Review 1A

Math 222

1. Find a formula for $\tan(3x)$ in terms of $\tan x$.
2. For the function $y = \sin(x)\cos(x) + 1$:
   (a) Find the derivative y'.
   (b) List all critical points on $[0, 2\pi]$.
   (c) Make a sign chart for y' over the interval $[0, 2\pi]$.
   (d) List all relative extrema on $[0, 2\pi]$.
   (e) Noting that $y \geq 0$ on $[0, 2\pi]$, find the area under the function over $[0, 2\pi]$.
3. Simplify the derivatives of the following functions:
   (a) $y = \ln(\csc x)$
   (b) $y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$
   (c) $y = \sinh(x^2)$
4. Evaluate the following integrals:
   (a) $\displaystyle\int_0^{\frac{\pi}{4}} \sin^3(x)\cos(x)dx$
   (b) $\displaystyle\int \frac{dx}{4+x^2}$
   (c) $\displaystyle\int \frac{2dx}{\sqrt{9-4x^2}}$
   (d) $\displaystyle\int \cosh^3(x)\sinh(x)dx$
5. Write $x = 3\sqrt{3}\sin t + 3\cos t$ as $x = A\sin(at+b)$.
6. Solve $\dfrac{d^2x}{dt^2} + \dfrac{1}{2}x = 0$ if $x = 3$ when $t = 0$ and $\left.\dfrac{dx}{dt}\right|_{t=0} = 5$
7. Prove the identity $\cosh^2(x) - \sinh^2(x) = 1$.

Answers to problems above: 1. $\tan(3x) = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$ 2. (a) $y' = \cos^2(x) - \sin^2(x)$, (b) $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, $x = \frac{7\pi}{4}$, (d) relative maxima at $(\frac{\pi}{4}, \frac{3}{2})$ and $(\frac{5\pi}{4}, \frac{3}{2})$, relative minima at $(\frac{3\pi}{4}, \frac{1}{2})$ and $(\frac{7\pi}{4}, \frac{1}{2})$, (e) 2π 3. (a) $y' = \cot(x)$, (b) $y' = \frac{1}{1+x^2}$, (c) $y' = 2x\cosh(x^2)$ 4. (a) $\frac{1}{16}$, (b) $\frac{1}{2}\tan^{-1}(\frac{x}{2}) + C$, (c) $\sin^{-1}\left(\frac{2x}{3}\right) + C$, (d) $\frac{1}{4}\cosh^4(x) + C$ 5. $x = 6\sin(t + \frac{\pi}{6})$ 6. $x = 5\sqrt{2}\sin\left(\frac{t}{\sqrt{2}}\right) + 3\cos\left(\frac{t}{\sqrt{2}}\right)$ 7. $\cosh^2(x) - \sinh^2(x) = \frac{\left(e^x+e^{-x}\right)^2}{4} - \frac{\left(e^x-e^{-x}\right)^2}{4} = \frac{1}{4}\left[(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})\right] = \frac{2+2}{4} = 1$

Review 1B

Math 222

1. Find a formula for $\sin(4x)$ in terms of $\sin x$ and $\cos x$.
2. For the function $y = \sin(x) + \cos(x) + 2$:
   (a) Find the derivative y'.
   (b) List all critical points on $[0, 2\pi]$.
   (c) Make a sign chart for y' over the interval $[0, 2\pi]$.
   (d) List all relative extrema on $[0, 2\pi]$.
   (e) Noting that $y \geq 0$ on $[0, 2\pi]$, find the area under the function over $[0, 2\pi]$.
3. Simplify the derivatives of the following functions:
   (a) $y = e^{\tan(x)}$
   (b) $y = \sin^{-1}\left(\frac{1}{x}\right)$ (assume $x \geq 0$)
   (c) $y = \ln(\sinh(x))$
4. Evaluate the following integrals:
   (a) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)}{1+\tan(x)}dx$
   (b) $\displaystyle\int \frac{dx}{\sqrt{1-4x^2}}$
   (c) $\displaystyle\int \frac{dx}{25+4x^2}$
   (d) $\displaystyle\int \sinh(3x) + 2\sinh(3x)\cosh(3x)dx$
5. Write $x = \sin(3t) - \cos(3t)$ as $x = A\sin(at+b)$.
6. Solve $\dfrac{d^2x}{dt^2} + 64x = 0$ if $x = -2$ when $t = 0$ and $\left.\dfrac{dx}{dt}\right|_{t=0} = 2$
7. Prove the identity $\sinh(2x) = 2\sinh(x)\cosh(x)$.

Answers to problems above: 1. $\sin(4x) = 2\sin(x)\cos(x)\left(\cos^2(x) - \sin^2(x)\right)$ 2. (a) $y' = \cos(x) - \sin(x)$, (b) $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$, (d) relative maximum at $(\frac{\pi}{4}, \sqrt{2}+2)$ and relative minimum at $(\frac{5\pi}{4}, -\sqrt{2}+2)$, (e) 4π 3. (a) $y' = \sec^2(x)e^{\tan(x)}$, (b) $y' = \frac{-1}{x\sqrt{x^2-1}}$, (c) $y' = \coth(x)$ 4. (a) $\ln(2)$, (b) $\frac{1}{2}\sin^{-1}(2x) + C$, (c) $\frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right) + C$, (d) $\frac{1}{3}\left[\cosh(3x) + \sinh^2(3x)\right] + C$ 5. $x = \sqrt{2}\sin(3t - \frac{\pi}{4})$ 6. $x = \frac{1}{4}\sin(8t) - 2\cos(8t)$ 7. $2\sinh(x)\cosh(x) = 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right) = 2\frac{e^{2x} - e^{-2x}}{4} = \frac{e^{2x} - e^{-2x}}{2} = \sinh(2x)$