Review 7A

1. Wolves either live in packs or wander alone. In any year 5% of those living in packs will leave to wander and 50% of those wandering will find a pack to accept them.
   (a) Write a transition matrix to represent this.
   (b) Assuming that all of the wolves started in packs, what percent would be living in packs in 1 year? in 2 years?
   (c) What is the steady-state distribution vector for the percentages of wolves in packs and wandering alone?

2. Find the steady-state vector for the transition matrix $\begin{bmatrix} .2 & .8 & .3 \\ .5 & .1 & .3 \\ .3 & .1 & .4 \end{bmatrix}$.

3. Rewrite the absorbing stochastic matrix $\begin{bmatrix} .6 & 0 & 0 & .2 \\ .1 & 1 & 0 & .2 \\ .1 & 0 & 1 & .2 \\ .2 & 0 & 0 & .4 \end{bmatrix}$ so the absorbing states appear first and partition it as $\left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$.
   Then compute the steady-state transition matrix.

4. Check one or more of the boxes that describe each matrix:
   (a) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} .1 & .9 \\ .3 & .7 \end{bmatrix}$
   (b) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} .4 & .1 & .3 \\ .2 & .8 & .4 \\ .4 & .1 & .3 \end{bmatrix}$
   (c) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} .5 & 0 & .3 \\ .2 & 1 & .2 \\ .3 & 0 & .4 \end{bmatrix}$
   (d) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

5. Determine the maximin and minimax strategies for the two-person zero-sum game whose payoff matrix is $\begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 3 & -3 \end{bmatrix}$.

6. Consider the two-person game with payoff matrix $\begin{bmatrix} -3 & 1 & -4 \\ -2 & 2 & -1 \end{bmatrix}$.
   (a) Find the saddle points of the game.
   (b) Find the optimal strategy for each player.
   (c) What is the value of the game and who does the game favor?

7. Find the expected payoff of the game with payoff matrix $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & -2 & -3 \end{bmatrix}$ when R plays the mixed strategy $P = \begin{bmatrix} .7 & .2 & .1 \end{bmatrix}$ and C plays the mixed strategy $Q = \begin{bmatrix} 0 \\ .2 \\ .8 \end{bmatrix}$.

8. Find the optimal mixed strategies for each player in the two-player game whose payoff matrix is $\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$. What is the value of the game and who does the game favor?

Answers to problems on other side: 1. $T = \begin{bmatrix} .75 & .4 & .15 \\ .05 & .3 & .35 \\ .2 & .3 & .5 \end{bmatrix}$, 52.5% does well, 18.5% does average, 29% does poor on the 2nd exam, 51.125% does well, 18.325% does average, 30.55% does poor on the 3rd exam, $X = \begin{bmatrix} 49/99 \\ 19/99 \\ 31/99 \end{bmatrix}$; 2. $X = \begin{bmatrix} .5 \\ .5 \end{bmatrix}$; 3. $\left[\begin{array}{cc|cc} 1 & 0 & .3 & .1 \\ 0 & 1 & .1 & .2 \\ \hline 0 & 0 & .6 & .2 \\ 0 & 0 & 0 & .5 \end{array}\right]$, $L = \begin{bmatrix} 1 & 0 & .75 & .5 \\ 0 & 1 & .25 & .5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; 4.(a) SR, (b) None, (c) None, (d) None; 5. maximin is row 1, minimax is column 3; 6.(a) $a_{32} = 1$ is a saddle point, (b) R should play row 3, C should play column 2, (c) The value of the game is 1 and favors R; 7. $E = -.65$; R should play $P = \begin{bmatrix} \frac{3}{5} & \frac{2}{5} \end{bmatrix}$, C should play $Q = \begin{bmatrix} 13/20 \\ 7/20 \end{bmatrix}$, $E = \frac{1}{5}$ and the game favors R.

Review 7B

1. A teacher ranked her student's test results into three general categories: They did *well* or they did *average* or they did *poor*. She found that of the students who did well on a particular exam, 75% continued to do well on the next exam, while 20% did poorly on the next exam. Of those whose performed average on an exam, 40% did well on the next exam, while 30% continued to do average. Of those who scored poorly on an exam, 15% scored well on the next exam, but 50% continued to do poorly on the next exam.
   (a) Write a transition matrix to represent this.
   (b) Assuming that on the first exam 50% did well and 30% did average, what would the teacher expect for the second and third exams?
   (c) What is the steady-state distribution vector in this situation?

2. Find the steady-state vector for the transition matrix $\begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$.

3. Rewrite the absorbing stochastic matrix $\begin{bmatrix} .6 & 0 & 0 & .2 \\ .3 & 1 & 0 & .1 \\ .1 & 0 & 1 & .2 \\ 0 & 0 & 0 & .5 \end{bmatrix}$ so the absorbing states appear first and partition it as $\left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$. Then compute the steady-state transition matrix.

4. Check one or more of the boxes that describe each matrix:
   (a) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} .1 & .3 \\ .9 & .7 \end{bmatrix}$
   (b) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} .4 & .5 & .1 \\ .1 & .3 & .1 \\ .4 & .2 & .8 \end{bmatrix}$
   (c) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} 0 & .5 & .3 \\ 0 & .5 & .4 \\ 1 & 0 & 0 \end{bmatrix}$
   (d) ☐ stochastic ☐ regular stochastic ☐ absorbing stochastic: $\begin{bmatrix} 1.3 & .5 \\ -.3 & .5 \end{bmatrix}$

5. Determine the maximin and minimax strategies for the two-person zero-sum game whose payoff matrix is $\begin{bmatrix} 5 & 10 & -1 \\ -12 & -2 & 2 \end{bmatrix}$.

6. Consider the two-person game with payoff matrix $\begin{bmatrix} 1 & -5 & -1 \\ 2 & 0 & -2 \\ 3 & 1 & 2 \end{bmatrix}$.
   (a) Find the saddle points of the game.
   (b) Find the optimal strategy for each player.
   (c) What is the value of the game and who does the game favor?

7. Find the expected payoff of the game with payoff matrix $\begin{bmatrix} 5 & 2 & -3 \\ -2 & 0 & 1 \\ 5 & -1 & 0 \end{bmatrix}$ when R plays the mixed strategy $P = \begin{bmatrix} .5 & 0 & .5 \end{bmatrix}$ and C plays the mixed strategy $Q = \begin{bmatrix} .1 \\ .1 \\ .8 \end{bmatrix}$.

8. Find the optimal mixed strategies for each player in the two-player game whose payoff matrix is $\begin{bmatrix} 3 & -5 \\ -4 & 8 \end{bmatrix}$. What is the value of the game and who does the game favor?

Answers to problems on other side: 1. $T = \begin{bmatrix} .95 & .5 \\ .05 & .5 \end{bmatrix}$, $X_1 = \begin{bmatrix} .95 \\ .05 \end{bmatrix}$, $X_2 = \begin{bmatrix} .9275 \\ .0725 \end{bmatrix}$, $X = \begin{bmatrix} 10/11 \\ 1/11 \end{bmatrix}$; 2. $X = \begin{bmatrix} 51/122 \\ 39/122 \\ 16/61 \end{bmatrix}$; 3. $\left[\begin{array}{cc|cc} 1 & 0 & .1 & .2 \\ 0 & 1 & .1 & .2 \\ \hline 0 & 0 & .6 & .2 \\ 0 & 0 & .2 & .4 \end{array}\right]$, $L = \begin{bmatrix} 1 & 0 & .5 & .5 \\ 0 & 1 & .5 & .5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; 4.(a) None, (b) SR, (c) SA, (d) S; 5. maximin is row 2, minimax is column 2; 6.(a) $a_{21} = -2$ is a saddle point, (b) R should play row 2, C should play column 1, (c) The value of the game is -2 and favors C; 7. $E = .24$; R should play $P = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}$, C should play $Q = \begin{bmatrix} 5/8 \\ 3/8 \end{bmatrix}$, $E = \frac{1}{8}$ and the game favors R.