Suggested Solution to Skill CD Report for "Oscillation of a compound pendulum"

Readings

Equation: $T^2 = 4\pi^2\left(\frac{h}{g} + \frac{1}{3}\frac{Ml^2}{mgh}\right) \quad \Rightarrow \quad T^2 h = \frac{4\pi^2}{g}h^2 + \frac{4\pi^2}{3}\frac{Ml^2}{mg}$

Gradient: $\frac{4\pi^2}{g}$ Intercept: $\frac{4\pi^2}{3}\frac{Ml^2}{mg}$

Mass of pendulum bob, $m = 0.070$ kg
Length of half the ruler, $l = 0.500$ m

h/ m	Time for 10 oscillations			T/ s	$T^2 h$/ s^2 m	h^2/ m^2
	t_1/ s	t_2/ s	t_{ave}/ s			
0.650	18.21	18.24	18.23	1.823	2.159	0.423
0.530	17.40	17.36	17.38	1.738	1.601	0.281
0.460	17.00	17.00	17.00	1.700	1.329	0.212
0.390	16.52	16.55	16.54	1.654	1.066	0.152
0.320	16.50	16.80	16.65	1.665	0.8871	0.102
0.250	16.86	16.89	16.88	1.688	0.7119	0.0625
				Average	1.292	0.205

Centroid is (0.205, 1.292)

Calculations and Results

Best-fit line

$$\text{gradient} = \frac{1.80 - 0.95}{0.333 - 0.120} = 3.991 \text{ s}^2 \text{ m}^{-1}$$

$$\text{y-intercept} = 1.80 - 3.991 \times 0.333 = 0.4710 \text{ s}^2 \text{ m}$$

$$g = \frac{4\pi^2}{m} = \frac{4\pi^2}{3.991} = 9.892 \text{ m s}^{-2}$$

$$M = \frac{3}{4\pi^2}\frac{mgc}{l^2} = \frac{3}{4\pi^2}\frac{0.070 \times 9.892 \times 0.4710}{0.500^2} = 0.099 \text{ kg}$$

Extreme line

$$\text{gradient} = \frac{1.78 - 0.76}{0.320 - 0.080} = 4.250 \ \text{s}^2\ \text{m}^{-1}$$

$$\begin{aligned}\text{y-intercept} &= 1.78 - 4.250 \times 0.320 \\ &= 0.4200 \ \text{s}^2\ \text{m}\end{aligned}$$

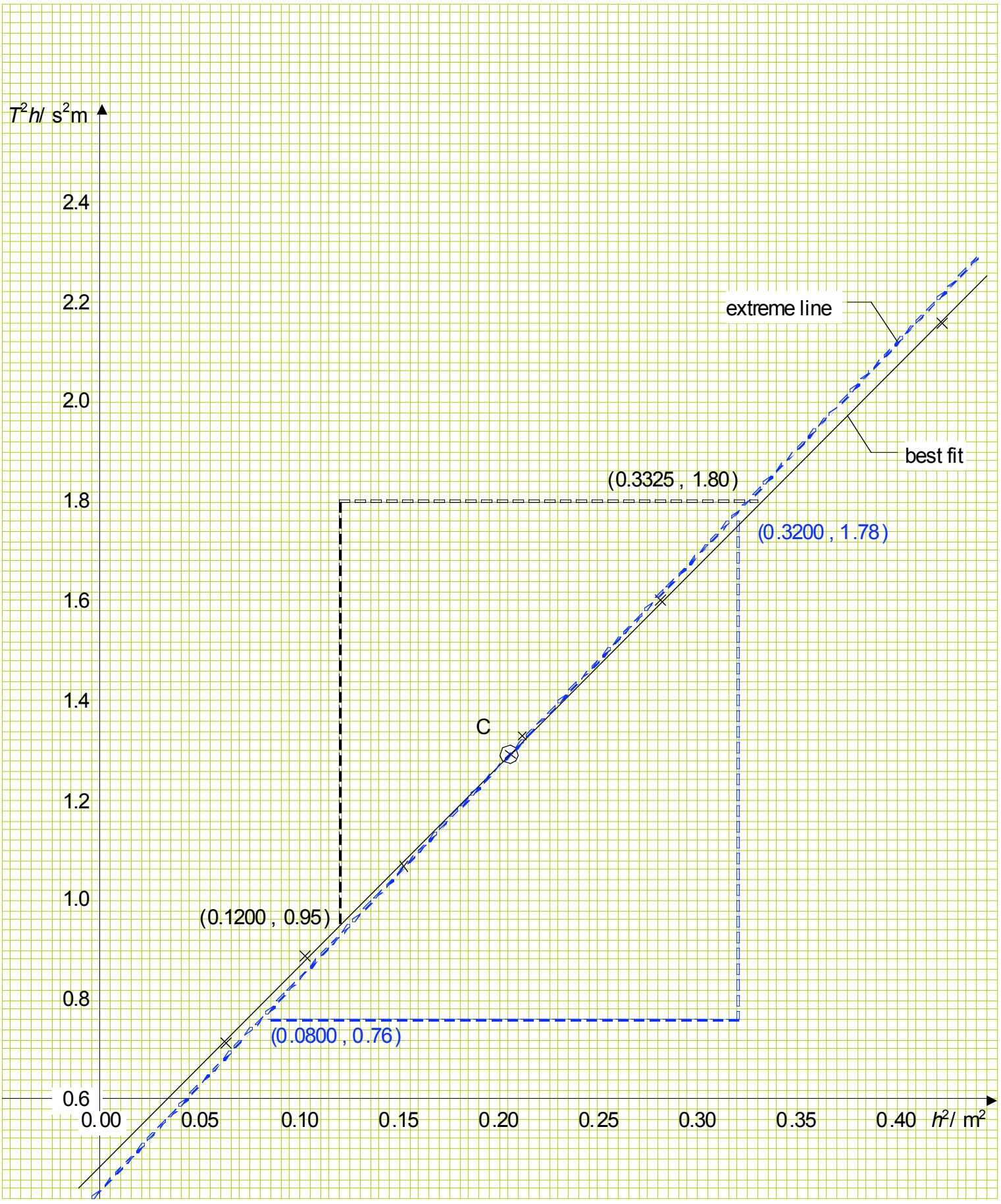
$$g = \frac{4\pi^2}{m} = \frac{4\pi^2}{4.250} = 9.289 \ \text{m s}^{-2}$$

$$\begin{aligned}M &= \frac{3}{4\pi^2}\frac{mgc}{l^2} \\ &= \frac{3}{4\pi^2}\frac{0.070 \times 9.289 \times 0.4200}{0.500^2} \\ &= 0.083 \ \text{kg}\end{aligned}$$

Conclusions

$m = 4.0 \pm 0.3 \ \text{s}^2\ \text{m}^{-1}$
$c = 0.47 \pm 0.05 \ \text{s}^2\ \text{m}$
$g = 9.9 \pm 0.6 \ \text{m s}^{-2}$
$M = 0.10 \pm 0.02 \ \text{kg}$

Since the graph of T^2h against h^2 is a straight line with gradient of $4.0 \ \text{s}^2\ \text{m}^{-1}$ and y-intercept of $0.47 \ \text{s}^2\ \text{m}$, the equation $T^2 = 4\pi^2\left(\frac{h}{g} + \frac{1}{3}\frac{Ml^2}{mgh}\right)$ is valid.


T^2h/ s^{2}m
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
h^2/ m^2
extreme line
best fit
(0.3325 , 1.80)
(0.3200 , 1.78)
C
(0.1200 , 0.95)
(0.0800 , 0.76)

Discussion

	Sources of Errors/ Limitations of Measurements	Methods to overcome errors/ Improvements
1.	**Human reaction time, about 0.2 s, is a random error which results in large percentage uncertainty in *t* (hence *T*) when *t* is short.**	**To reduce this uncertainty to the order of 1%, increase the number of oscillations to be timed to 20 so that *t* is longer than 20.0 s.**
2.	The pendulum bob is large and hence random error might result from the inability to start and stop the stopwatch accurately, when the bob passes through the equilibrium position. This increases the random error in *t*.	An optical pin (fiducial marker) is placed under/behind the bob at rest so that starting and stopping the stopwatch, when the bob passes through its equilibrium position, incur a smaller random error.
3.	The total times of oscillations *t* and period *T* for do not vary much for different *h*. Therefore range of T^2h is narrow and gradient and *y*-intercept would not be reliable.	A wider range for *h* is desirable, e.g. vary *h* from 0.150 m to 0.850 m, instead.
4.	**The pendulum bob tends to slide along the thread during the oscillations. This would introduce random error in *t*.**	**Use a small tape to secure the bob to the string.** **Small displacement for oscillations minimizes movement of bob.**
5.	*** The rule may not oscillate in a vertical plane and hence unwanted motion in the horizontal plane usually arises because the short loop is not rigid.** **This would introduce random error in *t*.**	**Replace the short loop with a horizontal optical pin as the pivot for the rule.** **The pin should be sandwiched between two split corks, which are secured with clamp.**
6.	The pivoting hole in the wooden metre rule may not be exactly at its centre of mass due to • wear and tear • non-uniform density along its length Hence, the metre rule is not horizontal if the pendulum bob is suspended directly below the centre of the rule. Measurement of *h* would not be accurate.	Use a metallic (e.g. Al) or alloy (e.g. Steel) metre rule which can have uniform density and is less susceptible to wear and tear.
7.	The equation assumes that the system oscillates in S.H.M. which is not true when the amplitude is large. A systematic error is introduced to *t* when amplitude is large.	Small displacement for oscillations ensures that oscillations is approximately S.H.M.
8.	The equation assumes that the moment of inertia of the ruler is given by $\frac{1}{12}Ml^2$ based on uniform distribution of mass along its length. The assumption may be inaccurate due to wear and tear and non-uniform density along its length. Hence systematic error is introduced to *y*-intercept.	Use a metallic (e.g. Al) or alloy (e.g. Steel) metre rule with uniform distribution of mass along its length and minimal wear and tear.