

Appendix B. Definitions

General

B.1 Accuracy of measurement

The closeness of the agreement between the result of a measurement and a *true value* of the measurand [H.4].

NOTE 1 "Accuracy" is a qualitative concept.

NOTE 2 The term "precision" should not be used for "accuracy".

B.2 Precision

The closeness of agreement between independent test results obtained under stipulated conditions [H.5].

NOTE 1 Precision depends only on the distribution of random errors and does not relate to the true value or the specified value.

NOTE 2 The measure of precision is usually expressed in terms of imprecision and computed as a standard deviation of the test results. Less precision is reflected by a larger standard deviation.

NOTE 3 "Independent test results" means results obtained in a manner not influenced by any previous result on the same or similar test object. Quantitative measures of precision depend critically on the stipulated conditions. Repeatability and reproducibility conditions are particular sets of extreme stipulated conditions.

B.3 True value

Value consistent with the definition of a given particular quantity [H.4].

NOTE 1 This is a value that would be obtained by a perfect measurement.

NOTE 2 True values are by nature indeterminate.

NOTE 3 The indefinite article "a" rather than the definite article "the" is used in

conjunction with "true value" because there may be many values consistent with the definition of a given particular quantity.

B.4 Conventional true value

Value attributed to a particular quantity and accepted, sometimes by convention, as having an uncertainty appropriate for a given purpose [H.4].

EXAMPLES

a) At a given location, the value assigned to the quantity realised by a reference standard may be taken as a conventional true value.

b) The CODATA (1986) recommended value for the Avogadro constant, N_A : $6.0221367 \times 10^{23} \text{ mol}^{-1}$

NOTE 1 "Conventional true value" is sometimes called *assigned value*, *best estimate* of the value, *conventional value* or *reference value*.

NOTE 2 Frequently, a number of results of measurements of a quantity is used to establish a conventional true value.

B.5 Influence quantity

A quantity that is not the measurand but that affects the result of the measurement [H.4].

EXAMPLES

1. Temperature of a micrometer used to measure length;

2. Frequency in the measurement of an alternating electric potential difference;

3. Bilirubin concentration in the measurement of haemoglobin concentration in human blood plasma.

Measurement**B.6 Measurand**

Particular quantity subject to measurement [H.4].

NOTE The specification of a measurand may require statements about quantities such as time, temperature and pressure..

B.7 Measurement

Set of operations having the object of determining a value of a quantity [H.4].

B.8 Measurement procedure

Set of operations, described specifically, used in the performance of measurements according to a given method [H.4].

NOTE A measurement procedure is usually recorded in a document that is sometimes itself called a "measurement procedure" (or a *measurement method*) and is usually in sufficient detail to enable an operator to carry out a measurement without additional information.

B.9 Method of measurement

A logical sequence of operations, described generically, used in the performance of measurements [H.4].

NOTE Methods of measurement may be qualified in various ways such as:

- substitution method
- differential method
- null method

B.10 Result of a measurement

Value attributed to a measurand, obtained by measurement [H.4].

NOTE 1 When the term "result of a measurement" is used, it should be made clear whether it refers to:

- The indication.
- The uncorrected result.
- The corrected result.

and whether several values are averaged.

NOTE 2 A complete statement of the result of a measurement includes information about the uncertainty of measurement.

Uncertainty**B.11 Uncertainty (of measurement)**

Parameter associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand [H.4].

NOTE 1 The parameter may be, for example, a standard deviation (or a given multiple of it), or the width of a confidence interval.

NOTE 2 Uncertainty of measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of a series of measurements and can be characterised by experimental standard deviations. The other components, which can also be characterised by standard deviations, are evaluated from assumed probability distributions based on experience or other information.

NOTE 3 It is understood that the result of the measurement is the best estimate of the value of the measurand and that all components of uncertainty, including those arising from systematic effects, such as components associated with corrections and reference standards, contribute to the dispersion.

B.12 Traceability

The property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties [H.4].

B.13 Standard uncertainty

$u(x_i)$ Uncertainty of the result x_i of a measurement expressed as a standard deviation [H.2].

B.14 Combined standard uncertainty

$u_c(y)$ Standard uncertainty of the result y of a measurement when the result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being

the variances or covariances of these other quantities weighted according to how the measurement result varies with these quantities [H.2].

B.15 Expanded uncertainty

U Quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand [H.2].

NOTE 1 The fraction may be regarded as the coverage probability or level of confidence of the interval.

NOTE 2 To associate a specific level of confidence with the interval defined by the expanded uncertainty requires explicit or implicit assumptions regarding the probability distribution characterised by the measurement result and its combined standard uncertainty. The level of confidence that may be attributed to this interval can be known only to the extent to which such assumptions can be justified.

NOTE 3 An expanded uncertainty *U* is calculated from a combined standard uncertainty *u_c* and a coverage factor *k* using

$$U = k \times u_c$$

B.16 Coverage factor

k Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty [H.2].

NOTE A coverage factor is typically in the range 2 to 3.

B.17 Type A evaluation (of uncertainty)

Method of evaluation of uncertainty by the statistical analysis of series of observations [H.2].

B.18 Type B evaluation (of uncertainty)

Method of evaluation of uncertainty by means other than the statistical analysis of series of observations [H.2]

Error

B.19 Error (of measurement)

The result of a measurement minus a true value of the measurand [H.4].

NOTE 1 Since a true value cannot be determined, in practice a conventional true value is used.

B.20 Random error

Result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions [H.4].

NOTE 1 Random error is equal to error minus systematic error.

NOTE 2 Because only a finite number of measurements can be made, it is possible to determine only an estimate of random error.

B.21 Systematic error

Mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand [H.4].

NOTE 1: Systematic error is equal to error minus random error.

NOTE 2: Like true value, systematic error and its causes cannot be known.

Statistical terms

B.22 Arithmetic mean

\bar{x} Arithmetic mean value of a sample of *n* results.

$$\bar{x} = \frac{\sum_{i=1,n} x_i}{n}$$

B.23 Sample Standard Deviation

s An estimate of the population standard deviation σ from a sample of *n* results.

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

B.24 Standard deviation of the mean

$s_{\bar{x}}$ The standard deviation of the mean \bar{x} of n values taken from a population is given by

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

The terms "standard error" and "standard error of the mean" have also been used to describe the same quantity.

B.25 Relative Standard Deviation (RSD)

RSD An estimate of the standard deviation of a population from a sample of n results divided by the mean of that sample. Often known as coefficient of variation (CV). Also frequently stated as a percentage.

$$\text{RSD} = \frac{s}{\bar{x}}$$

Appendix H. Bibliography

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TABLE 3.1

Values of z and the two-tailed probability that its absolute value will be exceeded in a normal population (see also Fig. 3.3a)

Second decimal in p										
p	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	∞	2.576	2.326	2.170	2.054	1.960	1.881	1.812	1.750	1.695
0.1	1.645	1.598	1.555	1.514	1.476	1.439	1.405	1.372	1.340	1.311
0.2	1.231	1.254	1.226	1.200	1.175	1.150	1.126	1.103	1.080	1.058
0.3	1.036	1.015	0.994	0.974	0.954	0.935	0.915	0.896	0.878	0.860
0.4	0.842	0.824	0.806	0.789	0.772	0.755	0.739	0.722	0.706	0.690
0.5	0.674	0.659	0.643	0.623	0.613	0.598	0.583	0.568	0.553	0.539
0.6	0.524	0.510	0.496	0.482	0.468	0.454	0.440	0.436	0.412	0.399
0.7	0.385	0.372	0.358	0.345	0.332	0.319	0.305	0.292	0.279	0.266
0.8	0.253	0.240	0.228	0.215	0.202	0.189	0.176	0.164	0.151	0.138
0.9	0.126	0.113	0.100	0.088	0.075	0.063	0.050	0.038	0.025	0.013
p	0.002		0.001	0.0001	0.00001	0.000001	0.0000001		0.00000001	
z	3.090		3.290	3.890	4.417	4.891	5.326		5.730	

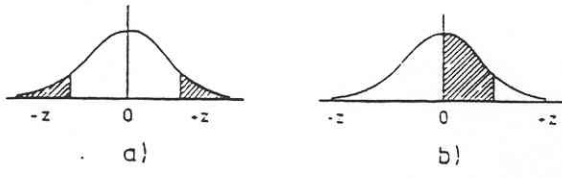


TABLE 3.2

Probability p to find a value between 0 and z (see also Fig. 3.3b)

Second decimal of z											
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	
0.0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036	
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075	
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114	
0.3	0.118	0.122	0.125	0.129	0.133	0.137	0.144	0.141	0.148	0.152	
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188	
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222	
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255	
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285	
0.8	0.288	0.291	0.294	0.297	0.299	0.302	0.305	0.308	0.311	0.313	
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339	
1.0	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362	
1.1	0.364	0.366	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383	
1.2	0.385	0.387	0.389	0.391	0.392	0.394	0.396	0.398	0.400	0.401	
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418	
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432	
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444	
1.6	0.445	0.446	0.447	0.448	0.449	0.450	0.451	0.452	0.453	0.454	
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463	
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471	
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477	
$z =$	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	
$F(z) =$	0.477	0.482	0.486	0.489	0.492	0.494	0.495	0.496	0.497	0.498	
$z =$	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
$F(z) =$	0.4987	0.4990	0.4993	0.4995	0.4997	0.4998	0.4998	0.4998	0.4999	0.49995	0.49997

TABLE 3.3
Probability to find a value lower than z (see also Fig. 3.3c)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.625	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.799	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.0	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.866	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.892	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.950	0.951	0.952	0.953	0.954
1.7	0.955	0.956	0.957	0.958	0.960	0.961	0.962	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977

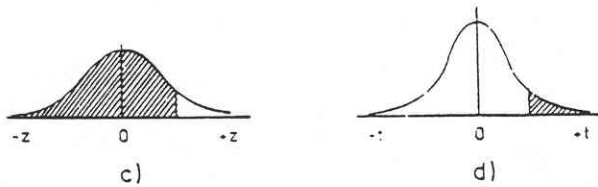


TABLE 3.4
One-sided t -table (see also Fig. 3.3d)

Area in upper tail of t -distribution							
df	0.10	0.05	0.025	0.01	0.005	0.0025	0.001
1	3.078	6.314	12.706	31.821	63.657	127.32	318.310
2	1.886	2.920	4.303	6.965	9.925	14.089	22.327
3	1.638	2.353	3.182	4.541	5.841	7.453	10.215
4	1.533	2.132	2.776	3.747	4.604	5.598	7.173
5	1.476	2.015	2.571	3.365	4.032	4.773	5.893
6	1.440	1.943	2.447	3.143	3.707	4.317	5.208
7	1.415	1.895	2.365	2.998	3.499	4.029	4.785
8	1.397	1.860	2.306	2.896	3.355	3.832	4.501
9	1.383	1.833	2.262	2.821	3.250	3.690	4.297
10	1.372	1.812	2.228	2.764	3.169	3.581	4.144
11	1.363	1.796	2.201	2.718	3.106	3.497	4.025
12	1.356	1.782	2.179	2.681	3.055	3.428	3.930
13	1.350	1.771	2.160	2.650	3.012	3.372	3.852
14	1.345	1.761	2.145	2.624	2.977	3.326	3.787
15	1.341	1.753	2.131	2.602	2.947	3.286	3.733
16	1.337	1.746	2.120	2.583	2.921	3.252	3.686
17	1.333	1.740	2.110	2.567	2.898	3.222	3.646
18	1.330	1.734	2.101	2.552	2.878	3.197	3.610
19	1.328	1.729	2.093	2.539	2.861	3.174	3.579
20	1.325	1.725	2.086	2.528	2.845	3.153	3.552
21	1.323	1.721	2.080	2.518	2.831	3.135	3.527
22	1.321	1.717	2.074	2.508	2.819	3.119	3.505
23	1.319	1.714	2.069	2.500	2.807	3.104	3.485
24	1.318	1.711	2.064	2.492	2.797	3.090	3.467
25	1.316	1.708	2.060	2.485	2.787	3.078	3.450
26	1.315	1.706	2.056	2.479	2.779	3.067	3.435
27	1.314	1.703	2.052	2.473	2.771	3.056	3.421
28	1.313	1.701	2.048	2.467	2.763	3.047	3.408
29	1.311	1.699	2.045	2.462	2.756	3.038	3.396
30	1.310	1.697	2.042	2.457	2.750	3.030	3.385
40	1.303	1.684	2.021	2.423	2.704	2.971	3.307
60	1.296	1.671	2.000	2.390	2.660	2.915	3.232
120	1.289	1.658	1.980	2.358	2.617	2.860	3.160
∞	1.282	1.645	1.960	2.326	2.576	2.807	3.090

ical F -values for a one-tailed test ($\alpha = 0.025$)

df ₁																			
1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
647	779	864	899	922	937	948	956	963	968	976	985	993	997	1001	1005	1010	1014	1018	
38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50	
17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90	
12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26	
10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02	
8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85	
8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14	
7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67	
7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33	
6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08	
6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72	
6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40	
5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09	
5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94	
5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79	
5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64	
5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48	
5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31	
5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00	

df₂
1
2
3
4
5
6
7
8
9
10
12
15
20
24
30
40
60
120
∞

Critical F -values for a one-tailed test ($\alpha = 0.05$)

df ₁																			
df ₂	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	199	215	224	230	234	237	239	240	242	244	246	248	249	250	251	252	253	254
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Critical values of Chi-square (the α -values represent the area to the right of the critical χ^2 in one tail of the distribution)

df^{α}	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.001
1	0.0002	0.0010	0.0039	0.0158	2.71	3.84	5.02	6.63	10.83
2	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	13.82
3	0.12	0.22	0.35	0.58	6.25	7.81	9.35	11.34	16.27
4	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28	18.47
5	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09	20.52
6	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	22.46
7	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.47	24.32
8	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	26.13
9	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	27.88
10	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	29.59
11	3.05	3.82	4.57	5.58	17.27	19.67	21.92	24.72	31.26
12	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	32.91
13	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	34.53
14	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	36.12
15	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	37.70
16	5.81	6.91	7.96	9.31	23.54	26.30	28.84	32.00	39.25
17	6.41	7.56	8.67	10.08	24.77	27.59	30.19	33.41	40.79
18	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.80	42.31
19	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	43.82
20	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	45.32
21	8.90	10.28	11.59	13.24	29.61	32.67	35.48	38.93	46.80
22	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	48.27
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	49.73
24	10.86	12.40	13.85	15.66	33.20	36.41	39.37	42.98	51.18
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	52.62
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64	54.05
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	55.48
28	13.57	15.31	16.93	18.94	37.92	41.34	44.46	48.28	56.89
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	58.30
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	59.70

