Review 4A

Math 222

Give your reasons for convergence or divergence.

1. Does the sequence whose nth term is $\sqrt{n+1} - \sqrt{n}$ converge? If so, to what?
2. Does the series $\displaystyle\sum_{n=0}^{\infty}(-3)^{-n}$ converge? If so, what is its sum?
3. Over what interval of x-values does the geometric series $\frac{a}{x} + \frac{a}{x^2} + \frac{a}{x^3} + \cdots$ converge? And what is its sum?
4. Use the comparison test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n}$.
5. Use any test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+2n+1}{n^3+n^2-n+2}$.
6. Use the integral test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^2+6}$.
7. Use the ratio test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n!}$. (If it fails, use some other test.)
8. Use the root test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{2n-1}\right)^n$. (If it fails, use some other test.)
9. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{1}{\ln n}$ as absolutely convergent, conditionally convergent, or divergent.
10. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{n^n}{2^n}$ as absolutely convergent, conditionally convergent, or divergent.
11. Use any test to establish the convergence or divergence of the series $\displaystyle\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}$

Answers to problems above: 1. Yes, it converges to 0 2. Yes, it converges to $\frac{3}{4}$ 3. $(-\infty,-1)\cup(1,\infty)$, $S = \frac{a}{x-1}$ 4. Since $0 \le \frac{1}{n} \le \frac{\ln n}{n}$ and $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ diverges, so does $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ 5. Use LCT with $\sum\frac{1}{n}$. Since $\displaystyle\lim_{n\to\infty}\frac{\frac{n^2+2n+1}{n^3+n^2-n+2}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n^3+2n^2+n}{n^3+n^2-n+2} = 1$ and $\sum\frac{1}{n}$ diverges, so does $\sum_{n=1}^{\infty}\frac{n^2+2n+1}{n^3+n^2-n+2}$ 6. $\int_1^{\infty}\frac{x}{x^2+6}dx = \frac{1}{2}\ln(x^2+6)\Big|_1^{\infty} = \infty$, so $\sum_{n=1}^{\infty}\frac{n}{n^2+6}$ also diverges 7. $\displaystyle\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{(n+1)^2}{(n+1)!}}{\frac{n^2}{n!}} = \lim_{n\to\infty}\frac{(n+1)^2 n!}{n^2(n+1)!} = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^2\frac{1}{n+1} = 0 < 1$. So the series converges. 8. $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{n}{2n-1} = \frac{1}{2} < 1$. So the series converges. 9. Conditionally convergence since $\lim_{n\to 0}\frac{1}{\ln n} = 0$, so $\sum(-1)^{n+1}\frac{1}{\ln n}$ converges, but since $\frac{1}{\ln n} > \frac{1}{n}$, and $\sum\frac{1}{n}$ diverges, so does $\sum\frac{1}{\ln n}$. 10. Divergent since $\lim_{n\to 0}\frac{n^n}{2^n} = \infty \ne 0$, by the divergence test $\sum(-1)^{n+1}\frac{n^n}{2^n}$ diverges 11. Converges by the ratio test, since $L = \lim_{n\to\infty}\sqrt{\frac{n!}{(n+1)!}} = \lim_{n\to\infty}\sqrt{\frac{1}{(n+1)}} = 0 < 1$

Review 4B

Math 222

1. Does the sequence whose nth term is $\frac{\ln(n^3)}{n}$ converge? If so, to what?

2. Does the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{\sqrt{5}}\right)^n$ converge? If so, what is its sum?

3. Over what interval of x-values does the geometric series $\frac{1}{1+x}-\frac{x}{1+x}+\frac{x^2}{1+x}-\cdots$ converge? And what is its sum?

4. Use the comparison test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n2^n}$.

5. Use any test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^3+3}}{\sqrt{n^5+5}}$.

6. Use the integral test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{1/n}{1+(\ln n)^2}$.

7. Use the ratio test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{n2^n}{3^n}$. (If it fails, use some other test.)

8. Use the root test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{(\ln 2)^n}$. (If it fails, use some other test.)

9. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{\cos^2 n}{n^2}$ as absolutely convergent, conditionally convergent, or divergent.

10. Use any test to classify the series $\displaystyle\sum(-1)^{n+1}\frac{2^n}{n!}$ as absolutely convergent, conditionally convergent, or divergent.

11. Use any test to establish the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^2}$.

Answers to problems above: 1. Yes, it converges to 0 2. Yes, it converges to $\frac{2}{\sqrt{5}-2}$ 3. $(-1,1)$, $S=\frac{1}{(1+x)^2}$ 4. Since $0\le\frac{1}{n2^n}\le\frac{1}{2^n}$, and $\sum_{n=1}^{\infty}\frac{1}{2^n}$ converges since it is a geometric series $\sum r^2$ with $|r|=\frac{1}{2}<1$, $\sum_{n=1}^{\infty}\frac{1}{n2^n}$ also converges. 5. Use LCT with $\sum\frac{1}{n}$. Since $\displaystyle\lim_{n\to\infty}\frac{\frac{\sqrt{n^3+3}}{\sqrt{n^5+5}}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{\sqrt{n^5+3n^2}}{\sqrt{n^5+5}}=1$ and $\sum\frac{1}{n}$ diverges, so does $\sum_{n=1}^{\infty}\frac{\sqrt{n^3+3}}{\sqrt{n^5+5}}$ 6. $\int_1^{\infty}\frac{1/x}{1+(\ln x)^2}dx=\tan^{-1}(\ln x)\Big|_1^{\infty}=\frac{\pi}{2}$, so $\sum_{n=1}^{\infty}\frac{1/n}{1+(\ln n)^2}$ also converges 7. $\displaystyle\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)2^{n+1}}{3^{n+1}}}{\frac{n2^n}{3^n}}=\lim_{n\to\infty}\frac{(n+1)2^{n+1}3^n}{n2^n3^{n+1}}=\lim_{n\to\infty}\frac{n+1}{n}\frac{2}{3}=\frac{2}{3}<1$. So the series converges. 8. $\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\frac{\sqrt[n]{n}}{\ln 2}=\frac{1}{\ln 2}>1$. So the series diverges. 9. Absolutely convergent since $\left|(-1)^{n+1}\frac{\cos^2 n}{n^2}\right|<\frac{1}{n^2}$ and the p-series $\sum\frac{1}{n^2}$ converges, so by the comparison test $\sum\left|(-1)^{n+1}\frac{\cos^2 n}{n^2}\right|$ also converges. 10. Absolutely convergent since $\displaystyle\sum\left|(-1)^{n+1}\frac{2^n}{n!}\right|=\sum\frac{2^n}{n!}$ which converges by the ratio test as in an earlier problem. 11. Diverges by many tests: In the ratio and root, both give $L=2>1$