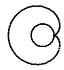
Review 8A

Math 222

1. Eliminate the parameter from $x = t^2$, $y = t^3$.

2. An epicycloid is generated by rolling a circle around the outside of another circle and following the path of a point. If a circle of radius b is rolled about a circle of radius a centered at the origin, then its path is given by $x = (a+b)\cos\theta - b\cos\left(\frac{a+b}{b}\theta\right)$ $y = (a+b)\sin\theta - b\sin\left(\frac{a+b}{b}\theta\right)$ where θ is the angle that the center of the outer circle makes with the positive x-axis. When $a = b$ this curve is called a *cardioid*. (a) Find the parametric equations of the cardioid where $a = b = 1$. (b) Find the derivative $\dfrac{dy}{dx}$. And (c) find the highest point on the cardioid (maximize y). 

3. Let $\vec{\mathbf{A}} = 4\vec{\mathbf{i}} - 8\vec{\mathbf{j}}$ and $\vec{\mathbf{B}} = 5\vec{\mathbf{i}} - 12\vec{\mathbf{j}}$. Find (a) $|\vec{\mathbf{B}}|$, (b) $2\vec{\mathbf{A}} - \vec{\mathbf{B}}$, (c) a unit vector in the same direction as $\vec{\mathbf{B}}$ and (d) a vector at right angles to $\vec{\mathbf{A}}$.

4. If $\vec{\mathbf{R}}(t) = \ln t\,\vec{\mathbf{i}} + t^2\vec{\mathbf{j}}$ is the position of a moving point at time t, find its (a) position, (b) velocity, (c) acceleration and (d) speed at time $t = 1$.

5. Find (a) the curvature of the curve $y = \sqrt{x}$ and (b) evaluate it at the point $(1, 1)$. (c) Locate the center of the osculating circle at this point

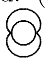
6. Find (a) the curvature of $\vec{\mathbf{R}} = \tan(t)\vec{\mathbf{i}} + \sec(t)\vec{\mathbf{j}}$ and (b) evaluate it where $t = \frac{\pi}{4}$.

7. Find the (a) position vector, (b) velocity vector, (c) acceleration vector, (d) speed and (e) tangential and (f) normal components of acceleration at $t = 1$ for the position vector $\vec{\mathbf{R}}(t) = t^2\vec{\mathbf{i}} + \ln t\vec{\mathbf{j}}$.

Answers to problems above: 1. $y^2 = x^3$ 2. (a) $x = 2\cos\theta - \cos(2\theta)$, $y = 2\sin\theta - \sin(2\theta)$ (b) $\frac{dy}{dx} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$ (c) maximum point is $(-\frac{1}{2}, \frac{3\sqrt{3}}{2})$ 3. (a) $|\vec{\mathbf{B}}| = 13$, (b) $2\vec{\mathbf{A}} - \vec{\mathbf{B}} = 3\vec{\mathbf{i}} - 4\vec{\mathbf{j}}$, (c) $\vec{\mathbf{u}}_B = \frac{5}{13}\vec{\mathbf{i}} - \frac{12}{13}\vec{\mathbf{j}}$, (d) $\vec{\mathbf{p}}_A = 8\vec{\mathbf{i}} + 4\vec{\mathbf{j}}$ or some multiple 4. (a) $\vec{\mathbf{R}}(1) = \vec{\mathbf{j}}$, (b) $\vec{\mathbf{v}}(1) = \vec{\mathbf{i}} + 2\vec{\mathbf{j}}$, (c) $\vec{\mathbf{a}}(1) = -1\vec{\mathbf{i}} + 2\vec{\mathbf{j}}$, (d) $v(1) = \sqrt{5}$ 5. (a) $k = -\frac{2}{(4x+1)^{\frac{3}{2}}}$, (b) $-\frac{2\sqrt{5}}{25}$, (c) $\left(6, \frac{3}{2}\right)$ 6. (a) $k = \frac{1}{(\sec^2(t) + \tan^2(t))^{\frac{3}{2}}}$, (b) $\frac{1}{3\sqrt{3}}$ 7. (a) $\vec{\mathbf{R}} = \vec{\mathbf{i}}$, (b) $\vec{\mathbf{v}} = 2\vec{\mathbf{i}} + \vec{\mathbf{j}}$, (c) $\vec{\mathbf{a}} = 2\vec{\mathbf{i}} - \vec{\mathbf{j}}$, (d) $v = \sqrt{5}$, (e) $a_t = \frac{3}{\sqrt{5}}$, (f) $a_n = \frac{4}{\sqrt{5}}$

Review 8B

Math 222

1. Eliminate the parameter from $x = 2\sin^2 t$, $y = \cos(2t)$.
2. An epicycloid is generated by rolling a circle around the outside of another circle and following the path of a point. If a circle of radius b is rolled about a circle of radius a centered at the origin, then its path is given by $x = (a+b)\cos\theta - b\cos\left(\frac{a+b}{b}\theta\right)$ $y = (a+b)\sin\theta - b\sin\left(\frac{a+b}{b}\theta\right)$ where θ is the angle that the center of the outer circle makes with the positive x-axis. When $a = 2b$ this curve is called a *nephroid*. (a) Find the parametric equations of the nephroid where $a = 2$ and $b = 1$. (b) Find the length of this nephroid. 
3. Let $\vec{\mathbf{A}} = 6\vec{\mathbf{i}} - 2\vec{\mathbf{j}}$ and $\vec{\mathbf{B}} = 2\vec{\mathbf{i}} - 3\vec{\mathbf{j}}$. Find (a) $|\vec{\mathbf{B}}|$, (b) $\vec{\mathbf{A}} - 2\vec{\mathbf{B}}$, (c) a unit vector in the same direction as $\vec{\mathbf{A}}$ and (d) a unit vector at right angles to $\vec{\mathbf{A}}$.
4. If $\vec{\mathbf{R}}(t) = t^3\vec{\mathbf{i}} + \cos(\pi t)\vec{\mathbf{j}}$ is the position of a moving point at time t, find its (a) position, (b) velocity, (c) acceleration and (d) speed at time $t = 2$.
5. Find (a) the curvature of the curve $y = \cos x$ and (b) evaluate it at the point $(0, 1)$. (c) Locate the center of the osculating circle at this point
6. Find (a) the curvature of $\vec{\mathbf{R}} = (t^2+1)\vec{\mathbf{i}} + (1+\cos t)\vec{\mathbf{j}}$ and (b) evaluate it where $t = \pi$.
7. Find the (a) position vector, (b) velocity vector, (c) acceleration vector, (d) speed and (e) tangential and (f) normal components of acceleration at $t = 1$ for the position vector $\vec{\mathbf{R}}(t) = t^5\vec{\mathbf{i}} + t^2\vec{\mathbf{j}}$.

Answers to problems above: 1. $y = 1 - x$ 2. (a) $x = 3\cos\theta - \cos(3\theta)$, $y = 3\sin\theta - \sin(3\theta)$ (b) $L = 4\int_0^{\frac{\pi}{2}} \sqrt{(3\sin\theta - \sin(3\theta))^2 + (3\cos\theta - \cos(3\theta))^2}\,d\theta = 24$ 3. (a) $|\vec{\mathbf{B}}| = \sqrt{13}$, (b) $\vec{\mathbf{A}} - 2\vec{\mathbf{B}} = 2\vec{\mathbf{i}} + 4\vec{\mathbf{j}}$, (c) $\vec{\mathbf{u}}_A = \frac{3}{\sqrt{10}}\vec{\mathbf{i}} - \frac{1}{\sqrt{10}}\vec{\mathbf{j}}$, (d) $\vec{\mathbf{p}}_A = \frac{1}{\sqrt{10}}\vec{\mathbf{i}} + \frac{3}{\sqrt{10}}\vec{\mathbf{j}}$ or $-\vec{\mathbf{p}}_A$ 4. (a) $\vec{\mathbf{R}}(2) = 8\vec{\mathbf{i}} + \vec{\mathbf{j}}$, (b) $\vec{\mathbf{v}}(2) = 12\vec{\mathbf{i}}$, (c) $\vec{\mathbf{a}}(2) = 12\vec{\mathbf{i}} + \pi^2\vec{\mathbf{j}}$, (d) $v(2) = 12$ 5. (a) $k = -\frac{\cos x}{(1+\sin^2 x)^{\frac{3}{2}}}$, (b) -1, (c) $(0,0)$ 6. (a) $k = \frac{2\sin t - 2t\cos t}{(\sin^2(t)+4t^2)^{\frac{3}{2}}}$, (b) $\frac{1}{4\pi}$ 7. (a) $\vec{\mathbf{R}} = \vec{\mathbf{i}} + \vec{\mathbf{j}}$, (b) $\vec{\mathbf{v}} = 5\vec{\mathbf{i}} + 2\vec{\mathbf{j}}$, (c) $\vec{\mathbf{a}} = 20\vec{\mathbf{i}} + 2\vec{\mathbf{j}}$, (d) $v = \sqrt{29}$, (e) $a_t = \frac{104}{\sqrt{29}}$, (f) $a_n = \frac{30}{\sqrt{29}}$