# Review 5A

Math 222

Know formula for $R_n(x)$. Bring a calculator

1. Find the radius of convergence and the interval of convergence of the series $\sum nx^n$.
2. Find the radius of convergence and the interval of convergence of the series $\sum \frac{(-1)^n(x+1)^n}{n3^n}$.
3. Starting with the geometric series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, derive a series for $\ln(1+x)$.
4. Find the first three nonzero terms of the Taylor series for $f(x) = x \csc x$.
5. Find the first four nonzero terms of the Taylor series for $f(x) = (1+x)^{\frac{13}{12}}$.
6. Find the first four nonzero terms of the Taylor series with center $a = 1$ of the function $f(x) = x^6$.
7. Using the Taylor series for $e^x = \sum \frac{1}{n!}x^n$, find the $n$ that guarantees that $1 + (-1) + \cdots + \frac{1}{n!}(-1)^n$ will approximate $e^{-1}$ to 5 decimals. Show how you get your answer.
8. Find the first four nonzero terms of the power series solution to the differential equation $y' = x^2 + y$ with $y(0) = 1$.
9. Use division to find the first three nonzero terms of the Taylor series for $\frac{\sin x}{1+\cos x}$.
10. Use substitution to find the first four nonzero terms of the Taylor series for $e^{\sin x}$.

Answers to problems above: 1. $R = 1$, $(-1, 1)$ 2. $R = 3$, $(-2, 4]$ 3. $\ln(1+x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1}$ 4. $1 + 0x + \frac{x^2}{6} + 0x^3 + \cdots$ 5. $1 + \frac{13\,x}{12} + \frac{13\,x^2}{288} - \frac{143\,x^3}{10368} + \cdots$ 6. $1 + 6\,(x-1) + 15\,(x-1)^2 + 20\,(x-1)^3 + \cdots$ 7. $|R_n(-1)| = \left|\frac{e^c}{(n+1)!}(-1)^{n+1}\right| = \frac{e^c}{(n+1)!} < \frac{\frac{1}{2}}{(n+1)!} \underset{\text{set}}{<} .00001$. We want $50000 < (n+1)!$. we have $8! < 50000 < 9!$. So $n = 8$ is the desired $n$. 8. $x + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{18}x^6 + \cdots$ 9. $\frac{x}{2} + \frac{x^3}{24} + \frac{x^5}{240} + \cdots$ 10. $1 + x + \frac{x^2}{2} - \frac{x^4}{8} \cdots$

# Review 5B

Math 222

Know series for $\dfrac{1}{1-x}$, $e^x$, $\sin x$, $\cos x$, $(1+x)^p$

1. Find the radius of convergence and the interval of convergence of the series $\sum \dfrac{2}{n^2} x^n$.
2. Find the radius of convergence and the interval of convergence of the series $\sum \dfrac{(x-2)^n}{n!}$.
3. Starting with the geometric series $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, derive a series for $\tan^{-1}(x)$.
4. Find the first four terms (zero and non-zero) of the Taylor series for $f(x) = \tan x$.
5. Find the first four nonzero terms of the Taylor series for $f(x) = \sqrt{1+x}$.
6. Find the first three nonzero terms of the Taylor series with center $a = \pi$ of the function $f(x) = \sin(x)$.
7. Using the Taylor series for $\cos x = \sum \frac{(-1)^n}{(2n)!} x^{2n}$, find the partial sum that guarantees that approximates $\cos(10^\circ)$ to 8 decimals. Show how you get your answer.
8. Find the first three nonzero terms of the power series solution to the differential equation $y' = x + xy$ with $y(0) = 0$.
9. Use division to find the first three nonzero terms of the Taylor series for $\dfrac{x}{\sin x}$.
10. Use multiplication to find the first three nonzero terms of the Taylor series for $e^x \cos x$.

Answers to problems above: 1. $R = 1$, $[-1, 1]$ 2. $R = \infty$, $(-\infty, +\infty)$ 3. $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ 4. $0 + x + +0x^2 + \frac{x^3}{3} + \cdots$ 5. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \cdots$ 6. $-(x-\pi) + \frac{(x-\pi)^3}{6} - \frac{(x-\pi)^5}{120} + \cdots$ 7. $10^\circ = \frac{\pi}{18}$. $\left|R_n\left(\frac{\pi}{18}\right)\right| = \left|\frac{f^{(n+1)}(c)}{(n+1)!}\left(\frac{\pi}{18}\right)^{n+1}\right| \le \frac{1}{(n+1)!}\left(\frac{\pi}{18}\right)^{n+1} \underset{\text{set}}{<} 10^{-8}$. We want $10^8 < (n+1)!\left(\frac{18}{\pi}\right)^{n+1}$. $n = 6$ is the first such integer, so $\cos(10^\circ) \approx 1 - \frac{1}{2}\left(\frac{\pi}{18}\right)^2 + \frac{1}{24}\left(\frac{\pi}{18}\right)^4 - \frac{1}{720}\left(\frac{\pi}{18}\right)^6$ to 8 decimals. 8. $\frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{48}x^6 + \cdots$ 9. $1 + \frac{x^2}{6} + \frac{7x^4}{360} + \cdots$ 10. $1 + x - \frac{x^3}{3} + \cdots$