Final Review

Math 222

1. Evaluate $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$.
2. Evaluate using partial fractions $\displaystyle\int \frac{6x-14}{x^2-2x-3}dx$.
3. Evaluate $\displaystyle\int x\cos x\,dx$.
4. Use L'Hospital's Rule to evaluate $\displaystyle\lim_{x\to 0}\frac{2\cos x-2+x^2}{3x^4}$.
5. Evaluate $\displaystyle\lim_{x\to 0^+}(\sin x)^{\tan x}$.
6. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{1}{\ln n}$ as absolutely convergent, conditionally convergent, or divergent.
7. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{n^n}{2^n}$ as absolutely convergent, conditionally convergent, or divergent.
8. Use any test to establish the convergence or divergence of the series $\displaystyle\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}$
9. Find the radius of convergence and the interval of convergence of the series $\displaystyle\sum nx^n$.
10. Find the radius of convergence and the interval of convergence of the series $\displaystyle\sum\frac{(-1)^n(x+1)^n}{n3^n}$.
11. Find the first four nonzero terms of the Taylor series with center $a=1$ of the function $f(x)=x^6$.
12. Complete the square to find the vertex, focus and directrix of the parabola $x^2-8y-2x-7=0$ and give a rough sketch of the graph.
13. Sketch the graph and state the foci and asymptotes of the hyperbola $\frac{x^2}{4}-y^2=1$.
14. Put into standard form and sketch the graph of $4x^2-y^2+16x+2y-1=0$.
15. Convert $r=4\sec\theta$ to rectangular coordinates and sketch its graph.
16. Sketch the graph of $r=2+3\sin\theta$.
17. Find the total area inside the loops of the 3-leaved rose $r=3\cos(3\theta)$.
18. Set up the integral(s) needed that give the area inside both the circle $r=2\sin\theta$ and the cardioid $r=2-3\sin\theta$.
19. Eliminate the parameter from $x=t^2$, $y=t^3$.
20. If $\vec{\mathbf{R}}(t)=\ln t\,\vec{\mathbf{i}}+t^2\vec{\mathbf{j}}$ is the position of a moving point at time t, find its (a) position, (b) velocity, (c) acceleration and (d) speed at time $t=1$.

Answers to problems above: 1. $2\sin\sqrt{x}+C$ 2. $\ln|x-3|+5\ln|x+1|+C$ 3. $x\sin x+\cos x+C$ 4. $\frac{1}{18}$ 5. 1 6. Conditionally convergence since $\lim_{n\to 0}\frac{1}{\ln n}=0$, so $\sum(-1)^{n+1}\frac{1}{\ln n}$ converges, but since $\frac{1}{\ln n}>\frac{1}{n}$, and $\sum\frac{1}{n}$ diverges, so does $\sum\frac{1}{\ln n}$. 7. Divergent since $\lim_{n\to 0}\frac{n^n}{2^n}=\infty\neq 0$, by the divergence test $\sum(-1)^{n+1}\frac{n^n}{2^n}$ diverges 8. Converges by the ratio test, since $L=\lim_{n\to\infty}\sqrt{\frac{n!}{(n+1)!}}=\lim_{n\to\infty}\sqrt{\frac{1}{(n+1)}}=0<1$ 9. $R=1$, $(-1,1)$ 10. $R=3$, $(-2,4]$ 11. $1+6\,(x-1)+15\,(x-1)^2+20\,(x-1)^3+\cdots$ 12. $(x-1)^2=8(y+1)$ vertex $(1,-1)$, focus $(1,1)$, directrix $y=-3$ 13. F: $(\pm\sqrt{5},0)$, A: $y=\pm\frac{1}{2}x$ 14. $\frac{(x+2)^2}{4}-\frac{(y-1)^2}{16}=1$ 15. $x=4$ (vertical line) 16. 17. $\frac{9\pi}{4}$ 18. $\int_0^{\arcsin(\frac{2}{5})}(2\sin\theta)^2d\theta+\int_{\arcsin(\frac{2}{5})}^{\frac{\pi}{2}}(2-3\sin\theta)^2d\theta$ 19. $y^2=x^3$ 20. (a) $\vec{\mathbf{R}}(1)=\vec{\mathbf{j}}$, (b) $\vec{\mathbf{v}}(1)=\vec{\mathbf{i}}+2\vec{\mathbf{j}}$, (c) $\vec{\mathbf{a}}(1)=-1\vec{\mathbf{i}}+2\vec{\mathbf{j}}$, (d) $v(1)=\sqrt{5}$

Final Review

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1. Evaluate $\displaystyle\int \frac{dx}{x\sqrt{\ln x}}$.
2. Evaluate using partial fractions $\displaystyle\int \frac{5x-12}{x^2-5x+6}dx$.
3. Evaluate $\displaystyle\int x\ln x\,dx$.
4. Use L'Hospital's Rule to evaluate $\displaystyle\lim_{x\to 0}\frac{e^{x^2}-1}{x\sin x}$.
5. Evaluate $\displaystyle\lim_{x\to 0^+}\left(1+\frac{2}{x}\right)^x$.
6. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{\cos^2 n}{n^2}$ as absolutely convergent, conditionally convergent, or divergent.
7. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{2^n}{n!}$ as absolutely convergent, conditionally convergent, or divergent.
8. Use any test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^2}$.
9. Find the radius of convergence and the interval of convergence of the series $\displaystyle\sum\frac{2}{n^2}x^n$.
10. Find the radius of convergence and the interval of convergence of the series $\displaystyle\sum\frac{(x-2)^n}{n!}$.
11. Find the first three nonzero terms of the Taylor series with center $a=\pi$ of the function $f(x)=\sin(x)$.
12. Complete the square to find the vertex, focus and directrix of the parabola $y^2-4\,y+2\,x+4=0$ and give a rough sketch of the graph.
13. Sketch the graph and state the foci and asymptotes of the hyperbola $\frac{(y-1)^2}{16}-\frac{(x+1)^2}{9}=1$.
14. Put into standard form and sketch the graph of $x^2+9y^2-4x-32=0$.
15. Convert $r=-4\sin\theta$ to rectangular coordinates and sketch its graph.
16. Sketch the graph of $r=2-\cos\theta$.
17. Find the total area inside both loops of the lemniscate $r^2=\cos(2\theta)$.
18. Set up the integral(s) needed that give the area inside the cardioid $r=1+\cos\theta$ but outside the circle $r=4\cos\theta$.
19. Eliminate the parameter from $x=2\sin^2 t$, $y=\cos(2t)$.
20. If $\vec{\mathbf{R}}(t)=t^3\vec{\mathbf{i}}+\cos(\pi t)\vec{\mathbf{j}}$ is the position of a moving point at time t, find its (a) position, (b) velocity, (c) acceleration and (d) speed at time $t=2$.

Answers to problems above: 1. $2\sqrt{\ln x}+C$ 2. $3\ln|x-3|+2\ln|x-2|+C$ 3. $\frac{x^2\ln x}{2}-\frac{x^2}{4}+C$ 4. 1 5. e^2 6. Absolutely convergent since $\left|(-1)^{n+1}\frac{\cos^2 n}{n^2}\right|<\frac{1}{n^2}$ and the p-series $\sum\frac{1}{n^2}$ converges, so by the comparison test $\sum\left|(-1)^{n+1}\frac{\cos^2 n}{n^2}\right|$ also converges. 7. Absolutely convergent since $\displaystyle\sum\left|(-1)^{n+1}\frac{2^n}{n!}\right|=\sum\frac{2^n}{n!}$ which converges by the ratio test as in an earlier problem. 8. Diverges by many tests: In the ratio and root, both give $L=2>1$ 9. $R=1$, $[-1,1]$ 10. $R=\infty$, $(-\infty,+\infty)$ 11. $-(x-\pi)+\frac{(x-\pi)^3}{6}-\frac{(x-\pi)^5}{120}+\cdots$ 12. $(y-2)^2=-2x$ vertex $(0,2)$, focus $(-\frac{1}{2},2)$, directrix $x=\frac{1}{2}$ 13. F: $(-1,1\pm 5)$, A: $y-1=\pm\frac{4}{3}(x+1)$ 14. $\frac{(x-2)^2}{36}+\frac{y^2}{4}=1$ 15. $x^2+(y+2)^2=2^2$ (circle) 16. 17. 1 18. $\int_{\arccos(\frac{1}{3})}^{\frac{\pi}{2}}(1+\cos\theta)^2-(4\cos\theta)^2d\theta+\int_{\frac{\pi}{2}}^{\pi}(1+\cos\theta)^2d\theta$ 19. $y=1-x$ 20. (a) $\vec{\mathbf{R}}(2)=8\vec{\mathbf{i}}+\vec{\mathbf{j}}$, (b) $\vec{\mathbf{v}}(2)=12\vec{\mathbf{i}}$, (c) $\vec{\mathbf{a}}(2)=12\vec{\mathbf{i}}+\pi^2\vec{\mathbf{j}}$, (d) $v(2)=12$