Final Review A

1. Find the equation of the circle with radius 3 and center $(2, 4)$.

2. Find the points on the line $y = 2$ whose distance from $(3, 5)$ is 6.

3. What is the equation of the line perpendicular to $2x + 3y = 2$ through the point $(2, -1)$?

4. A manufacturer of cockades has a fixed monthly cost of \$5000 and a production cost of \$1 for each cockade produced. If they are sold for \$14 each,

   (a) What is the cost function?

   (b) What is the revenue function?

   (c) What is the profit function?

   (d) What is the profit or loss corresponding to production levels of 100, 500 and 1000 cockades respectively

   (e) What is the break-even point.

5. The following data relates girls' length to age. State and solve the normal equations to find the least squares line.

Age (in months),x	3	6	9	18	30
Length (in cm),y	59	65	70	80	91

   Use the least squares line to determine the length at 24 months.

6. Solve the system $\begin{aligned} 3x + 2y &= 7 \\ 2x - y &= 0 \end{aligned}$.

7. Solve the system $\begin{aligned} 2x - 6y &= 2 \\ -3x + 9y &= -3 \end{aligned}$.

8. Combine $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 8 & 1 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

9. Find the inverse of $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & -1 & 0 \end{bmatrix}$.

10. Find the inverse of $\begin{bmatrix} 5 & 8 \\ 3 & 5 \end{bmatrix}$.

11. Use the inverse found in the last problem to solve $\begin{aligned} 5x + 8y &= 10 \\ 3x + 5y &= 8 \end{aligned}$.

12. Solve using the method of corners.

$$\begin{aligned} \text{Maximize} \quad & P = 3x - 4y \\ \text{subject to} \quad & x + y \le 4 \\ & y - x \le 2 \\ & y - x \ge -2 \\ & x \ge 0,\ y \ge 0 \end{aligned}$$

13. Solve using the Simplex method.

$$\begin{aligned} \text{Maximize} \quad & P = x + 2y \\ \text{subject to} \quad & 3x + y \le 24 \\ & x + y \le 14 \\ & x \ge 0,\ y \ge 0 \end{aligned}$$

14. What is the accumulated amount if \$5000 is invested at 4.5% compounded monthly over a period of 5 years?

15. Which has the higher effective interest rate: 5% compounded annually or 4% compounded daily? What are the effective rates?

16. If you deposit \$10 each month into a savings account with an interest rate of 5% compounded monthly, how much would be in the account after 40 years? How much would you have to put into the account now to have the same amount at the end of 40 years?

17. James and Mary take out a \$150,000 mortgage on their new house to be paid in monthly installments over a 30 year period. If they are charged 7% interest, what are their monthly payments?

18. From a box of 20 books, in how many ways may 10 be arranged on a bookshelf?

19. Out of a hat with 32 names, how many ways are there to draw out winners of a first, second and third prize (without replacing names).

20. Suppose a survey of 500 people showed that 210 had type O blood, 215 had type A, 55 had type B and 20 had type AB. Write a probability distribution for this frequency distribution.

21. Using the distribution from the last problem, answer the question: "What is the probability that a person surveyed does not have type O blood?"

22. A College survey showed that 63% of freshmen took english composition, 15% took calculus and 25% took neither class. What is the probability that a former freshman chosen at random took both English composition and calculus?

23. If 8 electronic games from each case of 24 are randomly selected and tested before shipping and the case is not shipped if any defective games are found, what is the probability that a case with 2 defective games is shipped.

24. Suppose that 70% of drivers are classed as being "careful" whereas 30% of the drivers are classed as being "reckless". Suppose further that a careful driver has a 0.05 probability of being in an accident in a given year, while for a reckless driver the probability is 0.2. What is the probability that a randomly selected driver will have an accident within a year?

25. In the last problem what is the probability that a driver involved in an accident is a reckless driver.

26. The number of freighters entering A Harbor between between 8am and 2pm on Fridays is given by the following probability distribution.

x	$P(X=x)$
0	0.10
1	0.20
2	0.40
3	0.15
4	0.10
5	0.05

Find the expected number of freighters entering the harbor during these hours. What is the variance and standard deviation of this distribution?

27. Two coins are tossed and the toss is judged a winning toss only if no tails occur. This is repeated five times.

(a) Find the probability of obtain exactly $0, 1, 2, 3, 4, 5$ winning tosses.

(b) Construct the binomial distribution and draw the histogram associated with this.

(c) Compute the mean and the standard deviation of the random variable associated with this.

28. Suppose weights of men are approximately normally distributed with a mean of 163 pounds and and standard deviation of 28 pounds. If the minimum and maximum weights in order to be a volunteer fireman are 128 and 254, what proportion of the men meet this qualification?

29. Suppose a coin is flipped 100 times. What is the probability of obtaining 65 or more heads?

30. Find the steady-state vector for the transition matrix $\begin{bmatrix} .2 & .8 & .3 \\ .5 & .1 & .3 \\ .3 & .1 & .4 \end{bmatrix}$.

31. Rewrite the absorbing stochastic matrix $\begin{bmatrix} .6 & 0 & 0 & .2 \\ .1 & 1 & 0 & .2 \\ .1 & 0 & 1 & .2 \\ .2 & 0 & 0 & .4 \end{bmatrix}$ so the absorbing states appear first and partition it as $\left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$. Then compute the steady-state transition matrix.

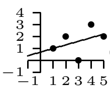
32. Consider the two-person game with payoff matrix $\begin{bmatrix} -3 & 1 & -4 \\ -2 & 2 & -1 \end{bmatrix}$.

(a) Find the saddle points of the game.

(b) Find the optimal strategy for each player.

(c) What is the value of the game and who does the game favor?

33. Find the optimal mixed strategies for each player in the two-player game whose payoff matrix is $\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$. What is the value of the game and who does the game favor?

Answers (Not Solutions) to the problems on the other side: 1. $(x-20)^2+(y+2)^2=144$; 2. $2\sqrt{37}$; 3. $y=-2x-3$; 4.(a) $C(x)=8x+500$, (b) $R(x)=15x$, (c) $P(x)=7x-500$, (d) loss of \$430, loss of \$150, profit of \$200 (e) about 71; 5. $5b+15m=8$, $15b+55m=27$, $y=.3x+.7$  6. $(16,7)$; 7. No Solution; 8. $\begin{bmatrix} 3 & -1 \\ 2 & 9 \\ -9 & 1 \end{bmatrix}$; 9. $\begin{bmatrix} -\frac{1}{7} & 0 & \frac{2}{7} \\ 0 & 1 & 0 \\ \frac{4}{7} & 0 & -\frac{1}{7} \end{bmatrix}$; 10. $\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$; 11. $(-293, 195)$. 12. $(4,0)$ produces a maximum of $P=12$; 13. $x=0$, $y=\frac{15}{2}$, $z=0$ produces a maximum of $P=\frac{135}{2}$; 14. \$1643.62; 15. 5.64% and 5.13%; 16. \$17,308.48, \$8612.62; 17. \$577.75; 18. $C(5,3)=10$; 19. $C(15,5)=3003$. 20.

Distance	$d<1$	$1<d<5$	$5<d<20$	$20<d$
Probability	.2857	.4615	.1978	.0549

21. .7473; 22. 1.8%; 23. $\frac{52}{59}\approx .8814$; 24. 75%; 25. $25\frac{1}{3}$%. 26. $E(X)=4.32$, $\text{Var}(X)=4.3408$, $\sigma=2.0835$; 27.(a) $P(X=0)=\frac{20736}{28561}$, $P(X=1)=\frac{6912}{28561}$, $P(X=2)=\frac{864}{28561}$, $P(X=3)=\frac{48}{28561}$, $P(X=4)=\frac{1}{28561}$,

(b)

x	0	1	2	3	4
$P(X=x)$	.7260	.2420	.0303	.0117	.0000

, (c) $\mu=0.3077$, $\sigma=.5329$; 28. .3085; 29. 18.67%. 30. $X=\begin{bmatrix} .5 \\ .5 \end{bmatrix}$; 31. $\left[\begin{array}{cc|cc} 1 & 0 & .3 & .1 \\ 0 & 1 & .1 & .2 \\ \hline 0 & 0 & .6 & .2 \\ 0 & 0 & 0 & .5 \end{array}\right]$, $L=\begin{bmatrix} 1 & 0 & .75 & .5 \\ 0 & 1 & .25 & .5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; 32.(a) $a_{32}=1$ is a saddle point, (b) R should play row 3, C should play column 2, (c) The value of the game is 1 and favors R; 33. R should play $P=\begin{bmatrix} \frac{3}{5} & \frac{2}{5} \end{bmatrix}$, C should play $Q=\begin{bmatrix} 13/20 \\ 7/20 \end{bmatrix}$, $E=\frac{1}{5}$ and the game favors R.

Final Review B

1. Find the equation of the circle with radius 12 and center $(20, -2)$.

2. Find the distance from $(-3, 5)$ to $(-1, -7)$.

3. What is the equation of the line parallel to $2x + y = 2$ through the point $(-2, 1)$?

4. A manufacturer of roulettes has a fixed monthly cost of \$500 and a production cost of \$8 for each roulette produced. If they are sold for \$15 each,

   (a) What is the cost function?

   (b) What is the revenue function?

   (c) What is the profit function?

   (d) What is the profit or loss corresponding to production levels of 10, 50 and 100 roulettes respectively

   (e) What is the break-even point.

5. State and solve the normal equations to find the least squares line for the following points.

x	1	2	3	4	5
y	1	2	0	3	2

   Graph the points and least squares line together.

6. Solve the system $\begin{aligned} x + y &= 23 \\ -2x + 3y &= -11 \end{aligned}$.

7. Solve the system $\begin{aligned} -4x + 2y &= 1 \\ 6x - 3y &= 0 \end{aligned}$.

8. Combine $\begin{bmatrix} 3 & 2 \\ -4 & 0 \\ -3 & 1 \end{bmatrix} - 3 \begin{bmatrix} 0 & 1 \\ -2 & -3 \\ 2 & 0 \end{bmatrix}$.

9. Find the inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$.

10. Find the inverse of $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$.

11. Use the inverse found in the last problem to solve $\begin{aligned} 2x + 3y &= 1 \\ 5x + 7y &= 100 \end{aligned}$.

12. Solve using the method of corners.

$$\begin{aligned} \text{Maximize} \quad & P = 3x + 2y \\ \text{subject to} \quad & y - x \leq 1 \\ & 3x + 4y \leq 12 \\ & x \geq 0,\ y \geq 0 \end{aligned}$$

13. Solve using the Simplex method.

$$\begin{aligned} \text{Maximize} \quad & P = 8x + 9y + 7z \\ \text{subject to} \quad & 3x + y + 4z \leq 12 \\ & 6x + 2y + 5z \leq 15 \\ & x \geq 0,\ y \geq 0,\ z \geq 0 \end{aligned}$$

14. What is the accumulated amount if \$1000 is invested at 5% compounded quarterly over a period of 10 years?

15. Which has the higher effective interest rate: 5.5% compounded monthly or 5% compounded daily? What are the effective rates?

16. If you deposit \$100 each month into a savings account with an interest rate of 7% compounded monthly, how much would be in the account after 10 years? How much would you have to put into the account now to have the same amount at the end of 10 years?

17. Suppose you want to accumulate \$100,000 over the next 10 years. How much must you put into your savings account, which pays 7% intrest compounded monthly, each month?

18. How many 3-element subsets are there of $\{a, b, c, d, e\}$?

19. How many ways are there to select a committee of 5 people from a pool of 15 people?

20. Suppose a survey of divers stopped along a street revealed that 26 had a destination that was less than a mile from their starting point, 26 had a destination between 1 and 5 miles from their starting point, 18 had a destination between 5 and 20 miles from their starting point and 5 had a destination more that 20 miles from their starting point. Write a probability distribution associated with this data.

21. Using the distribution from the last problem, answer the question: "What is the probability that a driver surveyed was traveling less than 5 miles?"

22. In a survey taken by 1000 students, 869 reported that their father was employed outside the home, 47 reported that their father was employed in the home, 59 reported that their father was not employed, and 43 reported having no father. What proportion of fathers were reported as working both at home and outside the home?

23. If 3 oranges are chosen out of a box of 60 oranges, where half have symptoms of *sour rot*. What is the probability that at least one orange will not have symptoms of sour rot?

24. Suppose for an elective next semester you will either take a basket weaving or a philosophy course depending on what your adviser decides. You estimate a probability of getting an A in basket weaving as 0.95, while in philosophy it is 0.70. However that chances of your adviser choosing basket weaving is only 20% while there is an 80% chance of being advised into the philosophy course. What is your probability of ending up with an A?

25. Suppose the person in the last problem got an A in the course. What is the probability that they took basket weaving?

26. An elementary school library finds that the number of books checked out each week by a fifth grade student has the following probability distribution.

x	$P(X=x)$
0	0.13
1	0.05
2	0.15
3	0.07
4	0.22
5	0.28
6	0.10

Find the expected number of books a randomly picked fifth grader will check out each week. What is the variance and standard deviation of this distribution?

27. A card is randomly selected from a deck of cards, examined to see if it is a ace and then returned to the deck. This is done four times.

    (a) Find the probability of drawing exactly $0, 1, 2, 3, 4$ spades.

    (b) Construct the binomial distribution and draw the histogram associated with this.

    (c) What is the expected number of spades for four draws? Compute the standard deviation of the random variable associated with this.

28. Suppose diameters of pinto bean are normally distributed with a mean of $\mu = 7.5$ mm and have a standard deviation of $\sigma = .4$ mm. What is the probability of a bean having a diameter of 8.5 mm or larger?

29. Suppose a die is thrown 100 times. What is the probability of obtaining 20 or more ones?

30. Find the steady-state vector for the transition matrix $\begin{bmatrix} .8 & .2 \\ .2 & .4 \end{bmatrix}$.

31. Rewrite the absorbing stochastic matrix $\begin{bmatrix} .6 & 0 & 0 & .2 \\ .3 & 1 & 0 & .1 \\ .1 & 0 & 1 & .2 \\ 0 & 0 & 0 & .5 \end{bmatrix}$ so the absorbing states appear first and partition it as $\left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$. Then compute the steady-state transition matrix.

32. Consider the two-person game with payoff matrix $\begin{bmatrix} 1 & -5 & -1 \\ 2 & 0 & -2 \\ 3 & 1 & 2 \end{bmatrix}$.

    (a) Find the saddle points of the game.

    (b) Find the optimal strategy for each player.

    (c) What is the value of the game and who does the game favor?

33. Find the optimal mixed strategies for each player in the two-player game whose payoff matrix is $\begin{bmatrix} 3 & -5 \\ -4 & 8 \end{bmatrix}$. What is the value of the game and who does the game favor?

Answers (Not Solutions) to the problems on the other side: 1. $(x-2)^2 + (y-4)^2 = 9$; 2. $(3 \pm 3\sqrt{3}, 2)$; 3. $y = \frac{3}{2}x - 4$; 4.(a) $C(x) = x + 5000$, (b) $R(x) = 14x$, (c) $P(x) = 13x - 5000$, (d) loss of \$3700, profit of \$1500, profit of \$8000, (e) about 385; 5. $5b + 66m = 365$, $66b + 1350m = 5367$, $y = 1.147x + 57.865$, 85.38cm; 6. $(1, 2)$; 7. $(3y+1, y)$; 8. $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 12 & 7 \end{bmatrix}$; 9. $\begin{bmatrix} 3 & -1 & -1 \\ 0 & 0 & -1 \\ -2 & 1 & 1 \end{bmatrix}$; 10. $\begin{bmatrix} 5 & -8 \\ -3 & 5 \end{bmatrix}$; 11. $(-14, 10)$. 12. $(2, 0)$ produces a maximum of $P = 6$; 13. $x = 0$, $y = 14$ produces a maximum of $P = 28$; 14. \$6258.98; 15. 5% and 4.08%; 16. \$15,260.20, \$2073.84; 17. \$997.95; 18. $P(20, 10) = \frac{20!}{10!} = 670,442,572,800$; 19. 29,760. 20.

Type	O	A	B	AB
Probability	.42	.43	.11	.04

21. .58; 22. .03; 23. $\frac{10}{23} \approx .4348$; 24. 9.5%; 25. $\frac{12}{19} \approx .6316$. 26. $E(X) = 2.1$, $\text{Var}(X) = 1.59$, $\sigma = 1.26$; 27.(a) $P(X=0) = \frac{243}{1024}$, $P(X=1) = \frac{405}{1024}$, $P(X=2) = \frac{135}{512}$, $P(X=3) = \frac{45}{512}$, $P(X=4) = \frac{15}{1024}$, $P(X=0) = \frac{1}{1024}$, (b)

x	0	1	2	3	4	5
$P(X=x)$	.2373	.3955	.2637	.0879	.0146	.0010

, (c) $\mu = 1.25$, $\sigma = .9682$; 28. 89.38%; 29. .0013; 30. $X = \begin{bmatrix} 51/122 \\ 39/122 \\ 16/61 \end{bmatrix}$; 31. $\left[\begin{array}{cc|cc} 1 & 0 & .1 & .2 \\ 0 & 1 & .1 & .2 \\ \hline 0 & 0 & .6 & .2 \\ 0 & 0 & .2 & .4 \end{array}\right]$, $L = \begin{bmatrix} 1 & 0 & .5 & .5 \\ 0 & 1 & .5 & .5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; 32.(a) $a_{21} = -2$ is a saddle point, (b) R should play row 2, C should play column 1, (c) The value of the game is -2 and favors C; 33. R should play $P = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}$, C should play $Q = \begin{bmatrix} 5/8 \\ 3/8 \end{bmatrix}$, $E = \frac{1}{8}$ and the game favors R.