

Review 2A

Math 113

The learning center is open for help.

1. Verify the identity $1 + \sin^2 x \sec^2 x = \sec^2 x$.
2. Verify the identity $\frac{\sin x}{1 - \sin x} + \frac{\cos x}{\sin x - 1} = \frac{1 - \cot x}{\csc x - 1}$
3. Find an exact value for $\tan 15^\circ$.
4. Given $\tan \alpha = \frac{1}{4}$, α in QI and $\sin \beta = \frac{2}{3}$, β in QII. Find the exact value for $\cos(\alpha + \beta)$.
5. Given $\cos \alpha = \frac{7}{9}$ with $0 < \alpha < 90^\circ$, find $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$.
6. Given $\tan \alpha = -3$ with $0^\circ < \alpha < -90^\circ$, find $\sin \frac{\alpha}{2}$, $\cos \frac{\alpha}{2}$ and $\tan \frac{\alpha}{2}$.
7. Verify the identity $\frac{2 \cos 2x}{\sin 2x} = \cot x - \tan x$.
8. Write $2 \cos 3x \cos 2x$ is the sum of two functions.
9. Write $y = 2 \sin x + 2 \cos x$ as $y = k \sin(x + \alpha)$.
10. Find the exact value of $\sin^{-1} \left(\sin \left(\frac{5\pi}{3} \right) \right)$.
11. Find the exact value of $\tan \left(\sin^{-1} \left(\frac{3}{4} \right) \right)$.
12. Find all exact solutions to the equation $2 \sin^3 x = \sin x$ in the interval $0 \leq x < 2\pi$.
13. Find all solutions to the equation $2 \sec x - 2 \cos x = 3$ in the interval $0 \leq x < 360^\circ$. (approximate to one decimal, if needed)

Answers to problems above: 1. $1 + \sin^2 x \sec^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \sec^2 x$ 2. $\frac{\sin x}{1 - \sin x} + \frac{\cos x}{\sin x - 1} = \frac{\sin x}{1 - \sin x} - \frac{\cos x}{1 - \sin x} = \frac{\sin x - \cos x}{1 - \sin x} = \frac{(\sin x - \cos x)/\sin x}{(1 - \sin x)/\sin x} = \frac{1 - \cot x}{\csc x - 1}$ 3. $2 - \sqrt{3}$ 4. $\frac{4\sqrt{5}-2}{3\sqrt{15}}$ 5. $\sin 2\alpha = \frac{56\sqrt{2}}{81}$, $\cos 2\alpha = \frac{17}{81}$, $\tan 2\alpha = \frac{56\sqrt{2}}{17}$ 6. $\sin \frac{\alpha}{2} = \sqrt{\frac{\sqrt{5}-1}{2\sqrt{5}}}$, $\cos \frac{\alpha}{2} = \sqrt{\frac{\sqrt{5}+1}{2\sqrt{5}}}$, $\tan \frac{\alpha}{2} = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ 7. $\frac{2 \cos 2x}{\sin 2x} = \frac{2}{\tan 2x} = \frac{2(1 - \tan^2 x)}{2 \tan x} = \frac{1 - \tan^2 x}{\tan x} = \frac{1}{\tan x} - \frac{\tan^2 x}{\tan x} = \cot x - \tan x$ 8. $\cos 5x + \cos x$ 9. $y = 2\sqrt{2} \sin \left(x + \frac{\pi}{4} \right)$ 10. $-\frac{\pi}{3}$ 11. $\frac{3\sqrt{7}}{7}$ 12. $x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$ 13. $x = 60^\circ, 300^\circ$

Review 2B

Math 113

1. Verify the identity $\sec x \tan^2 x + \sec x = \sec^3 x$.
2. Verify the identity $\frac{2 \cot x}{\cot x + \tan x} = 2 \cos^2 x$
3. Find an exact value for $\sin \frac{7\pi}{12}$.
4. Given $\cos \alpha = -\frac{1}{3}$, α in QII and $\tan \beta = 5$, β in QIII. Find the exact value for $\sin(\beta - \alpha)$.
5. Given $\tan \alpha = 5$ with $180^\circ < \alpha < 270^\circ$, find $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$.
6. Given $\sin \alpha = -\frac{1}{3}$ with $180^\circ < \alpha < 270^\circ$, find $\sin \frac{\alpha}{2}$, $\cos \frac{\alpha}{2}$ and $\tan \frac{\alpha}{2}$.
7. Verify the identity $1 - \tan^2 x = \frac{\cos 2x}{\cos^2 x}$.
8. Write $\sin 5x \cos x$ as the sum of two functions.
9. Write $y = 3 \sin x + 3\sqrt{3} \cos x$ as $y = k \sin(x + \alpha)$.
10. Find the exact value of $\sin^{-1}(\cos(\frac{\pi}{6}))$.
11. Find the exact value of $\csc(\tan^{-1}(2))$.
12. Find all exact solutions to the equation $3 \tan^2 x = 1$ in the interval $0 \leq x < 2\pi$.
13. Find all solutions to the equation $4 \csc x + 9 = 0$ in the interval $0 \leq x < 360^\circ$. (approximate to one decimal, if needed)

Answers to problems above: 1. $\sec x \tan^2 x + \sec x = \sec x (\tan^2 x + 1) = \sec x (\sec^2 x) = \sec^3 x$ 2. $\frac{2 \cot x}{\cot x + \tan x} = \frac{2 \frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} = \frac{(2 \frac{\cos x}{\sin x}) \sin x \cos x}{(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}) \sin x \cos x} = \frac{2 \cos^2 x}{\cos^2 x + \sin^2 x} = 2 \cos^2 x$ 3. $\frac{\sqrt{6}-\sqrt{2}}{4}$ 4. $\frac{5+2\sqrt{2}}{3\sqrt{26}}$ 5. $\sin 2\alpha = \frac{5}{13}$, $\cos 2\alpha = -\frac{1}{26}$, $\tan 2\alpha = -10$ 6. $\sin \frac{\alpha}{2} = \sqrt{\frac{3-2\sqrt{2}}{6}}$, $\cos \frac{\alpha}{2} = -\sqrt{\frac{3+2\sqrt{2}}{6}}$, $\tan \frac{\alpha}{2} = -\sqrt{\frac{3-2\sqrt{2}}{3+2\sqrt{2}}}$ 7. $\frac{\cos 2x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = 1 - \tan^2 x$ 8. $\frac{1}{2}(\sin 6x + \sin 4x)$ 9. $y = 6 \sin(x + \frac{\pi}{3})$

10. $\frac{\pi}{3}$ 11. $\frac{\sqrt{5}}{2}$ 12. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ 13. $x \approx 206.4^\circ, 333.6^\circ$