Complexity and mathematics incompleteness

Let us ask some questions about mathematics.

According to Gödel's theorem, a mathematical theory cannot be complete. There are always flaws within the formal system. This means that the knowledge or information introduced through the axioms is not sufficient to prove everything we can say about the formal system.

On the other hand, a mathematical theory is a description of the world. But the world is "perfect". There is no flaw in it. So Gödel's theorem revealed an *eternal hole* between mathematics and nature. The question is : why mathematics is incomplete by definition? More generally, why a formal system is necessarily incomplete by definition? What is the origin of this incompleteness of information?

A basis of mathematics is arithmetic. From the epistemological point of view, arithmetic is a theory based on a model of the world constructed from the crude data of the senses and direct intuition of human beings. This is a simple model for *a fragmented finite world containing only isolated, distinct, independent and finite parts*. These parts are numerically represented by the numbers $0, 1, 2...$ and related by the simple rules (axioms and theorems) like $1 = 1$, $1 + 0 = 1$, $1 + 1 = 2$ and so on.

This fragmented world is not wrong, but it is only partially true. It should be remembered that the world is an ensemble, an *infinite, messy* ensemble in which each part is correlated to other parts in space-time. There are indeed different parts, but they are more or less distinct, depending on the correlations. This is just the epistemological basis of the *decomposition* process of the world in our knowledge. From the time of Pythagoras up to now, this *decomposition and composition* method has allowed the marvellous development of scientific knowledge. But this method has its limits. And it is often forgotten that, during the decomposition and composition, a part of the world is rejected[1].

So in some sense, it can be said that mathematics is an approximate theory containing incomplete amount of information about the world it describes because some information is lost through the formation of the concepts and axioms. Any formation of axiomatic systems is necessarily made through a kind of decomposition of the world. In my opinion, this is the epistemological origin of the incompleteness theorem of Gödel.

A physical theory is in fact a mathematical system based on physical models. In physics, the incompleteness of mathematics becomes crucial and gets a practical importance because the calculations must be compared to the physical systems through observation. The logic flaws of mathematics may be now transformed into theoretical errors in the description, depending on

the nature of the system under consideration. For complex systems, one has to reject considerable information in order to construct a statistical model and to fit it to the mathematical framework, since there are infinite number of physical states, predictable or unpredictable. To my opinion, this is the natural way from Gödel's incompleteness theorem to the incompleteness of information in physics or in other probabilistic sciences.

The following section will be devoted to discuss this physical aspect of information incompleteness.

References

[1] The role played by the reductionist method (decomposition and composition approach) in the development of science is briefly discussed by Trinh Xuan Thuan in his book **Le chaos et l'harmonie** (p.410, Fayard, Paris, 1998). It is instructive to recall that in the traditional chinese philosophy, there is a methodology of reflection which is completely opposite to the reductionist one. Instead of dividing or decomposing the world in order to understand it, the chinese philosophers preferred to consider it first of all as an ensemble containing interacting parts in movement driven by two opposite forces : Yin and Yang. Furthermore, Yin and Yang are not completely independent and distinct from each other. As shown in the famous circular symbol of Taoism, Yin (black) and Yang (white) are two discernible things which come out from (and contained by) each other and are always willing to re-enter into each other. Although this way of reflection has not help to develop scientific knowledge in ancient China, but it indeed reflects another aspect of nature : there are different and discernible parts but they are not independent of each other.