Introduction:

The last few years witnessed major advancements in digital transceivers.Digital transceivers are now faster, cheaper and more efficient than analog transceivers.Digital modulation and detection consists of transferring information in the form of bits over a communications channel.The bits are binary digits taking on the values of either 1 or 0.

These information bits are derived from the information source, which may be a digital source or an analog source that has been passed through an A/D converter.Digital modulation consists of mapping the information bits into an analog signal for transmission over the channel.Detection consists of determining the original bit sequence based on the signal received over the channel.

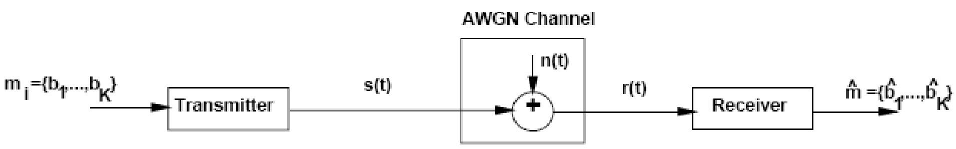
There are several advantages of digital modulation over analog modulation techniques such as higher data rates,powerful error correction techniques,resistance to channel impairments,more efficient multiple access strategies and better security and privacy.

Choosing a digital modulation technique depends on what the application requires such as high data rate,high spectral efficiency (minimum bandwidth occupancy),high power efficiency (minimum required transmit power),robustness to channel impairments (minimum probability of bit error)and low power (cost implementation).

The main categories of digital modulation are amplitude modulation,phase modulation and frequency modulation and these techniques will be explained later on in this report.

Ths signal space technique:

The signal space technique is a technique used to analyze the several modulation using simple vector analysis instead of complex equation yielding the same results. Digital modulation encodes a bit stream of finite length into one of several possible transmitted signals.The receiver minimizes the probability of detection error by decoding the received signal as the signal in the set of possible transmitted signals that is "closest" to the one received.



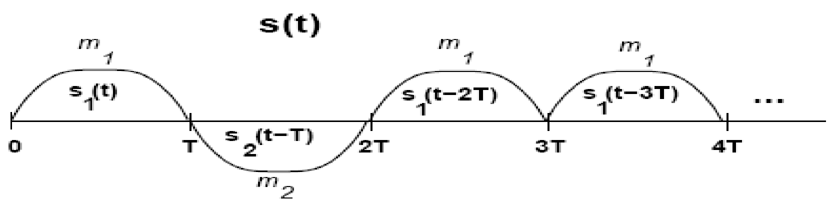
When considering the communication system model shown in the previous slide we'll find that.Every T seconds, the system sends $K = \log 2M$ bits of information through the channel for a data rate of $R = K/T$ bits per second (bps). There are $M = 2K$ possible sequences of K bits, and we say that each bit sequence of length K comprises a message $m_i = \{\mathbf{b}_1, \ldots, \mathbf{b}_K\} \in M$, where$M=\{m_1, \ldots, m_M\}$ is the set of all such messages. The messages have probability p_i of being selected for transmission, where the summation of all the probabilities is equal to one.

Determining the distance between the transmitted and received signals requires a metric for the distance between signals. By representing signals as projections onto a set of basis functions, we obtain a one-to-one correspondence between the set of transmitted signals and their vector representations.we can analyze signals in finite-dimensional vector space instead of infinite dimensional function space, using classical notions of distance for vector spaces.This general analysis can be applied to the modulation techniques discussed later on in this report.

Suppose message mi is to be transmitted over the channel during the time interval $[0, T)$. Since the channel is analog, the message must be embedded into an analog signal for channel transmission. Thus, each message $mi \in M$ is mapped to a unique analog signal $si(t) \in S = \{s1(t), . . . , sM(t)\}$ where $si(t)$ is defined on the time interval $[0, T]$ and has energy

$$E_{s_i} = \int_0^T s_i^2(t)dt, \qquad i = 1, \ldots, M.$$

Since each message represents a bit sequence, each signal $si(t) \in S$ also represents a bit sequence, and detection of the transmitted signal $si(t)$ at the receiver is equivalent to detection of the transmitted bit sequence. When messages are sent sequentially, the transmitted signal becomes a sequence of the corresponding analog signals over each time interval $[kT, (k + 1)T)$: $s(t) = k\ si(t - kT)$, where $si(t)$ is the analog signal corresponding to the message mi designated for the transmission interval $[kT, (k+1)T]$. This is illustrated in the next figure, where the transmitted signal $s(t) = s1(t) + s2(t - T) + s1(t - 2T) + s1(t - 3T)$ is shown corresponding to the string of messages $m1,m2,m1,m1$ with message mi mapped to signal $si(t)$.



The transmitted signal is sent through an AWGN channel, where a white Gaussian noise process $n(t)$ of power spectral density $No/2$ is added to form the received signal $r(t) = s(t) + n(t)$. Given $r(t)$ the receiver must determine the best estimate of which $si(t) \in S$ was transmitted during each transmission interval $[kT, (k + 1)T]$. This best estimate for $si(t)$ is mapped to a best estimate of the message $mi(t) \in M$ and the receiver then outputs this best estimate $m = \{\hat{}\mathbf{b}1, . . . ,\mathbf{b}K\} \in M$ of the transmitted bit sequence.The goal of the receiver design in estimating the transmitted message is to minimize the probability of message error

$$P_e = \sum_{i=1}^{M} p(\hat{m} \neq m_i | m_i \text{ sent}) p(m_i \text{ sent})$$

Geometric representation of signals in the signal space:

The basic premise behind a geometrical representation of signals is the notion of a basis set. Specifically, using a Gram-Schmidt orthogonalization procedure, it can be shown that any set of M real energy signals $S = (s1(t), \ldots, sM(t))$ defined on $[0, T]$ can be represented as a linear combination of $N \leq M$ real orthonormal basis functions $\{\varphi 1(t), \ldots, \varphi N(t)\}$. We say that these basis functions span the set S. Thus, we can write each $si(t) \in S$.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad 0 \leq t < T,$$

where

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt$$

The basis functions used to represent the signal have to be orthogonal such that

$$\int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} .$$

If the signals $\{si(t)\}$ are linearly independent then $N = M$, otherwise $N < M$. Moreover, the minimum number N of basis functions needed to represent any signal $si(t)$ of duration T and bandwidth B is roughly $2BT$ .The signal $si(t)$ thus occupies a **signal space** of dimension $2BT$.For linear passband modulation techniques, the basis set consists of the sine and cosine functions

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

The $\sqrt{2/T}$ factor is needed for normalization so that

$$\int_0^T \phi_i^2(t)dt = 1, i = 1, 2$$

In fact, with these basis functions we only get an approximation to zero.The carrier basis functions may have an initial phase offset Φ_0. The basis set may also include a baseband pulse-shaping filter $g(t)$ to improve the spectral characteristics of the transmitted signal:

$$\int_0^T \phi_i^2(t)dt = 1, i = 1, 2$$

The signal constellation:

We denote the coefficients $\{s_{ij}\}$ as a vector $\mathbf{s}_i = (s_{i1}, \ldots, s_{iN}) \in \mathcal{R}^N$ which is called the **signal constellation point** corresponding to the signal $s_i(t)$. The **signal constellation** consists of all constellation points $\{\mathbf{s}_1, \ldots, \mathbf{s}_M\}$.Given the basis functions $\{\varphi_1(t), \ldots, \varphi_N(t)\}$ there is a one-to-one correspondence between the transmitted signal $s_i(t)$ and its constellation point $\mathbf{s}_i$. $S_i(t)$ can be obtained from $\mathbf{s}_i$ and $\mathbf{s}_i$ can be obtained from $s_i(t)$.

The representation of $s_i(t)$ in terms of its constellation point $\mathbf{s}_i \in \mathcal{R}^N$ is called its **signal space representation** and the vector space containing the constellation is called the **signal space**. A two-dimensional signal space is illustrated in the next slide, where we show $\mathbf{s}_i \in \mathcal{R}^2$ with the i th axis $\mathcal{R}^2$ of corresponding to the basis function $\varphi_i(t)$, $i = 1, 2$.

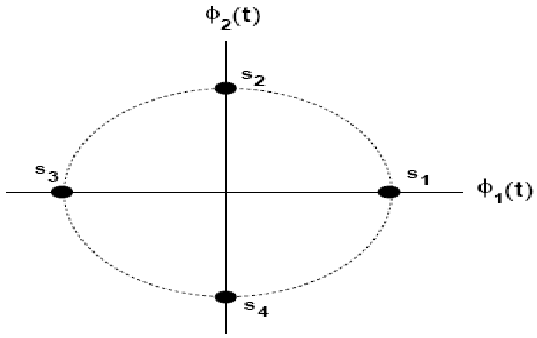
With this signal space representation we can analyze the infinite-dimensional functions $s_i(t)$ as vectors $\mathbf{s}_i$ in finite-dimensional vector space.This greatly simplifies the analysis of the system performance as well as the derivation of the optimal receiver design. Signal space representations for common modulation techniques like MPSK and MQAM are two-dimensional (corresponding to the in-phase and quadrature basis functions).

In order to analyze signals via a signal space representation, we require a few definitions for vector characterization in the vector space R^N. The length of a vector in R^N is defined as

$$||\mathbf{s}_i|| = \sqrt{\sum_{j=1}^{N} s_{ij}^2}.$$

The distance between two signal constellation points **s***i* and **s***k* is given by:

$$||\mathbf{s}_i - \mathbf{s}_k|| = \sqrt{\sum_{j=1}^{N}(s_{ij} - s_{kj})^2} = \sqrt{\int_0^T (s_i(t) - s_k(t))^2 dt},$$



The second equality is obtained by writing $si(t)$ and $sk(t)$ in their basis representation and using the orthonormal properties of the basis functions.

$$< s_i(t), s_k(t) >= \int_0^T s_i(t)s_k(t)dt.$$

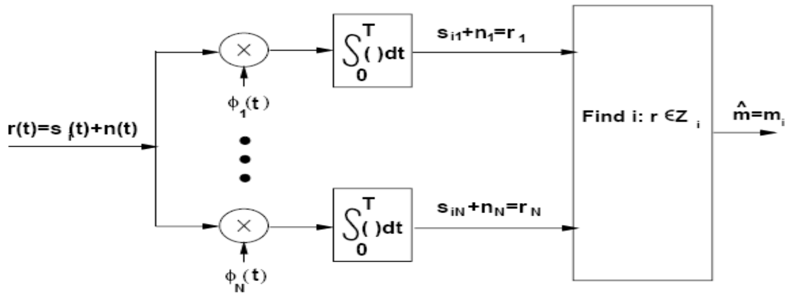
Finally, the inner product < $si(t)$, $sk(t)$ > between two real signals $si(t)$ and $sk(t)$ on the interval $[0, T]$ is

$$< \mathbf{s}_i, \mathbf{s}_k >= \mathbf{s}_i\mathbf{s}_k^T = \int_0^T s_i(t)s_k(t)dt =< s_i(t), s_k(t) >,$$

Receiver structure:

Given the channel output $r(t) = si(t) + n(t)$, $0 \leq t < T$, we now investigate the receiver structure to determine which constellation point **s***i* or, equivalently, which message *mi*, was sent over the time interval $[0, T)$. A similar procedure is done for each time interval $[kT, (k+1)T)$. We would like to convert the received signal $r(t)$ over each time interval into a vector, as it allows us to work in finite-dimensional vector space to estimate the transmitted signal. However, this conversion should not compromise the estimation accuracy.

For this conversion, consider the receiver structure shown below



where,

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt,$$

$$n_j = \int_0^T n(t)\phi_j(t)dt.$$

Decision regions and maximum likelihood decision criterion:

The optimal receiver minimizes error probability by selecting the detector output ˆ*m* that maximizes 1 − *Pe* = *p*(ˆ*m* sent / **r**). In other words, given a received vector **r**, the optimal receiver selects ˆ*m* = *mi* corresponding to the constellation **s***i* that satisfies *p*(**s***i* sent / **r**) > *p*(**s***j* sent / **r**)   *j* ≠ *i*. Let us define a set of **decisions regions** (*Z*1*, . . . , ZM*) that are subsets of the signal space *R^N* by

$$Z_i = (\mathbf{r} : p(\mathbf{s}_i \text{ sent}|\mathbf{r}) > p(\mathbf{s}_j \text{ sent}|\mathbf{r}) \forall j \neq i)$$

Clearly these regions do not overlap. Moreover, they partition the signal space assuming there is no **r** Є *RN* for which *p*(**s***i* sent/**r**) = *p*(**s***j* sent/**r**). If such points exist then the signal space is partitioned with decision regions by arbitrarily assigning such points to either decision region *Zi* or *Zj* . Once the signal space has been partitioned by decision regions, then for a received vector **r** Є *Zi* the optimal receiver outputs the message estimate ˆ*m* = *mi*.

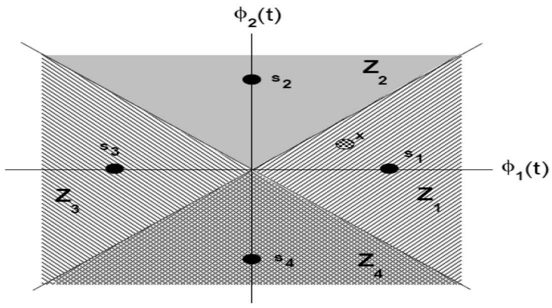
Thus, the receiver processing consists of computing the received vector **r** from *r*(*t*), finding which decision region *Zi* contains **r**, and outputting the corresponding message *mi*. This process is illustrated in the next figure, where we show a two-dimensional signal space with four decision regions *Z*1*, . . . , Z*4 corresponding to four constellations **s**1*, . . . ,* **s**4. The received vector **r** lies in region *Z*1, so the receiver will output the message *m*1 as the best message estimate given received vector **r**

We now examine the decision regions in more detail. We will abbreviate p(**s***i* sent/**r** received) as p(**s***i*/**r**) and p(**s***i* sent) as p(**s***i*). By Bayes rule,

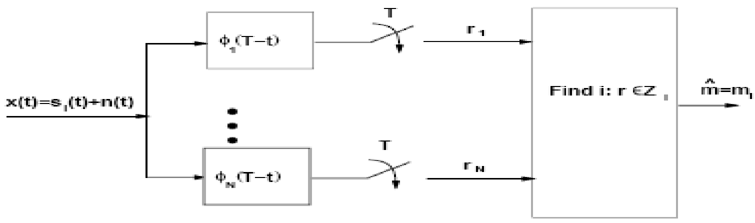
$$p(\mathbf{s}_i|\mathbf{r}) = \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})}.$$

To minimize error probability, the receiver output ˆm = mi corresponds to the constellation **s***i* that maximizes$p$(**s***i*/**r**), i.e. **s***i* must satisfy

$$\arg\max_{\mathbf{s}_i} \frac{p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i)}{p(\mathbf{r})} = \arg\max_{\mathbf{s}_i} p(\mathbf{r}|\mathbf{s}_i)p(\mathbf{s}_i), i = 1, \ldots, M$$



*T*he **matched filter** to the signal Φ(t), this receiver is also called a **matched filter receiver**. It can be shown that if a given input signal is passed through a filter matched to that signal, the output SNR is maximized



Probability of error:

$$\begin{aligned}
P_e &= \sum_{i=1}^{M} p(\mathbf{r} \not\in Z_i|m_i \text{ sent})p(m_i \text{ sent}) \\
&= \frac{1}{M}\sum_{i=1}^{M} p(\mathbf{r} \not\in Z_i|m_i \text{ sent}) \\
&= 1 - \frac{1}{M}\sum_{i=1}^{M} p(\mathbf{r} \in Z_i|m_i \text{ sent}) \\
&= 1 - \frac{1}{M}\sum_{i=1}^{M} \int_{Z_i} p(\mathbf{r}|m_i)d\mathbf{r} \\
&= 1 - \frac{1}{M}\sum_{i=1}^{M} \int_{Z_i} p(\mathbf{r} = \mathbf{s}_i + \mathbf{n}|\mathbf{s}_i)d\mathbf{n}. \\
&= 1 - \frac{1}{M}\sum_{i=1}^{M} \int_{Z_i - \mathbf{s}_i} p(\mathbf{n})d\mathbf{n}
\end{aligned}$$

Digital modulation techniques:

1.Amplitude and phase modulation:

In amplitude and phase modulation the information bit stream is encoded in the amplitude and/or phase of the transmitted signal. Specifically, over a time interval of Ts, $K = \log 2M$ bits are encoded into the amplitude and/or phase of the transmitted signal $s(t)$, $0 \leq t < Ts$. The transmitted signal over this period $s(t) = sI(t)\cos(2\pi fct) - sQ(t)\sin(2\pi fct)$ can be written in terms of its signal space representation as $s(t) = si1\varphi 1(t) + si2\varphi 2(t)$ with basis functions $\varphi 1(t) = g(t)\cos(2\pi fct + \varphi 0)$ and $\varphi 2(t) = -g(t)\sin(2\pi fct + \varphi 0)$, where $g(t)$ is a shaping pulse.

To send the ith message over the time interval $[kT, (k+1)T)$, we set $sI(t) = si1g(t)$ and $sQ(t) = si2g(t)$. These in-phase and quadrature signal components are baseband signals with spectral characteristics determined by the pulse shape $g(t)$. In particular, their bandwidth B equals the bandwidth of $g(t)$, and the transmitted signal $s(t)$ is a passband signal with center frequency fc and passband bandwidth $2B$. In practice we take $B = Kg/Ts$ where Kg depends on the pulse shape: for rectangular pulses Kg =0 .5 and for raised cosine pulses $0.5 \leq Kg \leq 1$

Thus, for rectangular pulses the bandwidth of $g(t)$ is $0.5/Ts$ and the bandwidth of $s(t)$ is $1/Ts$. Since the pulse shape $g(t)$ is fixed, the signal constellation for amplitude and phase modulation is defined based on the constellation point: $(si1, Si2) \in R^2$, $i = 1, \ldots, M$

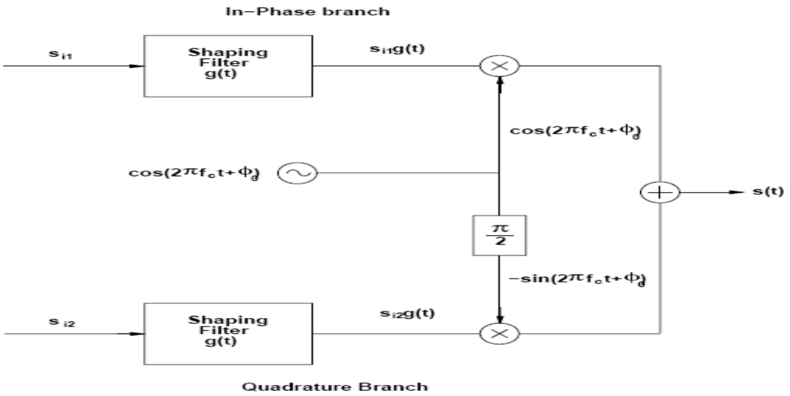
There are three main types of amplitude/phase modulation:

1. Pulse Amplitude Modulation (MPAM): information encoded in amplitude only.
2. Phase Shift Keying (MPSK): information encoded in p
phase only.
3.Quadrature Amplitude Modulation (MQAM): information encoded in both amplitude and phase.

Amplitude and phase modulation over a given symbol period can be generated using the modulator structure shown in the next slide. Note that the basis functions in this figure have an arbitrary phase $\varphi 0$ associated with the transmit oscillator.

Demodulation over each symbol period is performed using the demodulation structure shown in the next slide, which is equivalent to the matched filter receiver for
$\varphi 1(t) = g(t) \cos(2\pi fct + \varphi)$ and $\varphi 2(t) = -g(t) \sin(2\pi fct + \varphi)$.

Typically the receiver includes some additional circuitry for **carrier phase recovery** that matches the carrier phase φ at the receiver to the carrier phase $\varphi 0$ at the transmitter1, which is called **coherent detection**

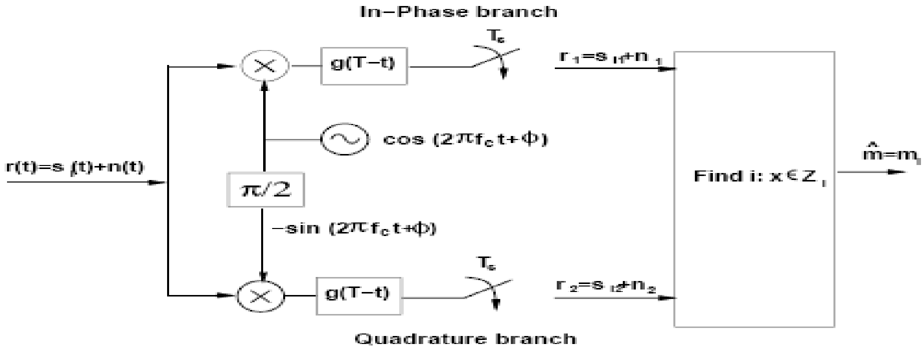


Pulse amplitude modulation:

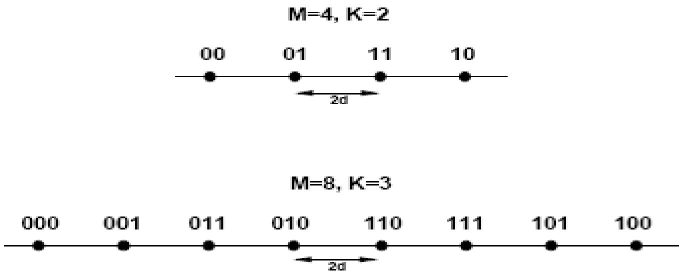
The simplest form of linear modulation is one dimensional MPAM, which has no quadrature component ($si2 = 0$). For MPAM all of the information is encoded into the signal amplitude Ai. The transmitted signal over one symbol time is given by

$$s_i(t) = \Re\{A_i g(t) e^{j2\pi f_c t}\} = A_i g(t)\cos(2\pi f_c t), \quad 0 \le t \le T_s >> 1/f_c,$$

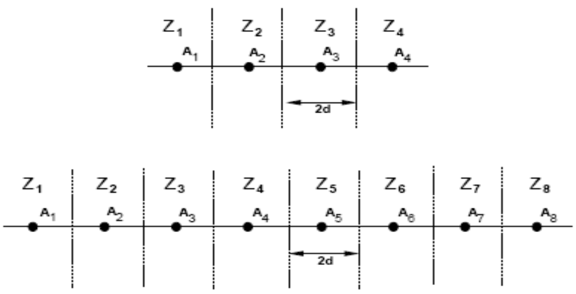
Amplitude/Phase Demodulator (Coherent: $\phi = \phi_0$).



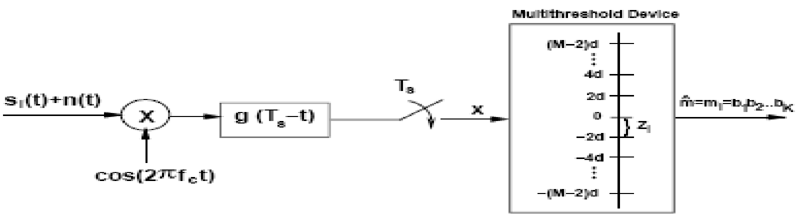
Gray Encoding for MPAM:



Decision regions for MPAM



MPAM coherent demodulator:



Phase shift keying(MPSK):

For MPSK all of the information is encoded in the phase of the transmitted signal. Thus, the transmitted signal over one symbol time is given by

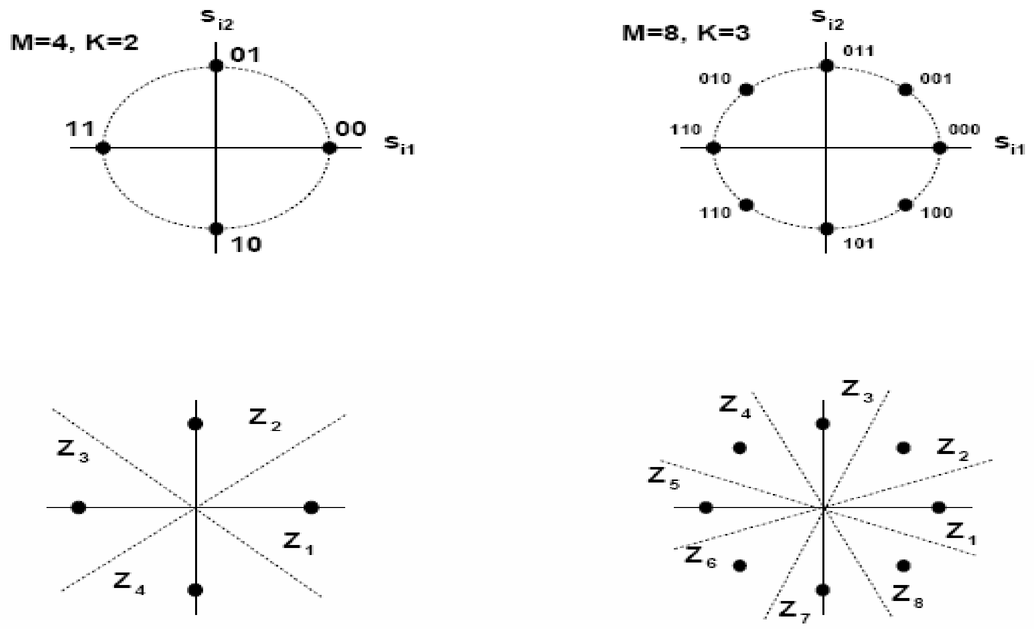
$$
\begin{aligned}
s_i(t) &= \Re\{Ag(t)e^{j2\pi(i-1)/M}e^{j2\pi f_c t}\}, \quad 0 \le t \le T_s \\
&= Ag(t)\cos\left[2\pi f_c t + \frac{2\pi(i-1)}{M}\right] \\
&= Ag(t)\cos\left[\frac{2\pi(i-1)}{M}\right]\cos 2\pi f_c t - Ag(t)\sin\left[\frac{2\pi(i-1)}{M}\right]\sin 2\pi f_c t.
\end{aligned}
$$

Thus, the constellation points or symbols ($si1$, $si2$) are given by $si1 = A\cos[\ 2\pi(i-1)M\ ]$ and $si2 = A\sin[\ 2\pi(i-1)M\ ]$ for $i = 1, \ldots, M$. $\theta i = 2\pi(i-1)\ M$, $i = 1, 2, \ldots, M = 2K$ are the different phases in the signal constellation points that convey the information bits. The minimum distance between constellation points is $dmin = 2A\sin(\pi/M)$, where A is typically a function of the signal energy. 2PSK is often referred to as binary PSK or BPSK, while 4PSK is often called quadrature phase shift keying (QPSK), and is the same as MQAM with $M = 4$ which is defined below.

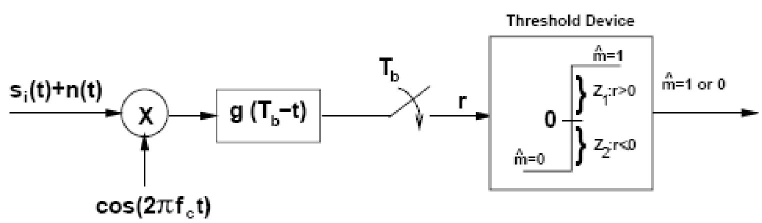
All possible transmitted signals $s_i(t)$ have equal energy:

$$E_{s_i} = \int_0^{T_s} s_i^2(t)dt = A^2$$

Gray encoding for MPSK:



Coherent demodulator for BPSK:



Differential modulation:

The information in MPSK and MQAM signals is carried in the signal phase. Thus, these modulation techniques require coherent demodulation, i.e. the phase of the transmitted signal carrier $\varphi 0$ must be matched to the phase of the receiver carrier φ. Techniques for phase recovery typically require more complexity and cost in the receiver and they are also susceptible to phase drift of the carrier. Moreover, obtaining a coherent phase reference in a rapidly fading channel can be difficult.

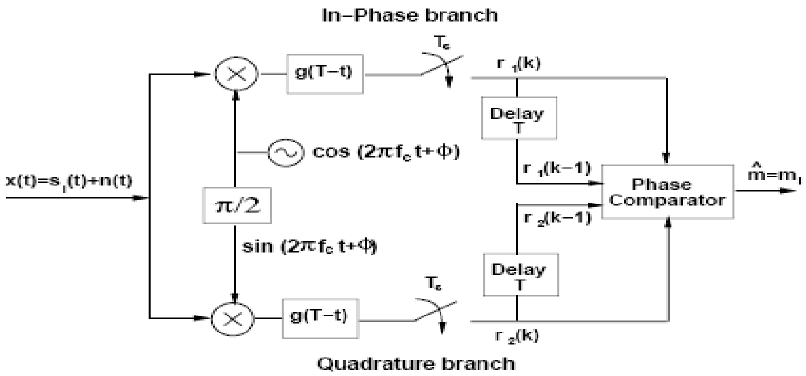
Differential modulation techniques, which do not require a coherent phase reference at the receiver, are generally preferred to coherent modulation for wireless applications. Differential modulation falls in the more general class of modulation with memory, where the symbol transmitted over time $[kTs, (k + 1)Ts)$ depends on the bits associated with the current message to be transmitted *and* the bits transmitted over prior symbol times.

The basic principle of differential modulation is to use the previous symbol as a phase reference for the current symbol, thus avoiding the need for a coherent phase reference at the receiver. Specifically, the information bits are encoded as the differential phase between the current symbol and the previous symbolFor example, in differential BPSK, referred to as DPSK, if the symbol over time $[(k-1)Ts, kTs)$ has phase $\theta(k - 1) = e\text{^}j\theta i$, $\theta i = 0, \pi$, then to encode a 0 bit over $[kTs, (k + 1)Ts)$, the symbol would have phase $\theta(k) = e\text{^}j\theta i$ and to encode a 1 bit the symbol would have phase $\theta(k) = e\text{^}j(\theta i+\pi)$

In other words, a 0 bit is encoded by no change in phase, whereas a 1 bit is encoded as a phase change of π. Similarly, in 4PSK modulation with differential encoding, the symbol phase over symbol interval $[kTs, (k + 1)Ts)$ depends on the current information bits over this time interval and the symbol phase over the previous symbol interval.The phase transitions for DQPSK modulation are summarized in the table below

Bit Sequence	Phase Transition
00	0
01	$\pi/2$
10	$-\pi/2$
11	π

The demodulator for differential modulation is shown below..

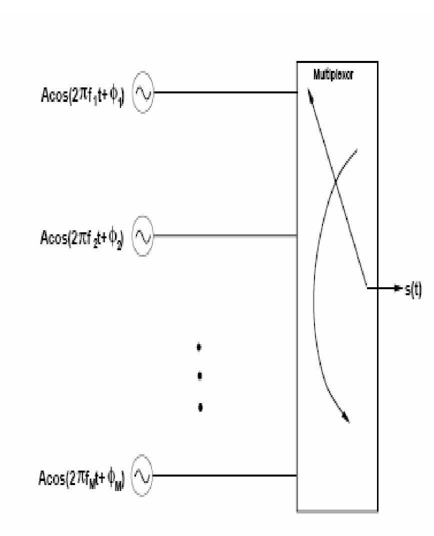


Frequency modulation:

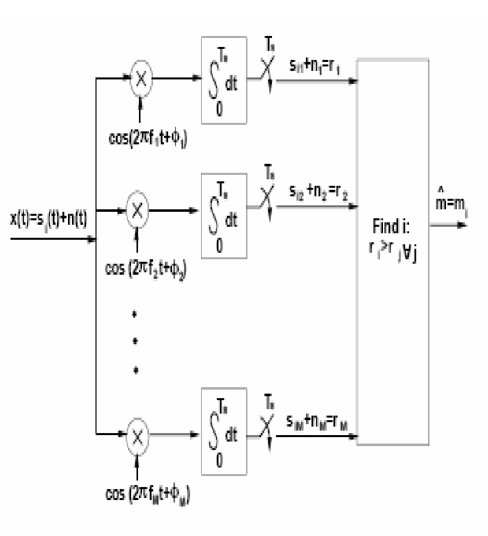
Frequency modulation encodes information bits into the frequency of the transmitted signal. Specifically, each symbol time $K = \log 2M$ bits are encoded into the frequency of the transmitted signal $s(t)$, $0 \leq t < Ts$, resulting in a transmitted signal $si(t) = A\cos(2\pi fit + \varphi i)$, where i is the index of the ith message corresponding to the $\log 2M$ bits and φi is the phase associated with the ith carrier.

The signal space representation is $si(t) =j\ sij\ \varphi j(t)$ where $sij = A\partial(i - j)$ and $\varphi j(t) = \cos(2\pi fjt + \varphi j)$, so the basis functions correspond to carriers at different frequencies and only one such basis function is transmitted in each symbol period. The orthogonality of the basis functions requires a minimum separation between different carrier frequencies of $\Delta f = \min ij\ |fj - fi| = 0.5/Ts$.

Frequency modulator



frequency demodulator



Phase shift keying and minimum shift keying:

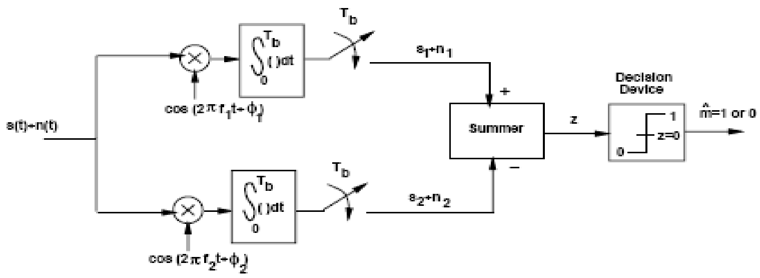
In MFSK the modulated signal is given by

$$s_i(t) = A\cos[2\pi f_c t + 2\pi\alpha_i \Delta f_c t + \phi_i], \quad 0 \leq t < T_s,$$

where $\alpha i = (2i - 1 - M)$, $i = 1, 2, \ldots, M = 2K$. The minimum frequency separation between FSK carriers is thus $2\Delta fc$. MFSK consists of M basis functions $\varphi i(t) = 2/Ts \cos[2\pi fct + 2\pi\alpha i\Delta fct + \varphi i]$, where the $2/Ts$ is a normalization factor to insure that $Ts0\ \varphi 2i\ (t) = 1$. Over a given symbol time only one basis function is transmitted through the channel.A simple way to generate the MFSK signal M oscillators are operating at the different frequencies $fi = fc + \alpha i\Delta fc$ and the modulator switches between these different oscillators each symbol time Ts.

However, with this implementation there will be a discontinuous phase transition at the switching times due to phase offsets between the oscillators MSK is a special case of FSK where the minimum frequency separation is 2Δfc = 0.5/Ts. Note that this is the minimum frequency separation so that < si(t), sj(t) >= 0 over a symbol time, for i = j. Since signal orthogonality is required for demodulation, 2Δfc = 0.5/Ts is the minimum possible frequency separation in FSK, and therefore it occupies the minimum bandwidth.

Demodulator for FSK:



Continuous phase FSK(CPFSK) :

A better way to generate MFSK that eliminates the phase discontinuity is to frequency modulate a single carrier with a modulating waveform, as in analog FM. In this case the modulated signal will be given by

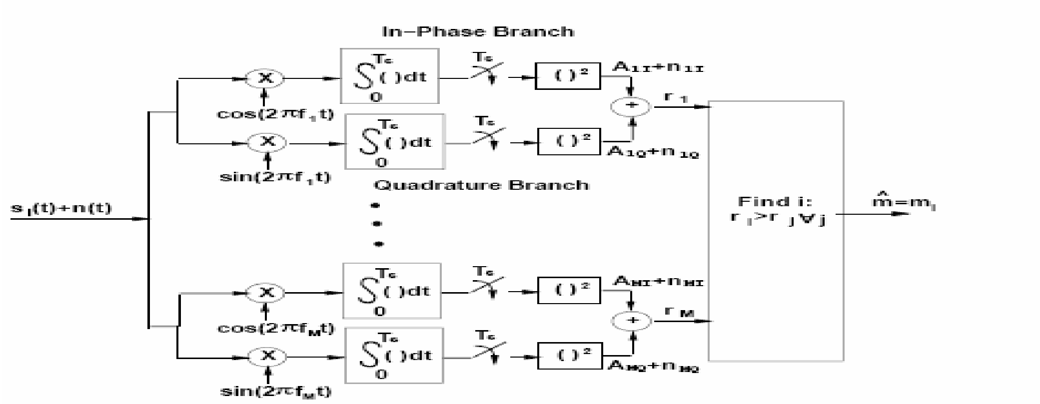
$$s_i(t) = A\cos\left[2\pi f_c t + 2\pi\beta \int_{-\infty}^{t} u(\tau)d\tau\right] = A\cos[2\pi f_c t + \theta(t)],$$

Non coherent detection of FSK:

The receiver requirement for a coherent phase reference associated with each FSK carrier can be difficult and expensive to meet. The need for a coherent phase reference can be eliminated by detecting the energy of the signal at each frequency and, if the *i*th branch has the highest energy of all branches, then the receiver outputs message *mi*. The modified receiver is shown below.Suppose the transmitted signal corresponds to frequency *fi*:

$$s(t) = A\cos(2\pi f_i t + \phi_i) = A\cos(\phi_i)\cos(2\pi f_i t) - A\sin(\phi_i)\sin(2\pi f_i t), \quad 0 \le t < T_s$$

Non coherent FSK demodulator:

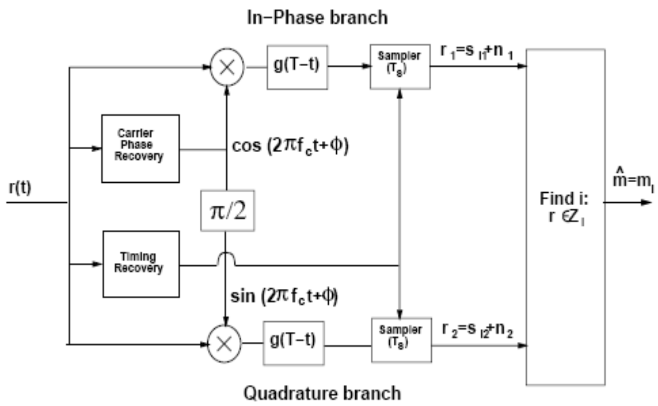


Symbol Synchronization and Carrier Phase Recovery:

One of the most challenging tasks of a digital demodulator is to acquire accurate symbol timing and carrier phase information.Timing information, obtained via synchronization, is needed to delineate the received signal associated with a given symbol. In particular, timing information is used to drive the sampling devices associated with the demodulators for amplitude, phase, and frequency demodulation Carrier phase information is needed in all coherent demodulators for both amplitude/phase and frequency modulation.

Receiver Structure with Phase and Timing Recovery:

The carrier phase and timing recovery circuitry for the amplitude and phase demodulator is shown next.For BPSK only the in-phase branch of this demodulator is needed. For the coherent frequency demodulator shown before a carrier phase recovery circuit is needed for *each* of the distinct *M* carriers, and the resulting circuit complexity motivates the need for the noncoherent demodulator. We'll see in the next figure that the carrier phase and timing recovery circuits operate directly on the received signal prior to demodulation.

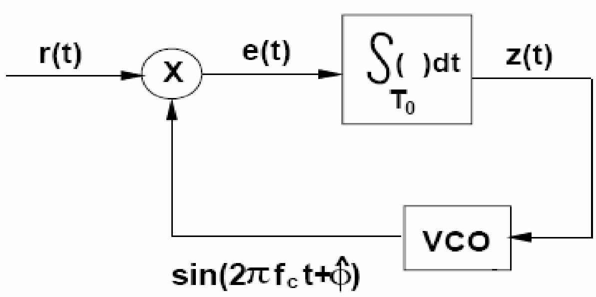


Maximum Likelihood Phase Estimation:

Using the equation shown below yields that maximizing $p(\mathbf{r}/\theta)$ is equivalent to maximizing the **likelihood function**

$$\Lambda(\theta) = \exp\left[-\frac{1}{N_0}\int_{T_0}[r(t) - s(t;\theta)]^2\right] dt$$

Phase locked loop for carrier phase recovery:



Maximum likelihood Timing estimation:

we derive the maximum likelihood estimate of delay τ assuming the carrier phase is known. Since we assume that the phase φ is known, the timing recovery will not affect down conversion by the carrier shown Thus, it suffices to consider timing estimation for the in-phase or quadrature baseband equivalent signals of $r(t)$ and $s(t;\tau)$. We denote the in-phase and quadrature components for $r(t)$ as $r1\ (t)$ and $rQ(t)$ and for $s(t;\tau)$ as $sI\ (t;\tau)$ and $sQ(t;\tau)$.We focus on the in-phase branch as the timing recovered from this branch can be used for the quadrature branch.

Early-Late Gate Synchronizer:

One structure for timing estimation is the **early-late gate synchronizer** shown below.