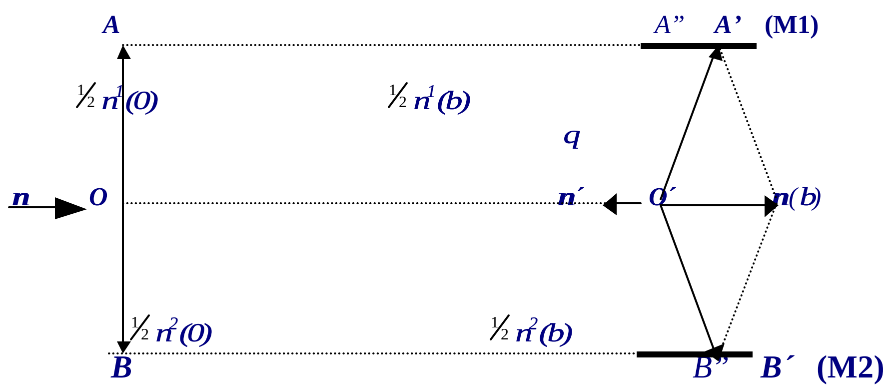

Fig. (3.1a) Fig.(3.1b)

[6] Notice that the ultimate reason for these conservative properties is just wave continuity, *i.e.*, the quantum waves, or the quantum cycles, can be exchanged but they cannot suddenly disappear or appear in the empty space, even during model interactions. In more conventional terms, *in a space free of gradients of the NL speed of light, the NL mass-energy and NL momentum are conserved* because, according to wave continuity *the net number of quantum cycles traveling along any direction, cannot suddenly disappear leaving nothing, or appear from nothing else*

In Fig. 3.1a, the system is made up of a single photon (coming from the left) and a transversal model (AB) at rest with r. In Fig. (3.1b), after the photon reflection, the model is moving towards the right with a velocity $b = V / c = sinq$.

In Fig. 3.1a, a transversal model at rest with respect to the observer reflects a photon coming from the left. The model is made up of *a single quantum of radiation, in the simplest kind of stationary state, within a wave cavity.* The wavelets, traveling up and down, are the mirror reflections of each other. Each one corresponds to energy[7] of *a half of that of quanta*, i.e., $E = \frac{1}{2} h \boldsymbol{n}^j(0)$. They may eventually be leptons[8]. They have been labeled with a super-index *j*. For simplicity, each one can be assigned with a constant *h' = 1/2h.* In this case, there are no external field gradients and, for simplicity, the positions of the object and of the observer have been omitted. Only their relative velocities ($\beta = V/c$), in a parenthesis, are explicitly stated.

Fig. (3.1b) is the same system, after the reflection of the external photon coming from the left. The values of the vectors are fixed by Quantum Vector Conservation.

a) Quantum vector conservation

According to it the initial quantum vectors of the system are equal to the final ones[9]:

$$\vec{\boldsymbol{n}} + \vec{\boldsymbol{n}}(0) = \vec{\boldsymbol{n}}' + \vec{\boldsymbol{n}}(\boldsymbol{b}) = \vec{Q}^{syst} \qquad \text{E3.6}$$

The rest model frequency vector is obviously zero. From Fig. (3.1b), the net quantum vector of the moving model is equal to the sum of its components.

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[7] Since this case does not involve a G field, the object and observer positions are omitted, for simplicity.

[8] This model is consistent with a *positronium atom or an electron-positron pair (e^- - $e+$).* Its annihilation normally releases two photons traveling in opposite directions.

[9] To start with the simplest case, in this example there are not external fields. Then the quantities used here do not depend on NL positions. For this reason, and for simplicity, the subscripts have been omitted. However they are implicit. Below, for G fields, the same relations are used but with explicit subscripts so as to check that all of the local quantities have been transformed to the same common unit system based on a standard in a fixed state of potential and velocity, *i.e.*, that the NL relations are *physically* homogeneous.

$$Q(\mathbf{b}) = \sum_{j=1}^{2} \frac{h}{2} \mathbf{n}^{j}(\mathbf{b}) sin \mathbf{q} = \sum_{j=1}^{2} \frac{h}{2} \frac{\mathbf{n}^{j}(0) sin \mathbf{q}}{\cos \mathbf{q}} = \frac{m(0)\mathbf{b}}{\sqrt{1-\mathbf{b}^2}} = m(\mathbf{b})\mathbf{b} \qquad \text{E3.7}$$

Where the model rest mass and the moving mass, with respect to the observer at rest, are, by definition and Fig. 3.1b:

$$m(\mathbf{b}) = \sum_{j=1}^{2} \frac{h}{2} \mathbf{n}^{j}(\mathbf{b}) = \sum_{j=1}^{2} \frac{h}{2} \frac{\mathbf{n}^{j}(0)}{\sqrt{1-\mathbf{b}^2}} = \frac{m(0)}{\sqrt{1-\mathbf{b}^2}} = \mathbf{g} m(0) \; ; \qquad \mathbf{g} = \frac{1}{\sqrt{1-\mathbf{b}^2}} \qquad \text{E3.8}$$

From E3.7, the momentum and the quantum vector turn out to be related by:

$$Q(\mathbf{b}) = m(\mathbf{b})\mathbf{b} = p(\mathbf{b})c \qquad p(\mathbf{b}) = \frac{m(\mathbf{b})}{c^2} V = m^{newt}(\mathbf{b}) V \qquad \text{E3.9}$$

where $m^{newt}(\mathbf{b}) = \dfrac{m(\mathbf{b})}{c^2}$ is the ordinary Newtonian mass[10]. E3.9b

During the reflection of the external photon, according to *quantum vector conservation*, the transversal projections of the moving model quantum vector do not change. According to this and Fig. (3.1b),

$$\mathbf{n}(0) = \mathbf{n}(\mathbf{b}) \cos \mathbf{q} = \mathbf{n}(\mathbf{b}) \sqrt{1-\mathbf{b}^2} \quad ; \quad \frac{m(\mathbf{b})}{m(0)} = \frac{\mathbf{n}(\mathbf{b})}{\mathbf{n}(0)} = \frac{1}{\sqrt{1-\mathbf{b}^2}} = \mathbf{g} \qquad \text{E3.10}$$

Then the model mass, with respect to the observer at rest, increases with the velocity just in the way predicted by special relativity.

b) Mass-Energy Conservation

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[10] Notice that the conventional concepts of forces (*dp/dt*) and inertial masses (*F/a*) can be obtained in a straightforward way from the last relationships.

According to *quantum cycle conservation*, (or mass-energy conservation) the initial and final quantum frequencies, with respect to the invariable clock, should not change, regardless on the orientations of their quanta. Thus the sum of the initial and the final frequencies with respect to the same observer must remain the same.

$$n + n(0) = n' + n(b) \qquad \text{E3.11}$$

According to this and E3.10, *the energy given up by the photon to the body* is

$$E - E' = hn - hn' = hn(b) - hn(0) = m(0)[g - 1] \qquad g = [1 - b^2]^{-1} \qquad \text{E3.12}$$

The consistency of the above relations with special relativity is evident.

c) Local longitudinal wavelength contraction

From above, the local speed of light is constant. This one is the scalar product of the frequency and wavelength vectors. Thus the increase of the average frequency of the longitudinal model is associated with an average wavelength contraction along the direction of movement:

$$l(b) = \frac{c}{n(b)} = \frac{c}{n(0)}\sqrt{1 - b^2} = l(0)\sqrt{1 - b^2} \qquad (\textit{Wavelength contraction}) \qquad \text{E3.13}$$

From quantum vector conservation, the changes of the longitudinal components of the dual vectors do not change the transversal ones. This means no model transversal contraction. The relation between these vectors is also clearer in Fig. 3.2, below.

3.3 Quantum Mechanical Waves

The model's position comes out from plain interference of its wavelets. In principle such position does not depend on the model orientation. This fact has been proved, below, after using longitudinal and transversal orientations. The more straightforward relations come out from the transversal orientations. The longitudinal orientations are used in the APPENDIX A.

3.3.1 Transversal model orientation

From the Huygen's principle, the model wavelets must interfere *constructively* but only *within it and its reflection boundary* (short-range fields). Thus the model relativistic quantum mechanical properties are ultimately fixed by the dual properties of its stationary radiation.

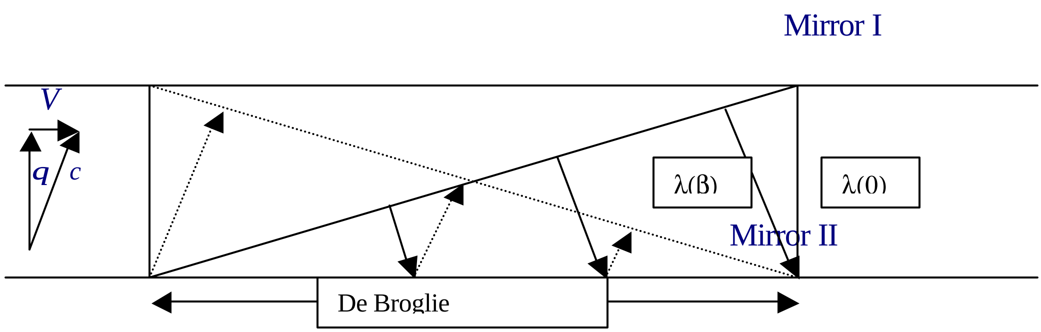

Fig. 3.2 In the transversal model moving towards the right, the wavelet-fronts propagate themselves in the direction of the vector $\boldsymbol{l}(\boldsymbol{b})$ *with the speed* **c***. After one wavelength the point* ***A*** **is** *translated up to* ***C****, which is the De Broglie wavelength.*

The De Broglie wavelengths, for example, can be derived from the inclination of the model wavefront of a transversal model. In Fig. (3.2), this is given by the angle $\boldsymbol{q}$ between AC and AB:

$$sin\boldsymbol{q} = \frac{V}{c} = \boldsymbol{b} = \frac{\boldsymbol{l}(\boldsymbol{b})}{\boldsymbol{l}*(\boldsymbol{b})} \qquad \text{E3.14}$$

From E3.14, E3.13, E3.10 and E3.9

$$\lambda^*(\beta) = \frac{\lambda(\beta)c}{V} = \frac{c^2}{\nu(\beta)V} = \frac{hc^2}{m(\beta)V} = \frac{h}{m^{nt}(\beta)\ V} = \frac{h}{p(\beta)} \qquad \text{E3.15}$$

This is just the *De Broglie wavelength.*

Within the model the model each wave front propagates itself with the speed of light. However their interference peaks move to the right with the velocity

$$w(\beta) = \frac{c}{\sin\theta} = \frac{c^2}{V} \qquad \text{E3.16}$$

This is just the wave velocity of the De Broglie waves.

The reason for these waves is even more clear in the next more detailed deduction.

In Fig. 3.3a, the model moving towards the right is made up of stationary waves traveling between the mirrors ***MI*** and ***MII***.

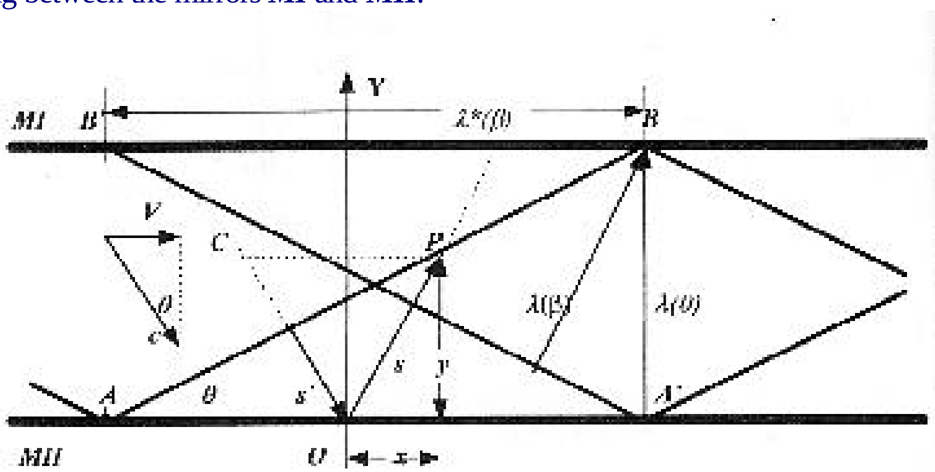

Fig. 3.3a. Wavelet fronts ***AB*** and ***A'B'****. of a model moving transversally towards the right with a velocity* $\beta = V/c = \sin\theta$*. They are inclined relative to the mirrors.*

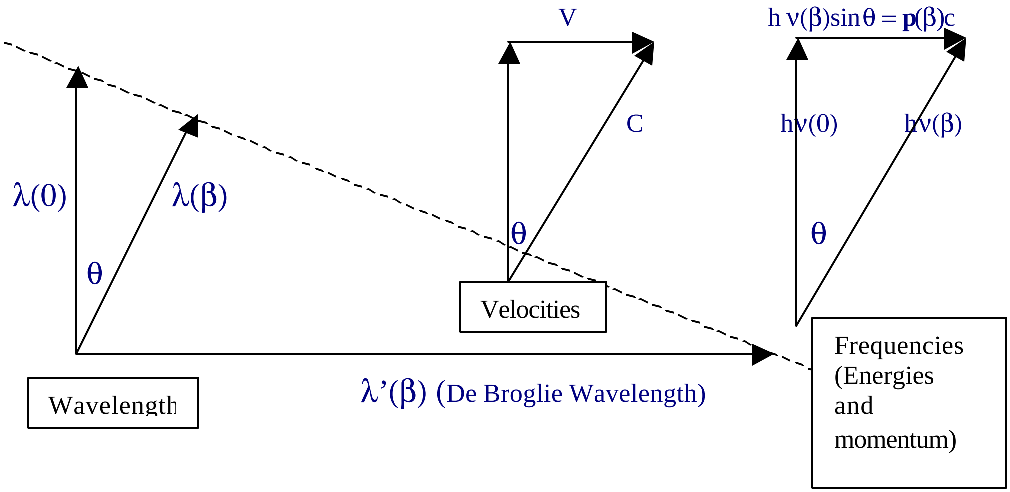

Fig. 3.3b. Vector diagrams for the model in Fig. 3.3a.

The wave amplitude at ***P***, with respect to the observer at ***O***, result from interference of the direct and reflected wavelets. They depend mainly on the phases of the original wavefront, front at a distance *s'* from the observer,

$$\mathbf{j}'(s') = 2\mathbf{p}\left\{\mathbf{n}(\mathbf{b})t + \frac{s'}{\mathbf{l}(\mathbf{b})}\right\}, \qquad \text{E3.17}$$

and the reflected one, at the intersection point **P,**

$$\mathbf{j}(s) = 2\mathbf{p}\left\{\mathbf{n}(\mathbf{b})t - \frac{s}{\mathbf{l}(\mathbf{b})}\right\} + \mathbf{p}. \qquad \text{E3.18}$$

The net amplitude resulting from interference is

$$\Psi \propto \sin 2\mathbf{p}\frac{s'}{\mathbf{l}(\mathbf{b})} + \sin\left[2\mathbf{p}\frac{s}{\mathbf{l}(\mathbf{b})} + \mathbf{p}\right]. \qquad \text{E3.19}$$

From *Fig. 3.3a,*

$$s = x\sin\theta + y\cos\theta \qquad \text{and} \qquad s' = P'O - P'C = y\cos\theta - x\sin\theta . \qquad \text{E3.20}$$

The result from E3.19, E3.20 and E3.18, after taking into account the relativistic wavelength contraction shown in Fig. 3.3, can be expressed in terms of functions that depend either on *x* or on *y*, respectively

$$\Psi \propto 2\sin\varphi(y)\cos\varphi(x) \qquad \text{E3.21}$$

$$\varphi(x) = 2\pi\left\{\nu(\beta)t - \frac{x\sin\theta}{\lambda(\beta)}\right\} \qquad \text{E3.22}$$

$$\varphi(y) = 2\pi\left\{\frac{y\cos\theta}{\lambda(\beta)}\right\} = 2\pi\left\{\frac{y}{\lambda(0)}\right\} \qquad \text{E3.23}$$

$$\Psi(x,y,t) = \psi(y)\cos 2\pi\nu(\beta)\left\{t - \frac{x\beta}{c}\right\} \qquad \psi(y) = \Psi^{max}\sin 2\pi\left\{\frac{y}{\lambda(0)}\right\} \qquad \text{E3.24}$$

From E3.24, these waves are modulated by a function of the model pattern given to the right. The maximum model height is *$\lambda(0)$, i. e., constant*. Notice that this does not depend on the velocity, in spite of the general contraction of the model front wavelengths observed in *Fig. 3.3*.

E3.24 shows a traveling wave whose *velocity and wavelength* can be derived by making Ψ(x, y, t) = constant.

$$w(\beta) = \left.\frac{\partial x}{\partial t}\right)_{\Psi} = \frac{c}{\beta} = \frac{c^2}{V} \qquad \text{E.3.25}$$

$$\lambda^{*}(\beta) = \frac{w(\beta)}{\nu(\beta)} = \frac{\lambda(\beta)}{\beta} = \frac{c^{2}}{\nu(\beta)V} = \frac{hc^{2}}{m(\beta)\,V} = \frac{h}{m^{nt}(\beta)\,V} = \frac{h}{p(\beta)} \qquad \text{E3.26}$$

The strict consistency with the De Broglie waves is obvious.

3.4 Some Quantum Mechanical Differential Equations

Ordinary wave equations are normally stated by using the real part of complex rotating vectors. For the model waves resulting from interference of the model waves its angular velocity is $w = 2pn$ For E3.24, the wave amplitude is the real part of:

$$\Psi(x,t) \propto e^{i[w(b)t - k^*(b)x]} \propto \left\{\cos[w(b)t - k^*(b)x] + i\,\mathrm{sen}[w(b)t - k^*(b)x]\right\}$$

E3.27

In which

$$w(b) = 2pn(b) \qquad \text{and} \qquad k^*(b) = 2p\,/\,l^*.$$

E3.28

According to Fig. (3.3b), these parameters are related to each other by

$$n^2(b) = n^2(0) + [n(b)b]^2 \qquad [m(b)]^2 = [m(0)]^2 + [p(b)c]^2$$

E3.29

$$w^2(b) = w^2(0) + [k(b)c]^2 \qquad \text{E3.30}$$

In a first order approximation,

$$w(b) \cong w(0) + \frac{[k^p(b)c]^2}{2w(b)} \qquad \text{E3.31}$$

From E3.27 and E328, by using $h = 2p\hbar$ and partial derivatives, we can get direct relations between the model variables and the partial derivatives.

$$w(b) = 2pn(b) = \frac{m(b)}{\hbar} = -i\frac{1}{\Psi(x,t)}\frac{\P\Psi(x,t)}{\P t} \qquad w^2(b) = \frac{-1}{\Psi(x,t)}\frac{\P^2\Psi(x,t)}{\P t^2} \qquad \text{E3.32}$$

$$k(\beta) = \frac{2\pi}{\lambda^{p}} = \frac{p(\beta)}{\hbar} = i\,\frac{1}{\Psi(x,t)}\frac{\partial\Psi(x,t)}{\partial x} \qquad \{k(\beta)\}^2 = \frac{-1}{\Psi(x,t)}\frac{\partial^2\Psi(x,t)}{\partial x^2} \qquad \text{E3.33}$$

From E3.30, E3.32, and E3.33 two kinds of differential equations are obtained:

$$\left[-\frac{i}{\Psi(x,t)}\frac{\partial\Psi(x,t)}{\partial t}\right]^2 = \left[\frac{m(0)}{\hbar}\right]^2 - \frac{c^2}{\Psi(x,t)}\frac{\partial^2\Psi(x,t)}{\partial x^2} \qquad \text{E3.34}$$

$$-\frac{1}{\Psi(x,t)}\frac{\partial^2\Psi(x,t)}{\partial t^2} = \left[\frac{m(0)}{\hbar}\right]^2 - \frac{c^2}{\Psi(x,t)}\frac{\partial^2\Psi(x,t)}{\partial x^2} \qquad \text{E3.35}$$

The last relation accounts for *the Klein-Gordon equation*.

The *Schroedinger equation* comes from the approximation E3.31 and the equations E3.32 and E3.33.

$$-\frac{i\hbar}{\Psi(x,t)}\frac{\partial\Psi(x,t)}{\partial t} \cong m(0) - \frac{1}{\Psi(x,t)}\frac{\partial^2\Psi(x,t)}{\partial x^2}\frac{\hbar^2c^2}{2m(\beta)} \qquad \text{E3.36}$$

$$\frac{\hbar^2c^2}{2m(\beta)}\frac{\partial^2\Psi(x,t)}{\partial x^2} - i\hbar\frac{\partial\Psi(x,t)}{\partial t} \cong m(0)\Psi(x,t) \qquad \text{E3.37}$$

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